Four-quark structure of the $Z_c(3900)$, $Z(4430)$, and $X_b(5568)$ states

Fabian Goerke,[1] Thomas Gutsche,[1] Mikhail A. Ivanov,[2] Jürgen G. Körner,[3]
Valery E. Lyubovitskij,[1,4,5] and Pietro Santorelli[6,7]

[1]*Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

[2]*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia*

[3]*PRISMA Cluster of Excellence, Institut für Physik, Thomas GutscheJohannes Gutenberg-Universität, D-55099 Mainz, Germany*

[4]*Department of Physics, Tomsk State University, 634050 Tomsk, Russia*

[5]*Laboratory of Particle Physics, Mathematical Physics Department, Tomsk Polytechnic University, Lenin Avenue 30, 634050 Tomsk, Russia*

[6]*Dipartimento di Fisica, Università di Napoli Federico II, Complesso Universitario di Monte Sant'Angelo, Via Cintia, Edificio 6, 80126 Napoli, Italy*

[7]*Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, 80126 Napoli, Italy*

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We examine the four-quark structure of the recently discovered charged $Z_c(3900)$, $Z(4430)$, and $X_b(5568)$ states. We calculate the widths of the strong decays $Z_c^+ \to J/\psi\pi^+$ ($\eta_c\rho^+$, $\bar{D}^0 D^{*+}$, $\bar{D}^{*0}D^+$), $Z(4430)^+ \to J/\psi\pi^+$ ($\psi(2s)\pi^+$), and $X_b^+ \to B_s\pi^+$ within a covariant quark model previously developed by us. We find that the tetraquark-type current widely used in the literature for the $Z_c(3900)$ leads to a significant suppression of the $\bar{D}D^*$ and $\bar{D}^*D$ modes. Contrary to this a molecular-type current provides an enhancement by a factor of 6–7 for the $\bar{D}D^*$ modes compared with the $Z_c^+ \to J/\psi\pi^+$, $\eta_c\rho^+$ modes in agreement with recent experimental data from the BESIII Collaboration. In the case of the $Z(4430)$ state we test a sensitivity of the ratio R_Z of the $Z(4430)^+ \to \psi(2s)\pi^+$ and $Z(4430)^+ \to J/\psi\pi^+$ decay rates to a choice of the size parameter $\Lambda_{Z(4430)}$ of the $Z(4430)$. Using the upper constraint for the sum of these two modes deduced from the LHCb Collaboration data we find that R_Z varies from 4.64 to 4.08 when $\Lambda_{Z(4430)}$ changes from 2.2 to 3.2 GeV. Also we make the prediction for the $Z(4430)^+ \to D^{*+}\bar{D}^{*0}$ decay rate.

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I. INTRODUCTION

In the course of experimentally establishing the heavy meson spectrum unusual states were observed that cannot be simply interpreted in the context of a minimal constituent quark-antiquark model. Among these new states are the $Z_c(3900)$, $Z(4430)$, and the $X_b(5568)$, where especially the last resonance still needs solid experimental confirmation. The flavor structure of these states is unusual as evident from their strong decay modes; a simple quark-antiquark interpretation is not feasible. In the following we focus on these special states. We first collect the experimental findings.

The process $e^+e^- \to \pi^+\pi^- J/\psi$ has been studied by the BESIII Collaboration [1]. A structure was observed at around 3.9 GeV in the $\pi^\pm J/\psi$ mass spectrum that was christened the $Z_c(3900)$ state. If interpreted as a new particle, it is unusual in that it carries an electric charge and couples to charmonium. A fit to the $\pi^\pm J/\psi$ invariant mass spectrum results in a mass of $M_{Z_c} = 3899.0 \pm 3.6(\mathrm{stat}) \pm 4.9(\mathrm{syst})$ MeV and a width of $\Gamma_{Z_c} = 46 \pm 10(\mathrm{stat}) \pm 20(\mathrm{syst})$ MeV.

The cross section for $e^+e^- \to \pi^+\pi^- J/\psi$ between 3.8 and 5.5 GeV was measured by the Belle Collaboration [2]. This measurement lead to the observation of the state $Y(4260)$, and its resonance parameters were determined. In addition, an excess of $\pi^+\pi^- J/\psi$ production around 4 GeV was observed. This feature can be described by a Breit-Wigner parametrization with properties that are consistent with the $Y(4008)$ state that was previously reported by Belle. In a study of the $Y(4260) \to \pi^+\pi^- J/\psi$ decays, a structure was observed in the $M(\pi^\pm J/\psi)$ mass spectrum with 5.2σ significance, with mass $M = 3894.5 \pm 6.6(\mathrm{stat}) \pm 4.5(\mathrm{syst})$ MeV and width $\Gamma = 63 \pm 24(\mathrm{stat}) \pm 26(\mathrm{syst})$ MeV, where the errors are statistical and systematic, respectively. This structure can be interpreted as a new charged charmoniumlike state.

Using 586 pb of e^+e^- annihilation data the CLEO-c detector made an analysis at $\sqrt{s} = 4170$ MeV at the peak of the charmonium resonance $\psi(4160)$. The subsequent decay $\psi(4160) \to \pi^+\pi^- J/\psi$ was analyzed [3], and the charged state $Z_c^\pm(3900)$ was observed, which decays into $\pi^\pm J/\psi$ at a significance level of $> 5\sigma$. The value of the mass $M_{Z_c} = 3886 \pm 4(\mathrm{stat}) \pm 2(\mathrm{syst})$ MeV and the width $\Gamma_{Z_c} = 37 \pm 4(\mathrm{stat}) \pm 8(\mathrm{syst})$ MeV were found to be in good agreement with the results for this resonance reported by the BES III and Belle collaborations in the decay of the resonance $Y(4260)$. In addition CLEO-c presented the first evidence for the production of the neutral member of this isospin triplet, $Z_c^0(3900)$ decaying into $\pi^0 J/\psi$ at a 3.5σ significance level.

A study of the process $e^+e^- \to \pi^\pm(D\bar{D}^*)^\mp$ was reported by the BESIII Collaboration [4] at $\sqrt{s} = 4.26$ GeV using a 525 pb^{-1} data sample collected with the BESIII detector at the BEPCII storage ring. A distinct charged structure was observed in the $(D\bar{D}^*)^\mp$ invariant mass distribution. When fitted to a mass-dependent-width Breit-Wigner line shape, the pole mass and width were determined to be $M_{\text{pole}} = 3883.9 \pm 1.5(\text{stat}) \pm 4.2(\text{syst})$ MeV and $\Gamma_{\text{pole}} = 24.8 \pm 3.3(\text{stat}) \pm 11.0(\text{syst})$ MeV. The mass and width of the structure referred to as $Z_c(3885)$ are 2σ and 1σ, respectively, below those of the $Z_c(3900) \to \pi^\pm J/\psi$ peak observed by BESIII and Belle in $\pi^+\pi^- J/\psi$ final states produced at the same center-of-mass energy. The angular distribution of the $\pi Z_c(3885)$ system favors a $J^P = 1^+$ quantum number assignment for the structure and disfavors the assignment 1^- or 0^-. The Born cross section times the DD^* branching fraction of the $Z_c(3885)$ is measured to be

$$\sigma(e^+e^- \to \pi^\pm Z_c^\mp(3885)) \times \mathcal{B}(Z_c^\mp(3885) \to (D\bar{D}^*)^\mp) = 83.5 \pm 6.6(\text{stat}) \pm 22.0(\text{syst})\ \text{pb}. \quad (1)$$

Assuming that the $Z_c(3885) \to D\bar{D}^*$ signal reported in [4] and the $Z_c(3900) \to \pi J/\psi$ signal are from the same source, the ratio of partial widths is determined as

$$\frac{\Gamma(Z_c(3885) \to D\bar{D}^*)}{\Gamma(Z_c(3885) \to \pi J/\psi)} = 6.2 \pm 1.1(\text{stat}) \pm 2.7(\text{syst}). \quad (2)$$

That means that the $Z_c(3900)$ state has a much stronger coupling to DD^* than to $\pi J/\psi$ [5]. An unbinned maximum likelihood fit gives a mass of $M = 3889.1 \pm 1.8$ MeV and a width of $\Gamma = 28.1 \pm 4.1$ MeV ($M = 3891.8 \pm 1.8$ MeV and $\Gamma = 27.8 \pm 3.9$ MeV) for the two data sets, respectively. The pole position of this peak is calculated to be $M_{\text{pole}} = 3883.9 \pm 1.5 \pm 4.2$ MeV and $\Gamma_{\text{pole}} = 24.8 \pm 3.3 \pm 11.0$ MeV. The mass and width of the peak observed in the DD^* final state agree with that of the $Z_c(3900)$. Thus, they are quite probably the same state.

