Form factor $A_0(q^2)$, nonleptonic $D(B) \to PV$ transitions, and rare $B \to K^*\gamma$ decays

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(Received 8 September 1994)

We use three-point function QCD sum rules to calculate the form factor $A_0(q^2)$ appearing in the matrix element of the flavor-changing axial vector current between the $D(B)$ state and a vector meson state. We describe the role of this form factor in nonleptonic $D(B) \to PV$ decays and analyze the light $SU(3)_F$-symmetry-breaking effects. We also discuss a proposal to relate the branching ratio of $B \to K^*\gamma$ to the spectrum of the semileptonic $B \to \rho\ell\nu$ decay.

PACS number(s): 11.55.Hx, 13.25.Ft, 13.25.Hw, 13.40.Hq

I. INTRODUCTION

The theoretical description of weak exclusive nonleptonic decays of heavy mesons is carried out, in the factorization approximation, in three steps. First, an effective Hamiltonian is constructed taking into account the effects of hard gluon exchanges [1]. Then, the hadronic matrix elements are factorized into the product of current-particle matrix elements that can be either inferred from experiment or calculated theoretically (notice that we only consider two body decays) [2,3]. Finally, strong rescattering effects (mainly for D meson decays) are included considering the coupling to intermediate resonances [4] or using the measured phase shifts [5].

The second step is dictated by our inability in reliably computing matrix elements of the effective Hamiltonian between the external states involved in the decay. Let us consider, for example, the transition $D^+(p) \to \overline{K}^{*0}(p')\pi^+(q)$, which is governed by the effective Hamiltonian

$$\mathcal{H}_W = -\frac{G_F}{\sqrt{2}} V_{ud}^* V_{cs} \left[\frac{c_+ + c_-}{2}\, O_1 + \frac{c_+ - c_-}{2}\, O_2 \right] \tag{1.1}$$

where G_F is the Fermi constant, V_{hk} are Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the local operators O_1 and O_2 are given by

$$O_1 = \bar{s}\gamma_\mu(1-\gamma_5)c\, \bar{u}\gamma^\mu(1-\gamma_5)d\ , \tag{1.2}$$
$$O_2 = \bar{s}\gamma_\mu(1-\gamma_5)d\, \bar{u}\gamma^\mu(1-\gamma_5)c\ . \tag{1.3}$$

The Wilson coefficients $c_+(\mu)$ and $c_-(\mu)$ account for the hard gluon effects in the renormalization of $\mathcal{H}_W$ from m_W to the low energy scale μ; they are known at the next-to-leading order approximation [6]. The amplitude $\mathcal{A}(D^+ \to \overline{K}^{*0}\pi^+)$ is written, in the factorization approximation, as

$$\begin{aligned}\mathcal{A}(D^+ \to \overline{K}^{*0}\pi^+)_{\rm fact} &= -\frac{G_F}{\sqrt{2}} V_{ud}^* V_{cs} \langle \overline{K}^{*0}\pi^+|\, [c_1 O_1 + c_2 O_2]\, |D^+\rangle_{\rm fact} \\ &\equiv -\frac{G_F}{\sqrt{2}} V_{ud}^* V_{cs} \Big\{ \left(c_1 + \frac{c_2}{N_c}\right) \langle \overline{K}^{*0}|\bar{s}\gamma_\mu(1-\gamma_5)c|D^+\rangle \langle \pi^+|\bar{u}\gamma^\mu(1-\gamma_5)d|0\rangle \\ &\quad + \left(c_2 + \frac{c_1}{N_c}\right) \langle \pi^+|\bar{u}\gamma^\mu(1-\gamma_5)c|D^+\rangle \langle \overline{K}^{*0}|\bar{s}\gamma_\mu(1-\gamma_5)d|0\rangle \Big\}\ ,\end{aligned} \tag{1.4}$$

where $c_1 = (c_+ + c_-)/2$, $c_2 = (c_+ - c_-)/2$. To obtain (1.4) we have used the identities

$$\begin{aligned} O_1 &= \frac{1}{N_c} O_2 + \frac{1}{2} \left[\bar{s}\gamma_\mu(1-\gamma_5)\lambda^a d\right] \left[\bar{u}\gamma^\mu(1-\gamma_5)\lambda^a c\right]\ , \\ O_2 &= \frac{1}{N_c} O_1 + \frac{1}{2} \left[\bar{s}\gamma_\mu(1-\gamma_5)\lambda^a c\right] \left[\bar{u}\gamma^\mu(1-\gamma_5)\lambda^a d\right]\ , \end{aligned} \tag{1.5}$$

where N_c is the number of colors and λ^a are the Gell-Mann $SU(3)_c$ matrices. Therefore, the amplitude is expressed in terms of current-particle matrix elements

$$\langle \overline{K}^{*0}(p',\lambda)|\bar{s}\gamma_\mu d|0\rangle = f_{K^*} m_{K^*} \epsilon^*_\mu(\lambda) \ , \tag{1.6}$$

$$\langle \pi^+(q)|\bar{u}\gamma_\mu\gamma_5 d|0\rangle = -i f_\pi q_\mu \ . \tag{1.7}$$

[$\epsilon(\lambda)$ is the K^* polarization vector] and of the matrix elements governing the semileptonic transition $D \to K^*(\pi)\ell\nu$:

$$\begin{aligned}\langle \overline{K}^{*0}(p',\lambda)|\bar{s}\gamma_\mu(1-\gamma_5)c|D^+(p)\rangle = \Big\{ & \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho p'^\sigma \frac{2V(q^2)}{m_D+m_{K^*}} \\ & -i\Big[(m_D+m_{K^*})A_1(q^2)\epsilon^*_\mu - \frac{A_2(q^2)}{m_D+m_{K^*}}(\epsilon^*\cdot p)(p+p')_\mu \\ & -(\epsilon^*\cdot p)\frac{2m_{K^*}}{q^2}q_\mu\left(A_3(q^2)-A_0(q^2)\right)\Big]\Big\} \ ,\end{aligned} \tag{1.8}$$

$$\langle \pi^+(q)|\bar{u}\gamma_\mu c|D^+(p)\rangle = (p+q)_\mu F_1(p'^2) + \frac{m_D^2-m_\pi^2}{p'^2}[F_0(p'^2)-F_1(p'^2)]p'_\mu \tag{1.9}$$

($q = p - p'$). Notice that we have used the Bauer-Stech-Wirbel (BSW) [7] parametrization of the form factors; in this parametrization A_1, A_2, and A_3 are not independent, but they satisfy the relation

$$A_3(q^2) = \frac{m_D+m_{K^*}}{2m_{K^*}}A_1(q^2) - \frac{m_D-m_{K^*}}{2m_{K^*}}A_2(q^2) \ , \tag{1.10}$$

with

$$A_3(0) = A_0(0) \tag{1.11}$$

in order to avoid the unphysical singularity at $q^2 = 0$ in Eq. (1.8); for the same reason $F_0(0) = F_1(0)$.