The charmoniumlike structure $Z_c^+(3900)$ was identified in Ref. [6] as the charged partner of the $X(3872)$ state. The $X(3872)$ meson is considered to be a four-quark state with quantum numbers $I^G(J^{PC}) = 0^+(1^{++})$. The $Z_c(3900)$ meson is interpreted as the isospin 1 partner of the $X(3872)$. As in Ref. [7] it was assumed that the quantum numbers for the neutral state in the isospin multiplet were $I^G(J^{PC}) = 1^+(1^{+-})$. Using standard QCD sum rules techniques, the coupling constants of the $Z_c^+ J/\psi \pi^+$, $Z_c^+ \eta_c \rho^+$, $Z_c^+ \bar{D}^0 D^{*+}$, and $Z_c^+ \bar{D}^{*0} D^+$ vertices and the corresponding decay widths were calculated with the following results:

$$\begin{aligned} \Gamma(Z_c^+ \to J/\psi + \pi^+) &= (29.1 \pm 8.2)\ \text{MeV}, \\ \Gamma(Z_c^+ \to \eta_c + \rho^+) &= (27.5 \pm 8.5)\ \text{MeV}, \\ \Gamma(Z_c^+ \to \bar{D}^0 + D^{*+}) &= (3.2 \pm 0.7)\ \text{MeV}, \\ \Gamma(Z_c^+ \to \bar{D}^{*0} + D^+) &= (3.2 \pm 0.7)\ \text{MeV}. \end{aligned} \quad (3)$$

The observation of a narrow structure, $X(5568)$, in the decay sequence $X(5568) \to B_s^0\pi^\pm$, $B_s^0 \to J/\psi\phi$, $J/\psi \to \mu^+\mu^-$, $\phi \to K^+K^-$ was reported in [8] by the D0 Collaboration. This would be the first observation of a hadronic state with valence quarks of four different flavors. The mass and width of the new state are measured to be $M = 5567.8 \pm 2.9(\text{stat})^{+0.9}_{-1.9}(\text{syst})\ \text{MeV}/c^2$, and $\Gamma = 21.9 \pm 6.4(\text{stat})^{+5.0}_{-2.5}(\text{syst})\ \text{MeV}/c^2$. However, in recent analysis performed by the LHCb Collaboration an existence of the claimed $X(5568)$ state has been not confirmed [9].

The observed strong decay mode of the $X^\pm(5586)$ implies flavor structures of the type $X^+(s\bar{b}u\bar{d})$. Since the $B_s^0\pi^+$ pair is produced in an S wave, its quantum numbers would be $J^P = 0^+$. As already pointed out in Ref. [8] the significant difference between the mass of the $X(5568)$ and $M_B + M_K$ threshold does not favor a hadronic molecular interpretation of the $X(5568)$. First qualitative considerations point to the $X(5568)$ state being a tetraquark state.

Structure issues of the $X(5568)$ have already been discussed in a number of theoretical papers [10–27] suggesting various tests both for the tetraquark and hadronic molecular structure of the $X(5586)$ state. In Ref. [23] a few options for the interpretation of the $X(5586)$ state have been checked. It was concluded that threshold, cusp, molecular, and tetraquark approaches for the explanation of the $X(5586)$ state are all disfavored. One of the important conclusions was that the mass of the $(bsqq)$ tetraquark state must be heavier than the $\Xi_b(5800)$ baryon. Also the authors of Ref. [23] deduced a lower limit for the masses of a possible $(bsud)$ tetraquark state: 6019 (6107) MeV. Complementary to Ref. [23,24] presented an analysis based on general properties of QCD to analyze the $X(5568)$ states. In particular, it was shown that the mass of the $(bsud)$ tetraquark state must be bigger than the sum of the masses of the B_s meson and the light quark-antiquark resonance leading to an estimate of the lower limit of $M_{bsud} \sim 5.9$ GeV. Reference [26] used a $B_s\pi$-$B\bar{K}$ coupled channel analysis with an interaction derived from heavy hadron chiral perturbation theory to implement the unitarity feature of the spectrum reported by the D0 Collaboration. The analysis lead to a T-matrix momentum cutoff of $\Lambda = 2.80 \pm 0.04$ GeV, which is much larger than a typical scale $\Lambda \simeq 1$ GeV.

Reference [28] estimated the mass of the lightest $(bsud)$, 0^+ tetraquark in the framework of a tightly bound diquark model. Their semiquantitative analysis leads to a mass of about 5770 MeV that lies approximately 200 MeV above the reported $X(5568)$ state, and 7 MeV below the $B\bar{K}$ threshold.

The $Z(4430)$ state with mass $M = 4433 \pm 4 \pm 2$ MeV and width $\Gamma = 45^{+18}_{-13}(\mathrm{stat})^{+30}_{-13}(\mathrm{syst})$ MeV has been discovered by the *BABAR* Collaboration in the $\pi^\pm\psi(2s)$ invariant mass distribution in $B \to K\pi^\pm\psi(2s)$ decay [29], where $\psi(2s)$ is the first radial excitation of the J/ψ. Later, in Ref. [30] the Belle Collaboration updated their predictions for the mass and width of the $Z(4430)$ resonance: $M = 4443^{+15}_{-12}(\mathrm{stat})^{+19}_{-13}(\mathrm{syst})$ MeV and $\Gamma = 107^{+86}_{-43}(\mathrm{stat})^{+74}_{-56}(\mathrm{syst})$ MeV.

The *BABAR* Collaboration studied the decays $\bar{B}^{-,0} \to K^{0,+}\pi^-\psi(2s)$ and $\bar{B}^{-,0} \to K^{0,+}\pi^- J/\psi$, but they did not see a $Z(4430)^-$ signal [31]. They derived upper limits for branching fractions that are yet compatible with the mentioned results of the Belle Collaboration. In Ref. [32] the Belle Collaboration reported on the spin and parity of the $Z(4430)^-$ state constrained from a full amplitude analysis of the $B^0 \to \psi(2s)K^+\pi^-$ decay with $\psi(2s) \to \mu^+\mu^-$ or e^+e^-. They found that the $Z(4430)^-$ being a $J^P = 1^+$-state was favored over the next likely state (0^-) with a significance of 3.4σ.

Furthermore, the Belle Collaboration did estimate for the product of branching fractions: $\mathcal{B}(\bar{B}^0 \to K^- Z(4430)^+) \times \mathcal{B}(Z(4430)^+ \to \pi^+\psi(2s)) = (3.2^{+1.8}_{-0.9}(\mathrm{stat})^{+5.3}_{-1.6}(\mathrm{syst})) \times 10^{-5}$. The *BABAR* Collaboration studied the decays $\bar{B}^{-,0} \to K^{0,+}\pi^-\psi(2s)$ and $\bar{B}^{-,0} \to K^{0,+}\pi^- J/\psi$, but they did not see a $Z(4430)^-$ signal [31]. They derived upper limits for branching fractions that are yet compatible with the mentioned results of the Belle Collaboration.

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In Ref. [33] the LHCb Collaboration confirmed the $Z(4430)^-$ signal in the $\psi(2s)\pi^-$-spectrum of the decay $B^0 \to \psi' K^+\pi^-$, and determined unambiguously the spin parity $J^P = 1^+$ of the $Z(4430)$ state [33]. They determined the following values for the mass and width of the $Z(4430)^-$ state: $M = 4475 \pm 7(\mathrm{stat})^{+15}_{-25}(\mathrm{syst})$ MeV and $\Gamma = 172 \pm 13(\mathrm{stat})^{+37}_{-34}(\mathrm{syst})$ MeV [33]. In Ref. [34] the LCHb Collaboration concluded that the only possible explanation for internal structure of the $Z(4430)$ state is a four-quark $ccud$ bound state.

In Ref. [35] the Belle Collaboration reported that they also found a $Z^+(4430)$ signal in the $J/\psi\pi^+$ spectrum of the decay $\bar{B}^0 \to J/\psi K^-\pi^+$. They report product branching fraction $\mathcal{B}(\bar{B}^0 \to K^- Z(4430)^+) \times \mathcal{B}(Z(4430)^+ \to \pi^+ J/\psi) = (5.4^{+4.0}_{-1.0}(\mathrm{stat})^{+1.1}_{-0.9}(\mathrm{syst})) \times 10^{-5}$. If one compares this value with the corresponding product branching fraction for the $\psi(2s)$ particle (see above) and assumes that the decay rates are invariant under charge conjugation, one can derive an estimation for the branching ratio of the two decay channels of the $Z(4430)^\pm$. We do not know how the errors of the values correlate, so we only do a rough estimation of the errors by dividing the upper limit of the one product branching fraction by the lower limit of the other one and vice versa, taking into account statistical and systematical error both in one step. We get

$$R_Z = \frac{\Gamma(Z(4430)^\pm \to \pi^\pm\psi(2s))}{\Gamma(Z(4430)^\pm \to \pi^\pm J/\psi)} \simeq 11.1^{+18}_{-8.6}. \qquad (4)$$