The amplitude of the process we are considering is therefore reduced to

$$\mathcal{A}(D^+ \to \overline{K}^{*0}\pi^+)_{\rm fact} = -\frac{G_F}{\sqrt{2}}V^*_{ud}V_{cs}2m_{K^*}(\epsilon^*\cdot p)\Big\{\left(c_1+\frac{c_2}{N_c}\right)f_\pi A_0(m_\pi^2) + \left(c_2+\frac{c_1}{N_c}\right)f_{K^*}F_1(m_{K^*}^2)\Big\} \ . \tag{1.12}$$

The leptonic constants f_π and f_{K^*} can be derived from experiment. As for the Wilson coefficients, it has been observed [3,8] that consistently neglecting all $1/N_c$ terms in the factorized amplitudes such as Eq. (1.12), and using the form factors, e.g., from the BSW model [7], the overall agreement with the available experimental data seems to improve. This is an appealing feature of this approach, since factorization becomes exact in the multicolor chromodynamics, in the limit $N_c \to \infty$ [9]. Dynamical justifications of the rule of discarding $1/N_c$ contributions have been investigated by analyzing the sign and the size of nonfactorizable matrix elements in some D and B meson nonleptonic decays [10,11].

However, in order to have a quantitative overview of the validity of the factorization approach we need a careful determination, either from experiment or by a QCD calculation, of the form factors appearing in the factorized amplitudes such as (1.12). This is an important preliminary study in the analysis of the decay channels where naive factorization seems to fail [12].

Early estimates of F_1, V, A_1, and A_2, evaluated at $q^2 = 0$ or at $q^2 = q^2_{\rm max}$, are available from constituent quark models [7,13,14]. More recently, these form factors have been computed, at a fixed value of q^2, by QCD sum rules [15–17] and by lattice QCD [18]; they have also been derived in the framework of models that incorporate heavy quark and chiral symmetries [19]. Moreover, QCD sum rules have been used to determine the q^2 dependence of F_1, V, A_1, A_2, and F_0 [15–17,20,21]; data on this functional dependence are also available from lattice QCD, but with large error bars [18].

As for $A_0(q^2)$, the procedure adopted so far consists in using Eqs. (1.10) and (1.11) to get $A_0(0)$ from $A_1(0)$ and $A_2(0)$, and then in assuming a suitable dependence on q^2, invoking the dominance of the nearest pole in the $t = q^2$ channel.

This procedure has two difficulties. The first one is that $A_1(0)$ and $A_2(0)$ are predicted by QCD sum rules and lattice QCD in a range of values; the uncertainty (larger for A_2) is determined by the variation of the parameters employed in the calculation (which are not accurately known) and by the statistical error in lattice simulations. This implies that $A_0(0)$ is determined with a large error (within an order of magnitude for the transition $B \to \rho$), an uncertainty that heavily affects the analysis of the accuracy of the factorization approach. This error is even more important in the study of light $SU(3)_F$-breaking effects, an argument which has recently prompted a number of interesting investigations [22].

The second point concerns the functional dependence of A_0 on q^2, which is needed in computing nonleptonic transitions. The assumption that, also for low values of q^2, the form factor A_0 is dominated by the nearest resonance requires an explicit check, since it is known that, for the transitions $B, D \to \rho, K^*$, the form factors A_1 and A_2 obtained by QCD sum rules appear to be nearly independent of q^2 up to rather large values of the squared momentum transferred [15,16,21].

Both these difficulties can be avoided by a direct calculation of $A_0(q^2)$. This is the problem we address in the present paper: we compute $A_0(q^2)$ for the transitions $B \to \rho$, $D \to \rho, K^*$, and $B \to D^*$ using three-point function QCD sum rules. In Sec. II we derive the sum rule for $A_0(q^2)$, and in Sec. III we collect our numerical results, together with a comparison with the outcome of other calculations.

The last point we analyze in this note is a proposal, put forward in Ref. [23–25], to relate the branching ratio of the rare transition $B \to K^*\gamma$ to the spectrum of the semileptonic decay $B \to \rho\ell\nu$. This connection could have interesting phenomenological consequences, e.g., for the measurement of V_{ub} ; it is based on the equality, obtained in the infinite heavy quark mass limit, $T_1(0) = A_0^{B\to\rho}(0)$, where T_1 is the form factor governing $B \to K^*\gamma$. We shall discuss the validity of this relation and show that there is a sizable deviation to be taken into account.

II. THE FORM FACTOR A_0 FROM THREE-POINT FUNCTION QCD SUM RULES

In order to calculate the form factor $A_0(q^2)$ in Eq. (1.8) using three-point sum rules, let us consider the correlator [26,27]

$$T_{\mu\nu}^{D\to K^*}(p,p',q) = i^2 \int dx\, dy\, e^{i(p'\cdot x - p\cdot y)} \times \langle 0|T\{j_\nu(x) A_\mu(0) j_5^\dagger(y)\}|0\rangle \ . \quad (2.1)$$

Here the weak current $A_\mu = \bar{s}\gamma_\mu\gamma_5 c$ represents the flavor-changing axial vector current in (1.8), while the quark currents $j_\nu = \bar{d}\gamma_\nu s$ and $j_5 = \bar{d}i\gamma_5 c$ interpolate K^* and D mesons, respectively, and have a nonvanishing matrix element between the vacuum and K^* and D states: Eq. (1.6) and

$$\langle 0|j_5|D(p)\rangle = f_D \frac{m_D^2}{m_c} \ , \quad (2.2)$$

where m_c is the mass of the charm quark (hereafter we put the masses of the light u and d quarks to zero). The product $q^\mu T_{\mu\nu}$ can be decomposed in two independent Lorentz structures:

$$q^\mu\, T_{\mu\nu} = -i \left[P_\nu\, T_1(p^2, p'^2, q^2) + q_\nu\, T_2(p^2, p'^2, q^2) \right] , \quad (2.3)$$

with $P_\nu = (p+p')_\nu$, and the two scalar functions T_i ($i = 1, 2$) can be represented by a double dispersion relation

$$T_i(p^2, p'^2, q^2) = \frac{1}{(2\pi)^2} \int ds\, ds' \frac{\rho_i^{(\mathrm{had})}(s, s', q^2)}{(s-p^2)(s'-p'^2)} + \text{subtractions} \ , \quad (2.4)$$

with the spectral functions $\rho_i^{(\mathrm{had})}$ expressed in terms of physical states:

$$\rho_i^{(\mathrm{had})} = \rho_i^{(\mathrm{res})} + \rho_i^{(\mathrm{hr})} \ . \quad (2.5)$$