In the present paper we critically check the tetraquark picture for both the $Z_c(3900)$ and $X(5568)$ states by analyzing their strong decays. In our consideration we use the covariant quark model proposed in [36] and used in Refs. [37,38] to describe the properties of the $X(3872)$ state as a tetraquark state. First, we employ an interpretation of the $Z_c(3900)$ state as the isospin 1 partner of the $X(3872)$ as was suggested in Refs. [6] and [7]. We calculate the partial widths of the decays $Z_c^+(3900) \to J/\psi\pi^+, \eta_c\rho^+$, and $\bar{D}^0 D^{*+}$, $\bar{D}^{*0}D^+$. We find that for a relatively small model size parameter $\Lambda_{Z_c} \sim 1.4$ GeV one can reproduce the central values for the partial widths of the decays $Z_c^+ \to J/\psi\pi^+$, $\eta_c\rho^+$ as they were also obtained in Refs. [6,7]. It turns out that, in our model, the leading Lorentz metric structure in the matrix elements describing the decays $Z_c(3900) \to \bar{D}D^*$ vanishes analytically. This results in a significant suppression of these decay widths by the smallness of the relevant phase space factor $|\mathbf{q}|^5$. Since the experimental data [4] show that the $Z_c(3900)$ has a much more stronger coupling to DD^* than to $J/\psi\pi$, one has to conclude that the tetraquark-type current for the $Z_c(3900)$ is in discord with experiment. As an alternative we employ a molecular-type four-quark current to describe the decays of the $Z_c(3900)$ state. In this case we find that for a relatively large size parameter $\Lambda_{Z_c} \sim 3.3$ GeV one can obtain the partial widths of the decays $Z_c^+(3900) \to \bar{D}D^*$ at the order ~ 15 MeV for each mode. At the same time the partial widths for decays $Z_c^+(3900) \to J/\psi\pi^+$, $\eta_c\rho^+$ are suppressed by a factor of 6–7 in accordance with experimental data [4].

Let us stress, that in our manuscript we consider exotic mesons in the four-quark picture with the use of two possible configuration of quarks in these states. Note that molecular configuration does not mean that a specific size of the state with such structure is more compact than the tetraquark configuration. In this sense it is differed from hadronic molecules—extended object with clear separation of two hadrons—the constituents building the exotic state. Such hadronic molecules (extended objects) with smaller size parameter (of order of 1–2 GeV) have been considered some of us in the phenomenological Lagrangian approach based on the composite structure of exotic states as bound states of separate hadrons [39]. In the present manuscript, for the first time in order to distinguish both configurations we vary the size parameter in the same region 3.2–3.4 GeV, which is guided by experimental data. Also we would like to mention that the size parameter is not directly related to the size of a hadron like e.g., in potential approaches. Indeed, our size parameter is related to the physical quantities like electromagnetic radii, slope of the form factors, etc.

Then, we test the tetraquark picture for the $X(5568)$ structure by analyzing its strong one-pion decay. We found that for a mass of 5568 MeV one can fit the experimental decay width by using the value of size parameter $\Lambda_{X_b}\sim$ 1.4 GeV. In the case of a larger mass of 5771 MeV [28] one finds $\Lambda_{X_b}\sim 1.7$ GeV. Finally, we consider the decays of the $Z(4430)$ state $Z(4430)^+\to J/\psi+\pi^+$, $Z(4430)^+\to\psi(2s)+\pi^+$, and $Z(4430)^+\to D^{*+}+\bar{D}^{*0}$ in the tetraquark picture.

The paper is organized as follows. In Sec. II, we consider the $Z_c(3900)$ state, a four-quark state, as a compact tetraquark bounded by color diquark and antidiquark. In Sec. III we test a molecular-type four-quark structure of the $Z_c(3900)$ state. In Sec. IV we present study of the $X(5568)$ exotic state as the tetraquark four-quark state. In Sec. V we apply the tetraquark model for the $Z(4430)$ state. Finally, in Sec. VI we summarize and conclude our results.

II. THE $Z_c(3900)$ AS A FOUR-QUARK STATE WITH A TETRAQUARK-TYPE CURRENT

Let us first interpret $Z_c(3900)$ as the isospin 1 partner of the $X(3872)$ as was suggested in Refs. [6] and [7]. Then the quantum numbers for the neutral state are $I^G(J^{PC})=1^+(1^{+-})$. Accordingly the interpolating current for the $Z_c^+(3900)$ state is given by

$$J^\mu=\frac{i}{\sqrt{2}}\varepsilon_{abc}\varepsilon_{dec}[(u_a^TC\gamma_5c_b)(\bar{d}_d\gamma^\mu C\bar{c}_e^T) - (u_a^TC\gamma^\mu c_b)(\bar{d}_d\gamma_5C\bar{c}_e^T)]. \tag{5}$$

We employ a charge conjugation matrix in the form of $C=\gamma^0\gamma^2$, i.e., without a factor "i" as is usually employed. This allows one to simplify the calculations because of $C=C^\dagger=C^{-1}=-C^T$, $C\Gamma^TC^{-1}=\pm\Gamma$ ("+"for $\Gamma=S$, P, A, and "−" for $\Gamma=V$, T). In what follows we drop the superscript "T" (transpose) from the spinors to avoid a complication of notation.

The nonlocal version of the four-quark interpolating current reads

$$J^\mu_{Z_c}(x)=\int dx_1\ldots\int dx_4\delta\left(x-\sum_{i=1}^4 w_ix_i\right)\times\Phi_{Z_c}\left(\sum_{i<j}(x_i-x_j)^2\right)J^\mu_{4q}(x_1,\ldots,x_4),$$
$$J^\mu_{4q}=\frac{i}{\sqrt{2}}\varepsilon_{abc}\varepsilon_{dec}\{[u_a(x_4)C\gamma_5c_b(x_1)][\bar{d}_d(x_3)\gamma^\mu C\bar{c}_e(x_2)] - [u_a(x_4)C\gamma^\mu c_b(x_1)][\bar{q}_d(x_3)\gamma_5C\bar{c}_e(x_2)]\}, \tag{6}$$

where $w_i=m_i/\sum_{j=1}^4 m_j$. The numbering of the coordinates x_i is chosen such that one has a convenient arrangement of vertices and propagators in the Feynman diagrams to be calculated. The effective interaction Lagrangian describing the coupling of the meson Z_c to its constituent quarks is written in the form

$$\mathcal{L}_{\rm int}=g_{Z_c}Z_{c,\mu}(x)\cdot J^\mu_{Z_c}(x)+{\rm H.c.} \tag{7}$$

The Fourier transform of the vertex function $\Phi_{Z_c}(\sum_{i<j}(x_i-x_j)^2)$ can be calculated by using appropriately chosen Jacobi coordinates

$$x_i=x+\sum_{j=1}^3 w_{ij}\rho_j, \tag{8}$$

where

$$w_{11}=+\frac{2w_2+w_3+w_4}{2\sqrt{2}}\quad w_{12}=-\frac{w_3-w_4}{2\sqrt{2}}\quad w_{13}=+\frac{w_3+w_4}{2}$$
$$w_{21}=-\frac{2w_1+w_3+w_4}{2\sqrt{2}}\quad w_{22}=-\frac{w_3-w_4}{2\sqrt{2}}\quad w_{23}=+\frac{w_3+w_4}{2}$$
$$w_{31}=-\frac{w_1-w_2}{2\sqrt{2}}\quad w_{32}=+\frac{w_1+w_2+2w_4}{2\sqrt{2}}\quad w_{33}=-\frac{w_1+w_2}{2}$$
$$w_{41}=-\frac{w_1-w_2}{2\sqrt{2}}\quad w_{42}=-\frac{w_1+w_2+2w_3}{2\sqrt{2}}\quad w_{43}=-\frac{w_1+w_2}{2}.$$

It is straightforward to check that $x=\sum_{i=1}^4 x_iw_i$, and $\sum_{1\le i<j\le 4}(x_i-x_j)^2=\sum_{i=1}^3\rho_i^2$. The vertex function is then written as

$$\Phi_{Z_c}\left(\sum_{i<j}(x_i-x_j)^2\right)=\int\frac{d\vec{\omega}}{(2\pi)^{12}}e^{-i\vec{\rho}\,\vec{\omega}}\tilde{\Phi}_{Z_c}(-\vec{\omega}^2), \tag{9}$$

where the vertex function in momentum space is chosen to have a Gaussian form

$$\tilde{\Phi}_{Z_c}(-\vec{\omega}^2)=\exp(\vec{\omega}^2/\Lambda^2_{Z_c}) \tag{10}$$

with the $\Lambda^2_{Z_c}$ being an adjustable size parameter.