In (2.5) $\rho_i^{(\mathrm{res})}$ represent the contribution of the lowest lying resonances (in the zero width approximation) and are expressed in terms of the form factor $A_0^{D\to K^*}$ we are interested in:

$$\rho_1^{(\mathrm{res})}(s, s', q^2) = (2\pi)^2 f_{K^*} f_D \frac{m_D^2}{2m_c}(m_{K^*}^2 - m_D^2 + q^2) \times A_0^{D\to K^*}(q^2)\delta(s - m_D^2)\delta(s' - m_{K^*}^2) \ , \quad (2.6)$$

$$\rho_2^{(\mathrm{res})}(s, s', q^2) = (2\pi)^2 f_{K^*} f_D \frac{m_D^2}{2m_c}(3m_{K^*}^2 + m_D^2 - q^2) \times A_0^{D\to K^*}(q^2)\delta(s - m_D^2)\delta(s' - m_{K^*}^2) \ , \quad (2.7)$$

while $\rho_i^{(\mathrm{hr})}$ take contributions from higher states and from the hadronic continuum.

The correlator (2.1) can also be calculated in QCD by the operator product expansion (OPE) in the region of large Euclidean momenta p^2, p'^2. In this expansion the most singular term is represented by the perturbative term, which can be obtained evaluating a triangle diagram. It has a dispersive representation similar to (2.4):

$$T_i^{(\mathrm{pert})}(p^2, p'^2, q^2) = \frac{1}{(2\pi)^2} \int ds\, ds' \frac{\rho_i^{(\mathrm{pert})}(s, s', q^2)}{(s-p^2)(s'-p'^2)} \ ; \quad (2.8)$$

the explicit form of the spectral functions $\rho_i^{(\mathrm{pert})}$ can be found in the Appendix. The next terms in the OPE can be read as

$$T_i^{(\mathrm{np})} = \sum_n d_i^{(n)}(p^2, p'^2, q^2)\langle 0|O_n|0\rangle \quad (2.9)$$

where the local operators O_n, written in terms of quark and gluon fields, are ordered according to their increasing dimension; their vacuum matrix elements (condensates) parametrize the effects of the nonperturbative QCD vacuum. Here we only consider the contribution of the operators of dimension $D = 3$ and $D = 5$:

$$T_i^{(\mathrm{np})} = d_i^{(3)}\langle\bar{q}q\rangle + d_i^{(5)}\langle\bar{q}g_s\sigma G q\rangle \ ; \quad (2.10)$$

$\langle\bar{q}q\rangle$ and $\langle\bar{q}g_s\sigma G q\rangle$ are the quark and the mixed quark-gluon condensates, respectively; the coefficients $d_i^{(3)}$ and $d_i^{(5)}$ can be calculated in perturbation theory.

The hadronic and the QCD representations of the correlator (2.1) can be used to derive two sum rules for $A_0(q^2)$. As a matter of fact, we invoke quark-hadron duality and assume the equality:

$$\int_{D'} ds\, ds' \frac{\rho_i^{(\mathrm{hr})}(s,s',q^2)}{(s-p^2)(s'-p'^2)}$$

$$= \int_{D'} ds\, ds' \frac{\rho_i^{(\mathrm{pert})}(s,s',q^2)}{(s-p^2)(s'-p'^2)} \quad (2.11)$$

in a region D' above some threshold s_0, s'_0. This assumption can be realized if we adopt the model

$$\rho_i^{(\mathrm{hr})}(s,s',q^2) = \rho_i^{(\mathrm{pert})}(s,s',q^2)[1-\Theta(s_0-s)\Theta(s'_0-s')]. \quad (2.12)$$

Using (2.12) the following sum rules for $A_0(q^2)$ can be written:

$$\frac{1}{(2\pi)^2}\int_D ds\, ds' \frac{\rho_i^{(\mathrm{res})}(s,s',q^2)}{(s-p^2)(s'-p'^2)} = \frac{1}{(2\pi)^2}\int_D ds\, ds' \frac{\rho_i^{(\mathrm{pert})}(s,s',q^2)}{(s-p^2)(s'-p'^2)} + d_i^{(3)}\langle\bar{q}q\rangle + d_i^{(5)}\langle\bar{q}g_s\sigma G q\rangle \quad (2.13)$$

where the region D is fixed by

$$m_c^2 \le s \le s_0 \quad (2.14)$$

and

$$m_s^2 \le s' \le s'_0\ , \qquad s'_-(s,q^2) \le s' \le s'_+(s,q^2)\ , \quad (2.15)$$

with (m_s is the strange quark mass)

$$s'_\pm(s,q^2) = \frac{1}{2}\left\{(s-m_c^2)\left(1-\frac{q^2}{m_c^2}\right) + \left(\frac{m_s}{m_c}\right)^2 (s+m_c^2) \pm (s-m_c^2)\left[\left(\frac{m_s}{m_c}\right)^4 + \left(1-\frac{q^2}{m_c^2}\right)^2 - 2\left(\frac{2m_s^2}{m_c^2}\right)\left(1+\frac{q^2}{m_c^2}\right)\right]^{1/2}\right\}. \quad (2.16)$$

In (2.13) we have omitted the subtraction terms, that can still be present. They are removed by performing a double Borel transform to both sides of (2.13) in the variables $P^2=-p^2$ and $P'^2=-p'^2$:

$$B_{P^2,P'^2}(M^2,M'^2) = \left[\frac{1}{n!}(-P^2)^{n+1}\left(\frac{d}{dP^2}\right)^{n+1}\right]\left[\frac{1}{m!}(-P'^2)^{m+1}\left(\frac{d}{dP'^2}\right)^{m+1}\right] \quad (2.17)$$

in the limit $P^2, P'^2 \to \infty$, $n,m\to\infty$, $M^2 = P^2/n$ $M'^2 = P'^2/m$ fixed. The resulting sum rules for $A_0^{D\to K^*}$ read

$$A_0^{D\to K^*}(q^2) = \frac{H_i}{(2\pi)^2}\int_D ds\, ds' \rho_i^{(\mathrm{pert})}(s,s',q^2)\ \exp\left(-\frac{(s-m_D^2)}{M^2} - \frac{(s'-m_{K^*}^2)}{M'^2}\right) + \left[\Gamma_i^{D=3}\langle\bar{q}q\rangle + \Gamma_i^{D=5}\langle q g_s \sigma^{\mu\nu} G^a_{\mu\nu}\frac{\lambda^a}{2}q\rangle\right] \exp\left(\frac{m_D^2-m_c^2}{M^2} + \frac{m_{K^*}^2-m_s^2}{M'^2}\right) \quad (2.18)$$

($i=1,2$); the coefficients H_i, $\Gamma_i^{D=3}$, and $\Gamma_i^{D=5}$ are collected in the Appendix.