The coupling constant g_{Z_c} in Eq. (7) is determined by the normalization condition called *the compositeness condition* (see Refs. [40] and [36] for details),

$$Z_{Z_c}=1-g^2_{Z_c}\tilde{\Pi}'_{Z_c}(m^2_{Z_c})=0, \tag{11}$$

where $\Pi_{Z_c}(p^2)$ is the scalar part of the vector-meson mass operator

$$\tilde{\Pi}^{\mu\nu}_{Z_c}(p) = g^{\mu\nu}\tilde{\Pi}_{Z_c}(p^2) + p^\mu p^\nu \tilde{\Pi}^{(1)}_{Z_c}(p^2),$$
$$\tilde{\Pi}_{Z_c}(p^2) = \frac{1}{3}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Pi^{\mu\nu}_{Z_c}(p). \tag{12}$$

The Fourier transform of the Z_c-tetraquark mass operator reads

$$\Pi^{\mu\nu}_{Z_c}(p) = 6\prod_{i=1}^{3}\int\frac{d^4k_i}{(2\pi)^4 i}\tilde{\Phi}^2_{Z_c}(-\vec{\omega}^2)$$
$$\times\{\mathrm{tr}[S_4(\hat{k}_4)\gamma_5 S_1(\hat{k}_1)\gamma_5]\mathrm{tr}[S_3(\hat{k}_3)\gamma^\mu S_2(\hat{k}_2)\gamma^\nu]$$
$$+\mathrm{tr}[S_4(\hat{k}_4)\gamma^\nu S_2(\hat{k}_2)\gamma^\mu]\mathrm{tr}[S_3(\hat{k}_3)\gamma_5 S_1(\hat{k}_1)\gamma_5]\}, \tag{13}$$

where $\hat{k}_1 = k_1 - w_1 p$, $\hat{k}_2 = k_2 - w_2 p$, $\hat{k}_3 = k_3 + w_3 p$, $\hat{k}_4 = k_1 + k_2 - k_3 + w_4 p$, and $\vec{\omega}^2 = 1/2(k_1^2 + k_2^2 + k_3^2 + k_1 k_2 - k_1 k_3 - k_2 k_3)$. Details of the calculation can be found in our previous papers, e.g., [37,38].

The matrix elements of the decays $Z_c^+ \to J/\psi + \pi^+$ and $Z_c^+ \to \eta_c + \rho^+$ are given by

$$M^{\mu\nu}(Z_c(p,\epsilon_p^\mu) \to J/\psi(q_1,\epsilon_{q_1}^\nu) + \pi^+(q_2))$$
$$= \frac{6}{\sqrt{2}} g_{Z_c} g_{J/\psi} g_\pi \int\frac{d^4k_1}{(2\pi)^4 i}\int\frac{d^4k_2}{(2\pi)^4 i}\tilde{\Phi}_{Z_c}(-\vec{\eta}^2)\tilde{\Phi}_{J/\psi}(-(k_1+v_2q_1)^2)\tilde{\Phi}_\pi(-(k_2+u_4q_2)^2)$$
$$\times\{\mathrm{tr}[\gamma_5 S_4(k_2)\gamma_5 S_3(k_2+q_2)\gamma^\mu S_2(k_1)\gamma^\nu S_1(k_1+q_1)] + \mathrm{tr}[\gamma^\mu S_4(k_2)\gamma_5 S_3(k_2+q_2)\gamma_5 S_2(k_1)\gamma^\nu S_1(k_1+q_1)]\}$$
$$= A_{J/\psi\pi} g^{\mu\nu} + B_{J/\psi\pi} q_1^\mu q_2^\nu, \tag{14}$$

$$M^{\mu\alpha}(Z_c(p,\epsilon_p^\mu) \to \eta_c(q_1) + \rho(q_2,\epsilon_{q_2}^\alpha))$$
$$= \frac{6}{\sqrt{2}} g_{Z_c} g_{\eta_c} g_\rho \int\frac{d^4k_1}{(2\pi)^4 i}\int\frac{d^4k_2}{(2\pi)^4 i}\tilde{\Phi}_{Z_c}(-\vec{\eta}^2)\tilde{\Phi}_{\eta_c}(-(k_1+v_2q_1)^2)\tilde{\Phi}_\rho(-(k_2+u_4q_2)^2)$$
$$\times\{\mathrm{tr}[\gamma_5 S_4(k_2)\gamma^\alpha S_3(k_2+q_2)\gamma^\mu S_2(k_1)\gamma_5 S_1(k_1+q_1)] + \mathrm{tr}[\gamma^\mu S_4(k_2)\gamma^\alpha S_3(k_2+q_2)\gamma_5 S_2(k_1)\gamma_5 S_1(k_1+q_1)]\}$$
$$= A_{\eta_c\rho} g^{\mu\alpha} - B_{\eta_c\rho} q_2^\mu q_1^\alpha. \tag{15}$$

The argument of the Z_c-vertex function is given by

$$\vec{\eta}^2 = \eta_1^2 + \eta_2^2 + \eta_3^2,$$
$$\eta_1 = +\frac{1}{2\sqrt{2}}(2k_1 + (1 - w_1 + w_2)q_1 - (w_1 - w_2)q_2),$$
$$\eta_2 = +\frac{1}{2\sqrt{2}}(2k_2 - (w_3 - w_4)q_1 + (1 - w_3 + w_4)q_2),$$
$$\eta_3 = +\frac{1}{2}((w_3 + w_4)q_1 - (w_1 + w_2)q_2). \tag{16}$$

The quark masses are specified as $m_1 = m_2 = m_c$, $m_3 = m_4 = m_d = m_u$, and the two-body reduced masses as $v_1 = m_1/(m_1 + m_2)$, $v_2 = m_2/(m_1 + m_2)$, $u_3 = m_3/(m_3 + m_4)$, and $u_4 = m_4/(m_3 + m_4)$.

The matrix elements of the decays $Z_c^+ \to \bar{D}^0 + D^{*+}$ and $Z_c^+ \to \bar{D}^{*0} + D^+$ read

$$M^{\mu\nu}(Z_c(p,\epsilon_p^\mu) \to \bar{D}^0(q_1) + D^{*+}(q_2,\epsilon_{q_2}^\nu))$$
$$= \frac{6}{\sqrt{2}} g_{Z_c} g_D g_{D^*} \int\frac{d^4k_1}{(2\pi)^4 i}\int\frac{d^4k_2}{(2\pi)^4 i}\tilde{\Phi}_{Z_c}(-\vec{\delta}^2)\tilde{\Phi}_D(-(k_2+v_2q_2)^2)\tilde{\Phi}_{D^*}(-(k_1+u_1q_2)^2)$$
$$\times\{\mathrm{tr}[\gamma_5 S_4(k_2+q_1)\gamma_5 S_1(k_1)\gamma^\nu S_3(k_1+q_2)\gamma^\mu S_2(k_2)] - \mathrm{tr}[\gamma_5 S_4(k_2+q_1)\gamma^\mu S_1(k_1)\gamma^\nu S_3(k_1+q_2)\gamma_5 S_2(k_2)]\}$$
$$= A_{\bar{D}D^*} g^{\mu\nu} - B_{\bar{D}D^*} q_2^\mu q_1^\nu, \tag{17}$$

$$
\begin{aligned}
&M^{\mu\alpha}(Z_c(p,\epsilon_p^\mu)\to\bar{D}^{*0}(q_1,\epsilon_{q_1}^\alpha)+D^+(q_2,))\\
&=\frac{6}{\sqrt{2}}g_{Z_c}g_{D^*}g_D\int\frac{d^4k_1}{(2\pi)^4i}\int\frac{d^4k_2}{(2\pi)^4i}\tilde{\Phi}_{Z_c}(-\vec{\delta}^2)\tilde{\Phi}_{D^*}(-(k_1+\hat{v}_1q_1)^2)\tilde{\Phi}_D(-(k_2+\hat{u}_4q_2)^2)\\
&\quad\times\{\mathrm{tr}[S_4(k_2+q_1)\gamma_5S_1(k_1)\gamma_5S_3(k_1+q_2)\gamma^\mu S_2(k_2)\gamma^\alpha]-\mathrm{tr}[S_4(k_2+q_1)\gamma^\mu S_1(k_1)\gamma_5S_3(k_1+q_2)\gamma_5S_2(k_2)\gamma^\alpha]\}\\
&=A_{D^*D}g^{\mu\alpha}+B_{D^*D}q_1^\mu q_2^\alpha. \qquad (18)
\end{aligned}
$$

The argument of the Z_c-vertex function is given by

$$
\begin{aligned}
&\vec{\delta}^2=\delta_1^2+\delta_2^2+\delta_3^2,\qquad \delta_1=-\frac{1}{2\sqrt{2}}(k_1-k_2+(w_1-w_2)(q_1+q_2)),\\
&\delta_2=+\frac{1}{2\sqrt{2}}(k_1-k_2-(1+w_3-w_4)q_1+(1-w_3+w_4)q_2),\\
&\delta_3=-\frac{1}{2}(k_1+k_2+(w_1+w_2)(q_1+q_2)). \qquad (19)
\end{aligned}
$$

The quark masses are specified as $m_1=m_2=m_c$, $m_3=m_4=m_d=m_u$, and the two-body reduced masses as $\hat{v}_2=m_2/(m_2+m_4)$, $\hat{v}_4=m_4/(m_2+m_4)$, $\hat{u}_1=m_1/(m_1+m_3)$, and $\hat{u}_3=m_3/(m_1+m_3)$.