Equation (2.18) represents two independent sum rules for $A_0^{D\to K^*}(q^2)$, that will be analyzed in the next section. Now, before discussing the parameters and the criteria we have used to compute $A_0^{D\to K^*}$, let us consider the t dependence in (2.18). In principle, the operator product expansion can be reliably applied to the evaluation of the correlator (2.1) in the region of large Euclidean values of q^2. However, as discussed in [15], we can also consider positive values of q^2 provided that the occurrence of non-Landau singularities either is avoided or is carefully taken into account. Such singularities remain far from the integration region of the dispersive integral if q^2 is small with respect to the physical threshold in the t channel: $t_{\mathrm{th}} = (m_c+m_s)^2$; their presence is shown up by the appearance of branch points (for large and positive q^2) in the spectral functions $\rho_i^{(\mathrm{pert})}$.

Taking into account the above considerations, we study the form factor A_0 using Eq. (2.18) for small positive values of q^2, and not in the whole physically accessible t region. However, the information we obtain on the t dependence of the form factor is enough to describe a large number of nonleptonic $D(B)$-meson decays.

III. NUMERICAL ANALYSIS OF THE SUM RULES

The numerical analysis of the sum rules (2.18) is performed using the following values for the quark condensates, taken at a low renormalization scale ($\mu \simeq 1$ GeV) [26]:

$$\langle\bar{q}q\rangle = (-230\ \mathrm{MeV})^3\ ,\qquad \left\langle \bar{q}g_s\sigma^{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}q\right\rangle = m_0^2\,\langle\bar{q}q\rangle\ , \quad (3.1)$$

with $m_0^2 = 0.8$ GeV^2. The condensates can be evaluated at higher scales using the leading-log approximation for the anomalous dimension of the quark and of the mixed quark-gluon condensates; however, this rescaling does not affect sensitively the numerical result for A_0.

As for the quark masses, we use $m_c = 1.35$ GeV and $m_s = 0.16$ GeV; moreover, we use $m_b = 4.6$ GeV in the calculation of form factor connected with the transition $B \to \rho$. The leptonic constant f_{K^*} can be derived from the measurement of the branching ratio $\tau \to K^*\nu_\tau$: $f_{K^*} = 0.22 \pm 0.01$ GeV; we use $f_D = 195 \pm 20$ MeV for the leptonic constant of the D meson.

A comment on this value of f_D is in order. This value comes from two-point QCD sum rules, including radiative corrections at the order $O(\alpha_s)$ [28,29]; these corrections are at level of 13–15 % for f_D, f_{D^*}, and play a major role in f_B; including the corrections the results are in agreement with lattice QCD. On the other hand, the QCD expression of the three-point correlator has been computed at zero order in α_s. Several authors [15], when computing the form factors, adopt the strategy of taking the ratio of three- and two-point functions, calculated at the same order in α_s in order to be consistent, trying to reduce the uncertainty deriving from higher order contributions. However, the sign of the radiative corrections to three-point functions is not known on general grounds; for this reason our attitude is to use the best known values of the leptonic constants, e.g., the values on which QCD sum rules and lattice QCD agree.

The effective threshold s_0 and s_0' must be chosen in a range of values between the mass squared of the lowest lying resonances and the first excited states. They have been fixed by studying two-point sum rules, for the calculation of static properties as the mass of the leptonic constant of the particles, and three-point functions sum rules in the calculation of the semileptonic $D \to K^*\ell\nu$ decay [15]: s_0 =7–8 GeV^2 for the channel of D meson, and s_0' =1.5–1.7 GeV^2 for the channel of K^*. The variation of the values of s_0, s_0' induces a variation of the predicted form factor A_0 that can be considered a theoretical uncertainty in the final result.

Using the parameters above, $A_0^{D\to K^*}(q^2)$ can be obtained from (2.18) as a function of the Borel parameters M^2 and M'^2. However, since these variables are unphysical, we search a region in M^2, M'^2 where A_0 does not depend on them (stability plateau). In this region other conditions must be verified. The first constraint consists in checking a hierarchical structure of the various terms of (2.18):

$$T^{(\text{pert})} \geq T^{(D=3)} \geq T^{(D=5)} \; ; \qquad (3.2)$$

only if the OPE displays this structure can we hope that higher order power corrections in (2.18) can be safely neglected.

Another condition is connected with the approximation (2.11). Since we are not guaranteed that quark-hadron duality starts already in correspondence to the first hadronic excitations, we must choose M^2, M'^2 small enough to enhance the contribution of the lowest lying states in the sum rule, and to suppress exponentially the contribution of the continuum. We check this condition by verifying that

$$\left| \int_D ds\, ds' \rho_i^{(\text{pert})}(s,s',q^2)\, e^{-\frac{(s-m_D^2)}{M^2} - \frac{(s'-m_{K^*}^2)}{M'^2}} \right| \geq \alpha \left| \int_{D_1} ds\, ds' \rho_i^{(\text{pert})}(s,s',q^2)\, e^{-\frac{(s-m_D^2)}{M^2} - \frac{(s'-m_{K^*}^2)}{M'^2}} \right| , \qquad (3.3)$$

where D_1 is an integration region larger than D (it extends to 10 s_0 and 10 s_0') and α =0.4–0.5. From the condition (3.3) an upper bound to M^2, M'^2 can be determined.

In principle both the sum rules ($i = 1,2$) in (2.18) can be used to calculate A_0. However, it turns out that only for the first sum rule ($i = 1$) the condition (3.3) is fulfilled. In the second sum rule the constraint (3.3) is never verified as a consequence, probably, of the major role played by higher states. For this reason we exclude the second sum rule in our analysis and use only the first one to determine A_0.

Let us consider the form factor $A_0^{D\to K^*}(q^2)$. Our results, obtained for $M^2 = 3$ GeV^2 and $M'^2 = 1.5$ GeV^2, are depicted in Fig. 1 for several sets of parameters. At $q^2 = 0$ we have $A_0^{D\to K^*}(0) = 0.58 \pm 0.05$, where the uncertainty comes from the variation of the parameters. For positive values of q^2 we observe an increasing of the form factor; if we use the simple pole formula:

$$A_0^{D\to K^*}(q^2) = \frac{A_0^{D\to K^*}(0)}{1 - q^2/m_P^2} \qquad (3.4)$$

to fit the q^2 dependence, we obtain that the fitted pole mass is: $m_P = 1.65$ GeV, which is not far from the ex-

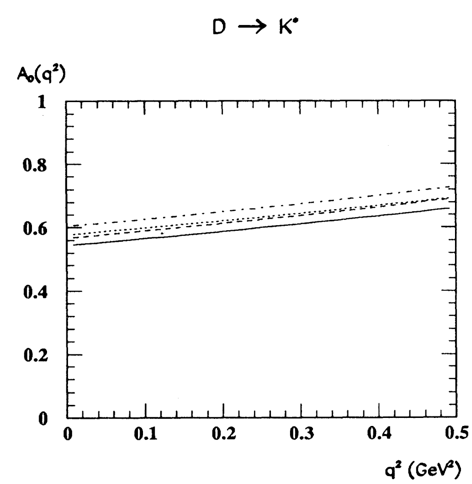

FIG. 1. The form factor $A_0^{D\to K^*}(q^2)$. The curves refer to different sets of parameters: $s_0 = 7$ GeV^2 and $s_0' = 1.5$ GeV^2 (continuous line), $s_0 = 7$ GeV^2 and $s_0' = 1.7$ GeV^2 (dashed line), $s_0 = 8$ GeV^2 and $s_0' = 1.5$ GeV^2 (dotted line), $s_0 = 8$ GeV^2 and $s_0' = 1.7$ GeV^2 (dashed-dotted line). $M^2 = 3$ GeV^2, $M'^2 = 1.5$ GeV^2.

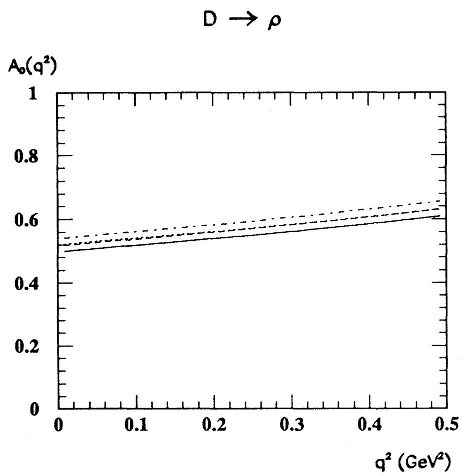

FIG. 2. The form factor $A_0^{D\to\rho}(q^2)$. The curves refer to the sets of parameters: $s_0 = 7$ GeV^2 and $s_0' = 1.3$ GeV^2 (continuous line), $s_0 = 7$ GeV^2 and $s_0' = 1.5$ GeV^2 (dashed line), $s_0 = 8$ GeV^2 and $s_0' = 1.3$ GeV^2 (dotted line), $s_0 = 8$ GeV^2 and $s_0' = 1.5$ GeV^2 (dashed-dotted line). $M^2 = 3$ GeV^2, $M'^2 = 1.5$ GeV^2.

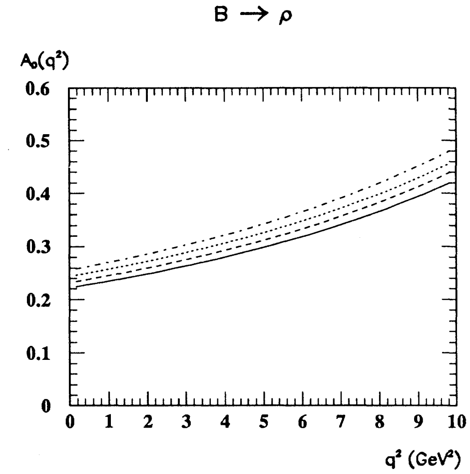

FIG. 3. The form factor $A_0^{B\to\rho}(q^2)$. The curves refer to the sets of parameters: $s_0 = 33$ GeV^2 and $s_0' = 1.3$ GeV^2 (continuous line), $s_0 = 33$ GeV^2 and $s_0' = 1.5$ GeV^2 (dashed line), $s_0 = 36$ GeV^2 and $s_0' = 1.3$ GeV^2 (dotted line), $s_0 = 36$ GeV^2 and $s_0' = 1.5$ GeV^2 (dashed-dotted line). The values of the Borel parameters are: $M^2 = 8$ GeV^2, $M'^2 = 2$ GeV^2.

perimental mass of the particle having the same quantum numbers of the pole: $m_{D_s} = 1.969$ GeV.

Let us now consider the transition $D \to \rho$. The constant f_{ρ^+}, computed from the decay $\rho^0 \to e^+e^-$ and assuming the isospin symmetry, is given by: $f_{\rho^+} = 216$ MeV. Using s_0' =1.3–1.5 GeV^2 we get: $A_0^{D\to\rho}(0) = 0.52 \pm 0.05$. The fitted pole mass is $m_P = 1.6$ GeV, to be compared with $m_D = 1.865$ GeV which is the mass of the nearest pole in the t channel (Fig. 2).

From the above results we would get: $r = A_0^{D\to K^*}(0)/A_0^{D\to\rho}(0) = 1.12 \pm 0.11$; however, the light SU(3)$_F$-breaking effects can be better estimated by studying the ratio of the sum rules for the transitions $D \to K^*$ and $D \to \rho$, which is stable with respect to the variation of the input parameters. We get $r = 1.10 \pm 0.05$, i.e., an SU(3)$_F$-breaking effect at the level of 10%.

We have also computed the form factor $A_0^{B\to\rho}$ appearing in nonleptonic B meson decays which are interesting due to their connection to the measurement of V_{ub}; using s_0 =33–36 GeV^2 and $f_B = 180 \pm 20$ MeV as obtained by two-point function QCD sum rules, we get $A_0^{B\to\rho}(0) = 0.24 \pm 0.02$ and a pole mass $m_P = 5$ GeV, to be compared to $m_B = 5.275$ GeV; the form factor is depicted in Fig. 3 in the range q^2 =0–10 GeV^2.