We finally calculate the two-body decay widths. The relevant spin kinematical formulas have been collected in the Appendix. Note that momentum of the daughter vector particle is chosen to be q_1 in Eq. (A1). In addition the matrix element is expressed through the dimensionless invariant amplitudes A_1, and A_2 in Eq. (A3). In order to adjust the notation in Eqs. (14), (15), (17), and (18) to those given in the Appendix, one has to replace $q_1\leftrightarrow q_2$ in Eqs. (15) and (17) and then introduce the dimensionless form factors $A_1=A/m$ and $A_2=\pm mB$ ($p^2=m^2$) where the sign "+" stands for Eqs. (14) and (18) and "−" for Eqs. (15) and (17), respectively. The expressions for helicity amplitudes via A_1 and A_2 are given in Eq. (A5). The two-body decay widths are now calculated using Eq. (A9).

As a consequence of the subtraction of the two traces in the matrix elements in Eqs. (17) and (18) we found that $A_{DD^*}=A_{D^*D}\equiv 0$ analytically. This results in a significant suppression of the decay widths due to the D-wave suppression factor of $|\mathbf{q_1}|^5$. In the calculation of the quark-loop diagrams we have only one free parameter Λ_{Z_c}, the size parameter of the Z_c state. The other model parameters have been fixed in previous papers [36–38,41] from analysis of hadron processes involving light and heavy quarks,

$m_{u/d}$	m_s	m_c	m_b	λ	
0.241	0.428	1.67	5.05	0.181	GeV.

(20)

Here m_q are the constituent quark masses and λ is an infrared cutoff parameter responsible for the quark confinement. The size parameters of the π ρ, D, D^*, J/ψ, and η_c have been fixed as

Λ_π	Λ_ρ	Λ_D	Λ_{D^*}	$\Lambda_{J/\psi}$	Λ_{η_c}	
0.871	0.624	1.600	1.529	1.738	3.777	GeV.

(21)

For the $Z_c(3900)$ mass we use the actual value 3.886 GeV. We adjust the size parameter Λ_{Z_c} in such a way as to be close to the central value for the decay $Z_c^+\to J/\psi+\pi^+$ obtained in Refs. [6,7]. If the parameter Λ_{Z_c} is varied in the region $\Lambda_{Z_c}=2.25\pm0.10$ GeV the numerical values of the decay widths vary as

$$
\begin{aligned}
\Gamma(Z_c^+\to J/\psi+\pi^+)&=(27.9^{+6.3}_{-5.0})\ \mathrm{MeV},\\
\Gamma(Z_c^+\to \eta_c+\rho^+)&=(35.7^{+6.3}_{-5.2})\ \mathrm{MeV},\\
\Gamma(Z_c^+\to \bar{D}^0+D^{*+})&\propto 10^{-8}\ \mathrm{MeV},\\
\Gamma(Z_c^+\to \bar{D}^{*0}+D^{+})&\propto 10^{-8}\ \mathrm{MeV}. \qquad (22)
\end{aligned}
$$

Here and in the following an increasing of the size parameter leads to a decreasing of the decay width. Since the experimental data [4] show that the $Z_c(3900)$ has a much more stronger coupling to DD^* than $J/\psi\pi$, one has to conclude that the tetraquark-type current for $Z_c(3900)$ is in discord with experiment.

Moreover, we expect that a realistic value of the size parameter Λ_{Z_c} is about 3 GeV. Using $\Lambda_{Z_c}=3.3\pm1.1$ GeV we get a significant suppression for the $Z_c^+\to J/\psi+\pi^+$ and $Z_c^+\to\eta_c+\rho^+$ modes, and the rates for the modes $Z_c^+\to\bar{D}^0+D^{*+}$ and $Z_c^+\to\bar{D}^{*0}+D^+$ become much more negligible,

$$
\begin{aligned}
\Gamma(Z_c^+\to J/\psi+\pi^+)&=(4.3^{+0.7}_{-0.6})\ \mathrm{MeV},\\
\Gamma(Z_c^+\to \eta_c+\rho^+)&=(8.0^{+1.2}_{-1.0})\ \mathrm{MeV},\\
\Gamma(Z_c^+\to \bar{D}^0+D^{*+})&\propto 10^{-9}\ \mathrm{MeV},\\
\Gamma(Z_c^+\to \bar{D}^{*0}+D^{+})&\propto 10^{-9}\ \mathrm{MeV}. \qquad (23)
\end{aligned}
$$

III. THE $Z_c(3900)$ AS A FOUR-QUARK STATE WITH A MOLECULAR-TYPE CURRENT

We describe the $Z_c^+(3900)$ as the charged particle in the isotriplet with a molecular-type current given by (see Ref. [42])

$$J^\mu = \frac{1}{\sqrt{2}}[(\bar{d}\gamma_5 c)(\bar{c}\gamma^\mu u) + (\bar{d}\gamma^\mu c)(\bar{c}\gamma_5 u)]. \tag{24}$$

Its nonlocal generalization is given by

$$J^\mu_{Z_c}(x) = \int dx_1 \ldots \int dx_4 \delta\left(x - \sum_{i=1}^4 w_i x_i\right) \Phi_{Z_c}\left(\sum_{i<j}(x_i - x_j)^2\right) J^\mu_{4q}(x_1, \ldots, x_4),$$
$$J^\mu_{4q} = \frac{1}{\sqrt{2}}\{(\bar{d}(x_3)\gamma_5 c(x_1))(\bar{c}(x_2)\gamma^\mu u(x_4)) + (\bar{d}(x_3)\gamma^\mu c(x_1))(\bar{c}(x_2)\gamma_5 u(x_4))\}. \tag{25}$$

The Fourier transform of the Z_c mass operator is written as

$$\Pi^{\mu\nu}_{Z_c}(p) = \frac{9}{2}\prod_{i=1}^3 \int \frac{d^4k_i}{(2\pi)^4 i} \tilde{\Phi}^2_{Z_c}(-\vec{\omega}^2)\{\mathrm{tr}[\gamma_5 S_1(\hat{k}_1)\gamma_5 S_3(\hat{k}_3)]\mathrm{tr}[\gamma^\mu S_4(\hat{k}_4)\gamma^\nu S_2(\hat{k}_2)]$$
$$+ \mathrm{tr}[\gamma^\mu S_1(\hat{k}_1)\gamma^\nu S_3(\hat{k}_3)]\mathrm{tr}[\gamma_5 S_4(\hat{k}_4)\gamma_5 S_2(\hat{k}_2)]\} \tag{26}$$

with $\hat{k}_i$ and $\vec{\omega}^2$ being defined as in the previous section.

The matrix elements of the decays $Z_c^+ \to J/\psi + \pi^+$ and $Z_c^+ \to \eta_c + \rho^+$ are given by

$$M^{\mu\nu}(Z_c(p, \epsilon^\mu_p) \to J/\psi(q_1, \epsilon^\nu_{q_1}) + \pi^+(q_2))$$
$$= \frac{3}{\sqrt{2}} g_{Z_c} g_{J/\psi} g_\pi \int \frac{d^4k_1}{(2\pi)^4 i} \int \frac{d^4k_2}{(2\pi)^4 i} \tilde{\Phi}_{Z_c}(-\vec{\eta}^2)\tilde{\Phi}_{J/\psi}(-(k_1 + v_1 q_1)^2)\tilde{\Phi}_\pi(-(k_2 + u_4 q_2)^2)$$
$$\times \{\mathrm{tr}[\gamma_5 S_1(k_1)\gamma^\nu S_2(k_1 + q_1)\gamma^\mu S_4(k_2)\gamma_5 S_3(k_2 + q_2)] + \mathrm{tr}[\gamma^\mu S_1(k_1)\gamma^\nu S_2(k_1 + q_1)\gamma_5 S_4(k_2)\gamma_5 S_3(k_2 + q_2)]\}$$
$$= A_{J/\psi\pi} g^{\mu\nu} + B_{J/\psi\pi} q_1^\mu q_2^\nu, \tag{27}$$

$$M^{\mu\alpha}(Z_c(p, \epsilon^\mu_p) \to \eta_c(q_1) + \rho(q_2, \epsilon^\alpha_{q_2}))$$
$$= \frac{3}{\sqrt{2}} g_{Z_c} g_{\eta_c} g_\rho \int \frac{d^4k_1}{(2\pi)^4 i} \int \frac{d^4k_2}{(2\pi)^4 i} \tilde{\Phi}_{Z_c}(-\vec{\eta}^2)\tilde{\Phi}_{\eta_c}(-(k_1 + v_1 q_1)^2)\tilde{\Phi}_\rho(-(k_2 + u_4 q_2)^2)$$
$$\times \{\mathrm{tr}[\gamma_5 S_1(k_1)\gamma_5 S_2(k_1 + q_1)\gamma^\mu S_4(k_2)\gamma^\alpha S_3(k_2 + q_2)] + \mathrm{tr}[\gamma^\mu S_1(k_1)\gamma_5 S_2(k_1 + q_1)\gamma_5 S_4(k_2)\gamma^\alpha S_3(k_2 + q_2)]\}$$
$$= A_{\eta_c\rho} g^{\mu\alpha} - B_{\eta_c\rho} q_2^\mu q_1^\alpha. \tag{28}$$

The argument of the Z_c-vertex function $\vec{\eta}^2$ and the specification of the quark masses are identical to those given in the previous section.

The matrix elements of the decays $Z_c^+ \to \bar{D}^0 + D^{*+}$ and $Z_c^+ \to \bar{D}^{*0} + D^+$ read

$$M^{\mu\nu}(Z_c(p, \epsilon^\mu_p) \to \bar{D}^0(q_1) + D^{*+}(q_2, \epsilon^\nu_{q_2}))$$
$$= \frac{9}{\sqrt{2}} g_{Z_c} g_D g_{D^*} \int \frac{d^4k_1}{(2\pi)^4 i} \int \frac{d^4k_2}{(2\pi)^4 i} \tilde{\Phi}_{Z_c}(-\vec{\delta}^2)\tilde{\Phi}_D(-(k_2 + v_4 q_1)^2)\tilde{\Phi}_{D^*}(-(k_1 + u_1 q_2)^2)$$
$$\times \{\mathrm{tr}[\gamma^\mu S_1(k_1)\gamma^\nu S_3(k_1 + q_2)]\mathrm{tr}[\gamma_5 S_4(k_2)\gamma_5 S_2(k_2 + q_1)]\}$$
$$= A_{\bar{D}D^*} g^{\mu\nu} - B_{\bar{D}D^*} q_2^\mu q_1^\nu, \tag{29}$$

$$\begin{aligned} M^{\mu\alpha}&(Z_c(p,\epsilon_p^\mu)\to \bar{D}^{*0}(q_1,\epsilon_{q_1}^\alpha)+D^+(q_2,)) \\ &= \frac{9}{\sqrt{2}} g_{Z_c} g_{D^*} g_D \int \frac{d^4k_1}{(2\pi)^4 i}\int \frac{d^4k_2}{(2\pi)^4 i} \tilde{\Phi}_{Z_c}(-\vec{\delta}^2)\tilde{\Phi}_{D^*}(-(k_1+\hat{v}_1 q_1)^2)\tilde{\Phi}_D(-(k_2+\hat{u}_4 q_2)^2) \\ &\quad \times \{\mathrm{tr}[\gamma_5 S_1(k_1)\gamma_5 S_3(k_1+q_2)]\mathrm{tr}[\gamma^\mu S_4(k_2)\gamma^\alpha S_2(k_2+q_1)]\} \\ &= A_{D^*D} g^{\mu\alpha} + B_{D^*D} q_1^\mu q_2^\alpha. \end{aligned} \tag{30}$$

The argument of the Z_c-vertex function is given by

$$\begin{aligned} \vec{\delta}^2 &= \delta_1^2+\delta_2^2+\delta_3^2, \\ \delta_1 &= -\frac{1}{2\sqrt{2}}(k_1+k_2+(1+w_1-w_2)q_1+(w_1-w_2)q_2)), \\ \delta_2 &= +\frac{1}{2\sqrt{2}}(k_1+k_2-(w_3-w_4)q_1+(1-w_3+w_4)q_2), \\ \delta_3 &= +\frac{1}{2}(-k_1+k_2+(1-w_1-w_2)q_1-(w_1+w_2)q_2)). \end{aligned} \tag{31}$$

The quark masses are specified as $m_1=m_2=m_c$, $m_3=m_4=m_d=m_u$, and the two-body reduced masses as $\hat{v}_2=m_2/(m_2+m_4)$, $\hat{v}_4=m_4/(m_2+m_4)$, $\hat{u}_1=m_1/(m_1+m_3)$, and $\hat{u}_3=m_3/(m_1+m_3)$.

As a guide to adjust the parameter Λ_{Z_c} we take the experimental values for decay widths given in Ref. [4]. If the parameter Λ_{Z_c} is varied in the limits $\Lambda_{Z_c}=3.3\pm 0.1$ GeV the numerical values of decay widths vary according to

$$\begin{aligned} \Gamma(Z_c^+\to J/\psi+\pi^+) &= (1.8\pm 0.3)\ \mathrm{MeV}, \\ \Gamma(Z_c^+\to \eta_c+\rho^+) &= (3.2^{+0.5}_{-0.4})\ \mathrm{MeV}, \\ \Gamma(Z_c^+\to \bar{D}^0+D^{*+}) &= (10.0^{+1.7}_{-1.4})\ \mathrm{MeV}, \\ \Gamma(Z_c^+\to \bar{D}^{*0}+D^{+}) &= (9.0^{+1.6}_{-1.3})\ \mathrm{MeV}. \end{aligned} \tag{32}$$

Thus a molecular-type current for the vertex function of the Z_c is in accordance with the experimental observation [4] that $Z_c(3900)$ has a much stronger coupling to DD^* than to $J/\psi\pi$.

IV. X_b AS A TETRAQUARK

Let us first interpret X_b as a tetraquark state with the quantum numbers $J^P=0^+$. Then the interpolating current for the $X_b(5568)$ is given by

$$J=\varepsilon_{abc}\varepsilon_{dec}(u_a^T C\gamma_5 b_b)(\bar{d}_d\gamma_5 C\bar{s}_e^T). \tag{33}$$

The nonlocal version of the four-quark interpolating current reads

$$\begin{aligned} J_{X_b}^+(x) &= \int dx_1 \ldots \int dx_4 \delta\left(x-\sum_{i=1}^4 w_i x_i\right) \\ &\quad \times \Phi_{X_b}\left(\sum_{i<j}(x_i-x_j)^2\right) J_{4q}^+(x_1,\ldots,x_4), \\ J_{4q}^+ &= \varepsilon_{abc}\varepsilon_{dec}[u_a(x_3)C\gamma_5 b_b(x_1)][\bar{d}_d(x_4)\gamma_5 C\bar{s}_e(x_2)], \end{aligned} \tag{34}$$

where $w_i=m_i/\sum_{j=1}^4 m_j$. The effective interaction Lagrangian describing the coupling of the meson X_b to its constituent quarks takes the form

$$\mathcal{L}_{\mathrm{int}} = g_{X_b} X_b^-(x)\cdot J_{X_b}^+(x)+\mathrm{H.c.} \tag{35}$$

The Fourier transform of the X_b-tetraquark mass operator is given by

$$\begin{aligned} \Pi_{X_b}(p^2) &= 6\prod_{i=1}^3\int \frac{d^4k_i}{(2\pi)^4 i}\tilde{\Phi}_{X_b}^2(-\vec{\omega}^2) \\ &\quad \times \mathrm{tr}[\gamma_5 S_1(\hat{k}_1)\gamma_5 S_3(\hat{k}_3)]\mathrm{tr}[\gamma_5 S_2(\hat{k}_2)\gamma_5 S_4(\hat{k}_4)], \end{aligned} \tag{36}$$

where $\hat{k}_1=k_1-w_1 p$, $\hat{k}_2=k_2-w_2 p$, $\hat{k}_3=k_3+w_3 p$, $\hat{k}_4=k_1+k_2-k_3+w_4 p$, and $\vec{\omega}^2=1/2(k_1^2+k_2^2+k_3^2+k_1k_2-k_1k_3-k_2k_3)$.

The matrix element of the decay $X_b^+(p)\to B_s(q_1)+\pi^+(q_2)$ reads

$$\begin{aligned} M(Z_b\to B_s+\pi^+) &= 6 g_{X_b} g_{B_s} g_\pi \int \frac{d^4k_1}{(2\pi)^4 i}\int \frac{d^4k_2}{(2\pi)^4 i}\tilde{\Phi}_{X_b}(-\vec{\eta}^2)\tilde{\Phi}_{B_s}(-(k_1+v_1 q_1)^2)\tilde{\Phi}_\pi(-(k_2+u_4 q_2)^2) \\ &\quad \times \mathrm{tr}[\gamma_5 S_1(k_1)\gamma_5 S_2(k_1+q_1)\gamma_5 S_4(k_2)\gamma_5 S_3(k_2+q_2)] \\ &= G_{X_b B_s \pi}, \end{aligned} \tag{37}$$

where the arguments of the X_b-vertex function are given by

$$\vec{\eta}^2 = \eta_1^2 + \eta_2^2 + \eta_3^2,$$
$$\eta_1 = -\frac{1}{2\sqrt{2}}(2k_1 + (1 + w_1 - w_2)q_1 + (w_1 - w_2)q_2),$$
$$\eta_2 = +\frac{1}{2\sqrt{2}}(2k_2 - (w_3 - w_4)q_1 + (1 - w_3 + w_4)q_2),$$
$$\eta_3 = +\frac{1}{2}((w_3 + w_4)q_1 - (w_1 + w_2)q_2). \quad (38)$$

The quark masses are specified as $m_1 = m_b$, $m_2 = m_s$, $m_3 = m_u$, $m_4 = m_d$, and the two-body reduced masses as $v_1 = m_1/(m_1 + m_2)$, $v_2 = m_2/(m_1 + m_2)$, $u_3 = m_3/(m_3 + m_4)$, and $u_4 = m_4/(m_3 + m_4)$.