Finally, for the heavy-to-heavy meson transition $B \to D^*$ we get (using $f_{D^*} = 250$ MeV [28]): $A_0^{B\to D^*}(0) = 0.65 \pm 0.05$.

Let us now compare our results with existing predictions for $A_0(0)$, obtained from the values of $A_1(0)$ and $A_2(0)$ in the corresponding channels (see Table I). The first observation is that our predictions appear to be smaller than from potential models [7,13,14] (the exception is $A_0^{B\to\rho}$ in [14]). On the other hand, the SU(3)-breaking effects are of the same size as in our approach.

The comparison with QCD sum rules predictions [15,16] shows the problem we have mentioned in the introduction: the uncertainties in the results obtained using $A_1(0)$ and $A_2(0)$ are so large that they obscure the real value of $A_0(0)$, whereas the error in our predictions is at the level of 10%, and therefore a substantial improvement has been obtained.

It is interesting to compare our predictions to the outcome of [19] obtained in a framework based on heavy-quark and chiral symmetries, using as an input the experimental data on $D \to K^*\ell\nu$. Although there is a remarkable agreement on $D \to K^*$ and $B \to \rho$, the light SU(3)$_F$-breaking corrections connecting $D \to K^*$ to $D \to \rho$ are of similar size than in our approach but with opposite sign.

The last point we would like to mention is that, in our calculation, the form factors A_0 in the different channels increase for positive values of q^2, and that their functional dependence is compatible with the simple pole behavior dominated by the nearest resonance in the t channel.

TABLE I. The form factor $A_0(0)$ of various transitions in different models.

Transition	[7]	[13]	[14]	[16]	[18]	[19]	This paper
$D \to K^*$	0.74	0.91	0.8	0.45 ± 0.30	0.77 ± 0.29	0.59	0.58 ± 0.05
$D \to \rho$	0.68	–	0.85	0.57 ± 0.40	–	0.74	0.52 ± 0.05
$B \to \rho$	0.28	–	0.14	0.79 ± 0.80	-0.57 ± 0.65	0.24	0.24 ± 0.02

IV. $B \to K^*\gamma$ VERSUS $B \to \rho\ell\nu$

In this section we discuss a procedure, proposed in Refs. [23,24], to relate the width of the radiative $B \to K^*\gamma$ transition to the spectrum of the Cabibbo suppressed semileptonic decay $B \to \rho\ell\nu$. This relation can be used to constrain the electroweak parameters involved in these processes and to reduce their dependence on the models for the hadronic form factors.

The radiative rare B meson decays, like $B \to K_i^*\gamma$ [$K_i^* = K^*(890), K_1(1400)$, etc.], have been extensively studied, from the theoretical standpoint, since they have a peculiar role in the precision tests of the quark sector in the standard model (SM) and probe the effects of new physics beyond the standard model [30]. Within the SM they are induced by the one-loop electromagnetic penguin operator $b \to s\gamma$ (for $m_s \ll m_b$) [31]:

$$\mathcal{H}_{\text{eff}}(b \to s) = \frac{G_F}{\sqrt{2}} C m_b \epsilon^\mu \bar{s}\sigma_{\mu\nu}\frac{1+\gamma_5}{2}q^\nu b \ , \tag{4.1}$$

where the constant C contains the dependence on the Cabibbo-Kobayashi-Maskawa matrix elements, on the QCD correcting terms and on the ratio m_{top}/m_W (the explicit formulae can be found in Ref. [31]).

The description of the exclusive decays requires the knowledge of form factors. For example, the relevant matrix element for the transition $B \to K^*(890)\gamma$ can be written as

$$\begin{aligned}\langle K^*(p',\lambda)|\bar{s}\sigma_{\mu\nu}q^\nu\frac{(1+\gamma_5)}{2}b|B(p)\rangle &= i\varepsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho p'^\sigma\, T_1(q^2) \\ &\quad + \left[(m_B^2 - m_{K^*}^2)\epsilon_\mu^* - (\epsilon^*\cdot q)(p+p')_\mu\right] T_2(q^2) \\ &\quad + (\epsilon^*\cdot q)\left[(p-p')_\mu - \frac{q^2}{(m_B^2-m_{K^*}^2)}(p+p')_\mu\right] T_3(q^2) \ . \end{aligned} \tag{4.2}$$

The form factors $T_i(q^2)$ require a nonperturbative evaluation. However, in the static limit for the b quark and using the $\text{SU}(3)_F$ symmetry for the light quarks, a number of interesting relations can be derived between T_i and the form factors V and A_i governing $B \to \rho\ell\nu$ [23,32]. Although such relations are obtained at zero recoil [$q^2_{\text{max}} = (m_B - m_{K^*,\rho})^2$], it has been argued that they continue to hold in the whole physically accessible q^2 range [23,33,34], and therefore that one is able to relate the width of $B \to K^*\gamma$ to observable quantities in $B \to \rho\ell\nu$. An example is represented by the ratio

$$\Gamma(B \to K^*\gamma)\left(\lim_{q^2\to 0,\text{curve}}\frac{1}{q^2}\frac{d\Gamma(B\to\rho e\bar{\nu})}{dE_\rho dE_e}\right)^{-1} = \frac{4\pi^2}{G_F^2}\frac{|\eta|^2}{|V_{ub}|^2}\frac{(m_B^2-m_{K^*}^2)^3}{m_B^4} \tag{4.3}$$

that Burdman and Donoghue [23] predict to be independent of any form factor. In (4.3) *curve* means the region in the Dalitz plot where $q^2 = 4\,E_e\,(m_B - E_\rho - E_e)$; the factor η includes the QCD corrections to the $b \to s\gamma$ decay and the other relevant electroweak parameters [23].

Equation (4.3) is interesting, both from the experimental and the theoretical point of view, since it would provide, for example, a method to measure V_{ub} without referring to calculations of the form factors, from the measurement $B(B \to K^*\gamma) = (4.5 \pm 1.0 \pm 0.9)\times 10^{-5}$ [35] and the spectrum of $B \to \rho\ell\nu$. The drawback, pointed out by O'Donnell and Tung [24], is that the semileptonic decay spectrum vanishes at $q^2 = 0$ on this *curve*, with the consequence that the number of experimental points required to measure the denominator in (4.3) could be insufficient for a reliable determination of V_{ub}.