The two-body decay width is given by

$$\Gamma(X_b \to B_s + \pi) = \frac{|\mathbf{q_1}|}{8\pi M_{X_b}^2} G_{X_b B_s \pi}^2, \quad (39)$$

where $|\mathbf{q_1}|$ is the momentum of the daughter particles in the rest frame of the X_b.

We adjust the parameter Λ_{X_b} for two values of the X_b mass, (i) $m_{X_b} = 5567.8$ MeV as reported by the D0 Collaboration [8], and (ii) $m_{X_b} = 5771$ MeV as was obtained in [28]. The numerical values of the decay widths can be calculated to be

$$m_{X_b} = 5.568\ \text{GeV}, \quad \Lambda_{X_b} = (1.36 \pm 0.05)\ \text{GeV},$$
$$\Gamma(X_b \to B_s\pi) = (21.9 \pm 3.5)\ \text{MeV},$$
$$m_{X_b} = 5.771\ \text{GeV}, \quad \Lambda_{X_b} = (1.66 \pm 0.05)\ \text{GeV},$$
$$\Gamma(X_b \to B_s\pi) = (21.7 \pm 3.5)\ \text{MeV}. \quad (40)$$

V. $Z(4430)$ AS A TETRAQUARK

The interpolating tetraquark current of the $Z(4430)$ state with $J^P = 1^+$ fixed by the LHCb Collaboration [33] has the same structure as the tetraquark current for the Z_c state [see Eqs. (5) and (6)]. Similarity of the $Z(4430)$ and Z_c states also concerns the effective interaction Lagrangian describing the coupling of $Z(4430)$ to its constituent quarks,

$$\mathcal{L}_{\text{int}} = g_Z Z_\mu(x) \cdot J_Z^\mu(x) + \text{H.c.}, \quad (41)$$

where $J_Z^\mu(x) = J_{Z_c}^\mu(x)$ with a specific value of the size parameter Λ_Z.

In the case of $Z(4430)$ we consider two strong decay modes $Z(4430) \to J/\psi + \pi$ and $Z(4430) \to \psi(2s) + \pi$, which are calculated by analogy with the case of $Z_c \to J/\psi + \pi$ in the tetraquark picture. A new feature is that we should specify the vertex function of the $\psi(2s)$ state. By analogy with the oscillator potential model it should emulate the node structure of the $\psi(2s)$. In our calculations we use the following form of the $\psi(2s)$-vertex function:

$$\tilde{\Phi}_{\psi(2s)}(-k^2) = \exp(k^2/\Lambda_{\psi(2s)})[1 - \alpha \exp(k^2/\Lambda_{\psi(2s)})], \quad (42)$$

where α is a free parameter, encoding the node structure of the $\psi(2s)$ meson. It is fixed at $\alpha = 1.0172$ from the description of the leptonic decay constant $f_{\psi(2s)} = 291$ MeV. For convenience, we use the same size parameter $\Lambda_{J/\psi} = \Lambda_{\psi(2s)} = 1.738$ GeV for J/ψ, and its radial excitation $\psi(2s)$ state. For the $Z(4430)$ mass we use the actual value 4.478 GeV.

TABLE I. $Z(4430)$ decay rates.

$\Lambda_{Z(4430)}$ (GeV)	$\Gamma_{J/\psi}$ (MeV)	$\Gamma_{\psi(2s)}$ (MeV)	Γ (MeV)	R_Z
2.2	37.4	173.7	211.1	4.64
2.3	31.7	144.7	176.4	4.56
2.4	26.9	120.6	147.5	4.48
2.5	22.9	100.8	123.7	4.40
2.6	19.4	84.4	103.8	4.35
2.7	16.5	70.9	87.4	4.30
2.8	14.1	59.7	73.8	4.23
2.9	12.0	50.4	62.4	4.20
3.0	10.3	42.7	53.0	4.15
3.1	8.8	36.3	45.1	4.13
3.2	7.6	31.0	38.6	4.08

Now let us turn to the discussion of our results for the $Z(4430) \to J/\psi + \pi$ and $Z(4430) \to \psi(2s) + \pi$ decay widths. We have a single free parameter: the $\Lambda_{Z(4430)}$-size parameter of the $Z(4430)$ state. We use the present upper limit for the total width of the $Z(4430)$ state $\Gamma \leq 212$ MeV deduced from the averaged value $\Gamma = 181 \pm 31$ MeV in Particle Data Group [43] as the upper limit for the sum of the widths of two modes $Z(4430) \to J/\psi + \pi$ and $Z(4430) \to \psi(2s) + \pi$. It constrains the choose of the size parameter Λ_Z. In particular, we found that $\Lambda_Z \geq 2.2$ GeV, which supports the compact $(c\bar{c}d\bar{u})$ tetraquark interpretation of the $Z(4430)$ state. In Table I we present our numerical results for the partial decay widths $\Gamma_{J/\psi} \doteq \Gamma(Z(4430) \to J/\psi + \pi)$, and $\Gamma_{\psi(2s)} \doteq \Gamma(Z(4430) \to \psi(2s) + \pi)$ decay widths, their sum $\Gamma = \Gamma_{J/\psi} + \Gamma_{\psi(2s)}$, and their ratio $R_Z = \Gamma_{\psi(2s)}/\Gamma_{J/\psi}$ for variation of Λ_Z from 2.2 to 3.2 GeV. One can see that the decay width of the $Z(4430) \to \psi(2s) + \pi$ process dominates over the one of $Z(4430) \to J/\psi + \pi$ by a factor $R_Z \simeq (4.36 \pm 0.28)$.

Finally, we make the prediction for the $Z(4430)^+ \to D^{*+} + \bar{D}^{*0}$ decay rate. This process is described by the invariant matrix element, which is expressed in terms of three relativistic amplitudes B_i, $(i = 1, 2, 3)$ as

$$M^{\mu\alpha\beta}(Z(4430)(p,\mu) \to D^*(q_1,\alpha) + \bar{D}^*(q_2,\beta)) = B_1 q^\mu \epsilon^{q_1 q_2 \alpha\beta} + B_2 \epsilon^{q_1 \mu\alpha\beta} + B_3 \epsilon^{q_2 \mu\alpha\beta}. \quad (43)$$

The $Z(4430)^+ \to D^{*+} + \bar{D}^{*0}$ decay rate is calculated according to the formula

$$\begin{aligned}\Gamma(Z(4430)^+ \to D^{*+} + \bar{D}^{*0}) &= \frac{|\mathbf{q_1}|}{12\pi M_Z^2}\bigg[B_1^2 M_Z^2 |\mathbf{q_1}|^4 + B_2^2\bigg(3M_{D^{*+}}^2 + \bigg(1+\frac{M_Z^2}{M_{D^{*0}}^2}\bigg)|\mathbf{q_1}|^2\bigg) \\ &\quad + B_3^2\bigg(3M_{D^{*0}}^2 + \bigg(1+\frac{M_Z^2}{M_{D^{*+}}^2}\bigg)|\mathbf{q_1}|^2\bigg) \\ &\quad + B_1 B_2 |\mathbf{q_1}|^2 (M_Z^2 + M_{D^{*+}}^2 - M_{D^{*0}}^2) \\ &\quad + B_1 B_3 |\mathbf{q_1}|^2 (M_Z^2 + M_{D^{*0}}^2 - M_{D^{*+}}^2) \\ &\quad + B_2 B_3 (3(M_Z^2 - M_{D^*+}^2 - M_{D^{*0}}^2) - |\mathbf{q_1}|^2)\bigg]. \end{aligned} \tag{44}$$

Our numerical result for $\Lambda_{Z(4430)}$ varied from 2.2 to 3.2 GeV is $\Gamma(Z(4430)^+ \to D^{*+} + \bar{D}^{*0}) = 23.5 \pm 15.6$ MeV.

VI. SUMMARY AND CONCLUSIONS

Let us summarize the main results of our paper. Presently two possible four-quark configurations for exotic states are tested experimentally and theoretically: the tetraquark (compact) configuration corresponding to the coupling of color diquark and antidiquark and molecular (extended) configuration corresponding to the coupling of two separate mesons. We have critically checked both possible four-quark pictures (tetraquark and molecular scenario) in the case of the $Z_c(3900)$ state. For the case of the $X(5568)$ and $Z(4430)$ states we considered only the tetraquark picture. Our study has been done by analyzing strong decays of the exotic state. The strong decays have been calculated in the framework of the covariant quark model previously developed by us. First, we have interpreted the $Z_c(3900)$ state as the isospin 1 partner of the $X(3872)$. We have calculated the partial widths of the decays $Z_c^+(3900) \to J/\psi\pi^+, \eta_c\rho^+$, and $\bar{D}^0 D^{*+}$, $\bar{D}^{*0} D^+$. It turned out that the leading metric Lorentz structure in the matrix elements describing the decays $Z_c(3900) \to \bar{D}D^*$ vanishes analytically. This results in a significant D-wave suppression of these decays through the appearance of the phase space factor proportional to $|\mathbf{q}|^5$. Since the experimental data from the BESIII Collaboration show that $Z_c(3900)$ has a much more stronger coupling to DD^* than to $J/\psi\pi$, we have concluded that the tetraquark-type current for the $Z_c(3900)$ is in disaccord with experiment. As an alternative we have employed a molecular-type four-quark current to describe the $Z_c(3900)$ state. In this case we found that for a relatively large model size parameter of $\Lambda_{Z_c} \sim 3.3$ GeV one can obtain partial widths for the decays $Z_c(3900) \to \bar{D}D^*$ that are close to $\sim$15 MeV for each mode. At the same time the partial widths for the decays $Z_c(3900) \to J/\psi\pi, \eta_c\rho$ are suppressed by a factor of 6–7 in accordance with experimental data.