A more convenient observable [25,36] is represented by the ratio

$$R(B \to K^*\gamma)\left(\left.\frac{d\Gamma(\bar{B}\to\rho l\bar{\nu}_l)}{dq^2}\right|_{q^2=0}\right)^{-1} = \frac{192\pi^3}{G_F^2}\frac{1}{|V_{bu}|^2}\frac{(m_B^2-m_{K^*}^2)^3}{(m_B^2-m_\rho^2)^3}\frac{m_b^3}{(m_b^2-m_s^2)^3}|\mathcal{I}|^2 \ . \tag{4.4}$$

In (4.4) $R(B\to K^*\gamma) = \Gamma(B\to K^*\gamma)/\Gamma(b\to s\gamma)$ is independent of the electroweak parameters appearing in the effective Hamiltonian, and only depends on $T_1(0)$:

$$R(B\to K^*\gamma) = \frac{m_b^3(m_B^2-m_{K^*}^2)^3}{m_B^3(m_b^2-m_s^2)^3}\,|T_1(0)|^2 \ , \tag{4.5}$$

having used the relation $T_2(0) = \frac{1}{2}T_1(0)$. The factor $\mathcal{I}$ is given by [25]

$$\mathcal{I} = \frac{(m_B+m_\rho)}{(m_B+m_{K^*})}\frac{T_1(0)}{A_0^{B\to\rho}(0)} \ . \tag{4.6}$$

The main reason for studying the ratio (4.4) is that $d\Gamma(\bar{B}\to\rho l\bar{\nu}_l)/dq^2$ in (4.4) does not vanish at $q^2 = 0$. Moreover, the factor $\mathcal{I}$ should be close to one. As a matter of fact, in the framework of the Bauer-Stech-Wirbel model [7] the value $\mathcal{I} = 1.12$ can be derived, whereas the constituent quark model [37] provides the value $\mathcal{I}$ =1.09–1.18. The relation $\mathcal{I} = 1$ has been obtained by the authors in Ref. [24] by considering the transition $b\bar{q} \to Q\bar{q}$ in the limit where both the b and Q quarks are heavy, applying spin symmetry relations; then, it has been argued that this relation still holds for a light Q quark in the weak binding limit for the meson.

The factor $\mathcal{I}$ can be derived by three-point QCD sum rules, using realistic values of the quark masses in a fully relativistic approach. Using the determination $T_1(0) = 0.35 \pm 0.05$ [33] and the value $A_0^{B\to\rho}(0)$ derived in the previous section, we get: $\mathcal{I} = 1.43 \pm 0.32$. Notice that, in evaluating the error for $\mathcal{I}$, we have taken into account the correlated errors in the numerator and in the denominator of $\mathcal{I}$.

The uncertainty can be reduced by studying the ratio of the sum rules determining $T_1(0)$ (the formulas can be found in [33]) and $A_0^{B\to\rho}(0)$. We find a wide stability window in correspondence to the result: $\mathcal{I} = 1.3 \pm 0.1$.

Therefore, the QCD sum rules prediction differs by $10-30\%$ from the values derived in [24,25]; this deviation still is model dependent, in the sense that it comes out from an explicit QCD sum rules calculation and not from symmetry arguments, and therefore it should be checked in a different approach, e.g., by lattice QCD.

In any case, we feel that, modulo this uncertainty on the value of $\mathcal{I}$, the idea of relating $B \to K^*\gamma$ to $B \to \rho\ell\nu$ provides us with an interesting alternative method to measure V_{ub}.

V. CONCLUSIONS

We have applied three-point function QCD sum rules to calculate the form factor A_0 appearing in the matrix element of the flavor-changing axial vector current between the D and B mesons and a vector meson state. This form factor plays an important role in the calculation of nonleptonic B and D decays to PV states in the hypothesis of factorization.

The explicit calculation has allowed us to determine A_0 at zero momentum transferred, with better accuracy than in the indirect determination from A_1 and A_2. We have also computed the q^2 dependence, finding an increase for positive values of the squared momentum transferred, compatible with a simple polar behavior.

Finally, we have considered the role of A_0 in relating the rare $B \to K^*\gamma$ decay with the semileptonic $B \to \rho\ell\nu$ spectrum, in the search of methods for measuring the matrix element V_{ub}.

ACKNOWLEDGMENTS

We thank F. Buccella, G. Nardulli, and N. Paver for discussions.

APPENDIX: PERTURBATIVE AND NONPERTURBATIVE TERMS

The perturbative spectral functions $\rho_i^{(\mathrm{pert})}(s,s',q^2)$ in (2.8), at the lowest order in α_s, can be obtained by the Cutkosky rules applied to a triangle diagram:

$$\rho_1(s,s',q^2) = \frac{3}{4\sqrt{\lambda}}\Big\{(m_c+m_s)\,(\Delta'-\Delta) + \frac{2(\Delta\, s' + \Delta'\, s) - u\,(\Delta'+\Delta)}{\lambda} \times \left[-(m_c+m_s)(s-s'+q^2) + 2m_c(m_c^2-m_s^2)\right]\Big\}\,,$$

$$\rho_2(s,s',q^2) = \frac{3}{4\sqrt{\lambda}}\Big\{(m_c+m_s)\,(\Delta'+\Delta) + \frac{2(\Delta\, s' - \Delta'\, s) + u\,(\Delta-\Delta')}{\lambda} \times \left[(m_c+m_s)(-s+s'-q^2) + 2m_c(m_c^2-m_s^2)\right]\Big\}\,,$$

where $\Delta = s - m_c^2$, $\Delta' = s' - m_s^2$, $u = s + s' - q^2$, and $\lambda = u^2 - 4\,s\,s'$.

The Borel transformed coefficients of the nonperturbative $D = 3$ terms read

$$\Gamma_1^{(D=3)} = \frac{m_c^2 - m_s^2}{2}\,, \tag{A1}$$

$$\Gamma_2^{(D=3)} = -\frac{(m_c+m_s)^2}{2}\,; \tag{A2}$$

the coefficients of the condensate of dimension $D = 5$ terms are

$$\Gamma_1^{(D=5)} = \frac{m_c^2(m_s^2-m_c^2)}{8\,M^4} + \frac{m_s^2(m_s^2-m_c^2)}{8\,M'^4} + \frac{(m_c^2-m_s^2)}{6\,M^2} + \frac{(m_c^2-m_s^2)}{12\,M'^2} + \frac{(m_s^2-m_c^2)\,(2m_s^2-m_c^2) - q^2(m_s+m_c)(2m_s-m_c)}{12\,M^2\,M'^2}$$

and

$$\Gamma_2^{(D=5)} = \frac{m_c^2(m_s+m_c)^2}{8\,M^4} + \frac{m_s^2(m_s+m_c)^2}{8\,M'^4} - \frac{(m_c^2+3m_s^2+4m_sm_c)}{12\,M'^2} - \frac{(3m_c^2+m_s^2+4m_sm_c)}{6\,M^2} + \frac{(m_s^2+m_c^2-q^2)\,(2m_s^2+m_c^2) + m_sm_c(4m_c^2+2m_s^2-3q^2)}{12\,M^2\,M'^2}\,.$$