Finally, we have tested a tetraquark picture for the $X(5568)$ and $Z(4430)$ structure by analyzing their strong one-pion decay. In the analysis of the $B_s\pi$ decay mode of the $X(5568)$, we found that one can fit the experimental decay width using a mass of 5568 MeV by taking the value of the parameter to be $\Lambda_{X_b} \sim 1.4$ GeV. In the case of a larger mass 5771 MeV one finds $\Lambda_{X_b} \sim 1.7$ GeV. In the case of the $Z(4430)$ state we considered the modes with J/ψ, and its first radial excitation $\psi(2s)$. We showed that the decay width of the $Z(4430) \to \psi(2s) + \pi$ process dominates over the one of $Z(4430) \to J/\psi + \pi$ by a factor $R_Z = (4.36 \pm 0.28)$ and the sum of the two decay rates of the $Z(4430)$ satisfies the upper limit for the total width of the $Z(4430)$ if the size parameter $\Lambda_{Z(4430)} \geq 2.2$ GeV. It means that the $Z(4430)$ state is a good candidate for the compact tetraquark state. Our prediction for the $Z(4430)^+ \to D^{*+} + \bar{D}^{*0}$ decay width is $\Gamma(Z(4430)^+ \to D^{*+} + \bar{D}^{*0}) = 23.5 \pm 15.6$ MeV.

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APPENDIX: SPIN KINEMATICS FOR THE DECAY $1^+ \to 1^- + 0^-$

The matrix element

$$M = \langle 1^-(q_1;\rho), 0^-(q_2)|T|1^+(p;\mu)\rangle \tag{A1}$$

can be described by the three sets of amplitudes: (i) invariant amplitudes, (ii) helicity amplitudes, and (iii) (LS) amplitudes. In this Appendix we derive the relations between the three sets of amplitudes.

The product of the parities of the two final state mesons is (+1), which matches the parity of the initial state. Thus the two final state mesons must have even relative orbital momenta. In the present case these are $L = 0$, 2. The spins s_1 and s_2 of the two final state mesons couple to the total spin $S = 1$. Thus one has the two (LS) amplitudes

$$A_{01}, \qquad A_{21}. \tag{A2}$$

There are two covariants $\mathcal{K}_1^{\mu\rho} = m g^{\mu\rho}$ and $\mathcal{K}_2^{\mu\rho} = \frac{1}{m} q_1^\mu q_2^\rho$ that describe the matrix element. These define the invariant amplitudes A_1 and A_2 according to

$$M = (A_1\mathcal{K}_1^{\mu\rho} + A_2\mathcal{K}_2^{\mu\rho})\varepsilon_\mu \varepsilon^*_{1\rho}. \tag{A3}$$

There are two independent helicity amplitudes $H_{\lambda\lambda_1}$ $(\lambda = \lambda_1)$,

$$H_{+1+1}, \qquad H_{00}. \tag{A4}$$

From parity one has $H_{-1-1} = H_{+1+1}$. In order to relate the helicity amplitudes to the invariant amplitudes we work in the rest system of the decay meson and define the z-direction to be along the momentum direction of meson 1. The helicity amplitudes can be related to the invariant amplitudes using the momenta and polarization vectors $\epsilon_1^\rho(\pm) = (0; \mp 1, -i, 0)/\sqrt{2}, \epsilon_1^\rho(0) = (|\mathbf{q_1}|; 0, 0, E_1)/m_1, \epsilon^\mu(\pm) = (0; \mp 1, -i, 0)/\sqrt{2}, \epsilon^\mu(0) = (0; 0, 0, 1), q_1^\mu = (E_1; 0, 0, |\mathbf{q_1}|), q_2^\mu = (E_2; 0, 0, -|\mathbf{q_1}|)$. One can then express the helicity amplitudes in terms of the invariant amplitudes. The relations can be calculated to be

$$\begin{aligned} H_{00} &= -\frac{m}{m_1}E_1 A_1 - \frac{1}{m_1}|\mathbf{q_1}|^2 A_2, \\ H_{+1+1} &= H_{-1-1} = -mA_1, \end{aligned} \tag{A5}$$

where the magnitude of the final state three-momentum is given by $|\mathbf{q_1}| = \sqrt{Q_+Q_-}/2m$ with $Q_\pm = m^2 - (m_1 \pm m_2)^2 = 2(q_1q_2 \mp m_1m_2)$.

The coefficients of the matrix relating the (LS) and helicity amplitudes can be calculated from the product of two C.G. coefficients according to [44]

$$\begin{aligned} &\langle JM; LS|JM; \lambda_1\lambda_2\rangle \\ &\quad = \left(\frac{2L+1}{2J+1}\right)^{1/2} \langle LS; 0\mu|J\mu\rangle\langle s_1s_2; \lambda_1, -\lambda_2|S\mu\rangle, \end{aligned} \tag{A6}$$

where $\mu = \lambda_1 - \lambda_2$. One obtains

$$\begin{pmatrix} A_{01} \\ A_{21} \end{pmatrix} = \sqrt{\frac{1}{3}}\begin{pmatrix} 2 & 1 \\ \sqrt{2} & -\sqrt{2} \end{pmatrix}\begin{pmatrix} H_{+1+1} \\ H_{00} \end{pmatrix}. \tag{A7}$$

We can thus relate the (LS) amplitudes to the invariant amplitudes A_i. The relations read

$$\begin{aligned} A_{01} &= -\sqrt{\frac{1}{3}}\frac{1}{m_1}(m(2m_1 + E_1)A_1 + |\mathbf{q_1}|^2 A_2), \\ A_{21} &= \sqrt{\frac{2}{3}}\frac{1}{m_1}(m(E_1 - m_1)A_1 + |\mathbf{q_1}|^2 A_2). \end{aligned} \tag{A8}$$

The (LS) amplitude A_{21} can be seen to have the correct D-wave threshold behavior proportional to $|\vec{q}_1|^2$ by taking the relation $(E_1 - m_1) = |\mathbf{q_1}|^2/(E_1 + m_1)$ into account.

The rate for the decay process $1^+(p) \to 1^-(q_1) + 0^-(q_2)$ is given by

$$\begin{aligned} \Gamma &= \frac{1}{8\pi}\frac{1}{2s+1}\frac{|\mathbf{q_1}|}{m^2}(|H_{+1+1}|^2 + |H_{-1-1}|^2 + |H_{00}|^2) \\ &= \frac{1}{8\pi}\frac{1}{2s+1}\frac{|\mathbf{q_1}|}{m^2}(|A_{01}|^2 + |A_{21}|^2), \end{aligned} \tag{A9}$$

where $2s + 1 = 3$.

We assume that the (1^-) meson decays into two pseudoscalar mesons as in the cascade decay $Z_c \to D + D^*(\to D + \pi)$. We treat the cascade decay in the narrow width approximation. The differential decay distribution for the cascade decay is given by

$$\begin{aligned} &\frac{d\Gamma(Z_c \to D + D^*(\to D + \pi))}{d\cos\theta} \\ &\quad = B(D^* \to D + \pi)\frac{1}{24\pi}\frac{|\mathbf{q_1}|}{m^2} \\ &\qquad \times \left(\frac{3}{8}(1 + \cos^2\theta)\mathcal{H}_T + \frac{3}{4}\sin^2\theta\mathcal{H}_L\right), \end{aligned} \tag{A10}$$

where $\mathcal{H}_T = |H_{+1+1}|^2 + |H_{-1-1}|^2$, $\mathcal{H}_L = |H_{00}|^2$. For the cascade decay $Z_c \to \pi + J/\psi(\to \ell^+\ell^-)$ we again have

$$\begin{aligned} &\frac{d\Gamma(Z_c \to \pi + J/\psi(\to \ell^+\ell^-))}{d\cos\theta} \\ &\quad = B(J/\psi \to \ell^+\ell^-)\frac{1}{24\pi}\frac{|\mathbf{q_1}|}{m^2} \\ &\qquad \times \left(\frac{3}{8}(1 + \cos^2\theta)\mathcal{H}_T + \frac{3}{4}\sin^2\theta\mathcal{H}_L\right). \end{aligned} \tag{A11}$$

In the latter cascade decay we have set $m_\ell = 0$.

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