Finally, the coefficients H_i in (2.18) read

$$H_1 = \frac{2m_c}{f_{K^*} f_D m_D^2 (m_{K^*}^2 - m_D^2 + q^2)} \tag{A3}$$

and

$$H_2 = \frac{2m_c}{f_{K^*} f_D m_D^2 (3m_{K^*}^2 + m_D^2 - q^2)} \,. \tag{A4}$$

The formulas for the transitions $D \to \rho$, $B \to \rho$ and $B \to D^*$ can be easily obtained.

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[1] M.K. Gaillard and B.W. Lee, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351 (1974).

[2] J. Schwinger, Phys. Rev. Lett. **12**, 630 (1964).

[3] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).

[4] F. Buccella, M. Lusignoli, G. Miele, and A. Pugliese, Z. Phys. C **55**, 243 (1992); F. Buccella, M. Lusignoli, G. Mangano, G. Miele, A. Pugliese, and P. Santorelli, Phys. Lett. B **302**, 319 (1993); A. Pugliese and P. Santorelli, in *Proceedings of the Third Workshop on the Tau-Charm Factory*, Marbella, Spain, 1993, edited by J. Kirkby and R. Kirkby (Editions Frontieres, Gif-sur-Yvette, France, 1994); F. Buccella, M. Lusignoli, G. Miele, A. Pugliese, and P. Santorelli, Report No. Roma 1026 and DSF-T-18/94 (unpublished).

[5] An example of the use of measured phase shifts in heavy-meson physics (long distance effects in D^0-$\overline{D}^0$ mixing) can be found in P. Colangelo, G. Nardulli, and N. Paver, Phys. Lett. B **242**, 71 (1990).

[6] G. Altarelli, G. Curci, G. Martinelli, and S. Petrarca, Nucl. Phys. **B187**, 461 (1981); A.J. Buras, M. Jamin, M.E. Lautenbacher, and P.E. Weisz, *ibid.* **B370**, 69 (1992); **B375**, 501 (1992) (addendum); M. Ciuchini, E. Franco, G. Martinelli, and L. Reina, Nucl. Phys. **B415**, 403 (1994).

[7] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985).

[8] A.J. Buras, J.M. Gerard, and R. Ruckl, Nucl. Phys. **B268**, 16 (1986).

[9] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974); E. Witten, *ibid.* **B149**, 285 (1979).

[10] B. Block and M. A. Shifman, Yad. Fiz. **45**, 211 (1987) [Sov. J. Nucl. Phys. **45**, 135 (1987)]; **45**, 478 (1987) [**45**, 301 (1987)]; **45**, 841 (1987) [**45**, 522 (1987)]; B. Block and M. A. Shifman, Nucl. Phys. **B389**, 534 (1993).

[11] A. Khodjamirian and R. Ruckl, Report No. MPI-PhT/94-26, 1994 (unpublished).

[12] CLEO Collaboration, M.S. Alam *et al.*, Phys. Rev. D **50**, 43 (1994).

[13] F.G. Gilman and R.L. Singleton, Phys. Rev. D **41**, 142 (1990).

[14] B. Grinstein, N. Isgur, and M.B. Wise, Phys. Rev. Lett. **56**, 298 (1986); B. Grinstein, N. Isgur, D. Scora, and M.B. Wise, Phys. Rev. D **39**, 799 (1989); N. Isgur and D. Scora, *ibid.* **40**, 1491 (1989).

[15] P. Ball, V.M. Braun, and H.G. Dosch, Phys. Rev. D **44**, 3567 (1991).

[16] P. Ball, Phys. Rev. D **48**, 3190 (1993).

[17] V.M. Belyaev, A. Kodjamirian, and R. Ruckl, Z. Phys. C **60**, 349 (1993).

[18] A.S. Abada *et al.*, Nucl. Phys. **B416**, 675 (1994).

[19] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, F. Feruglio, R. Gatto, and G. Nardulli, Phys. Lett. B **299**, 139 (1993).

[20] P. Colangelo and P. Santorelli, Phys. Lett. B **327**, 123 (1994).

[21] For a review see P. Colangelo, in *Proceedings of the Third Workshop on the Tau-Charm Factory* [4].

[22] See, for example, L.L. Chau and H.Y. Cheng, Report No. UTP-SB-93-49, hep-ph/9404207 (unpublished).

[23] G. Burdman and J.F. Donoghue, Phys. Lett. B **270**, 55 (1991).

[24] P.J. O'Donnell and H.K.K. Tung, Phys. Rev. D **48**, 2145 (1993).

[25] P.J. O'Donnell and Q.P. Xu, Phys. Lett. B **325**, 219 (1994).

[26] M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. **B147**, 385 (1979). For a review see the reprints volume: "Vacuum structure and QCD Sum Rules," edited by M.A. Shifman (North-Holland, Amsterdam, 1992).

[27] B.L. Ioffe and A.V. Smilga, Nucl. Phys. **B216**, 373 (1983); A.V. Nesterenko and A.V. Radyushkin, Phys. Lett. **115B**, 410 (1982).

[28] P. Colangelo, G. Nardulli, A.A. Ovchinnikov, and N. Paver, Phys. Lett. B **269**, 204 (1991).

[29] For a review, see P. Colangelo, G. Nardulli, and N. Paver, in *Proceedings of the ECFA Workshop on a European B-Meson Factory*, edited by R. Aleksan and A. Ali (ECFA, Hamburg, 1993), p. 155; C.A. Dominguez, in *Proceedings of the Third Workshop on the Tau-Charm Factory* [4].

[30] For a review, see N.G. Deshpande, in *B decays*, edited by S. Stone (World Scientific, Singapore, 1992), p. 430; S. Bertolini, F. Borzumati, and A. Masiero, in *ibid.*, p. 458; A. Ali, C. Greub, and T. Mannel, in *Proceedings of the ECFA Workshop on a European B-meson Factory*, Ref. [29].

[31] S. Bertolini, F. Borzumati, and A. Masiero, Phys. Rev. Lett. **59**, 180 (1987); N.G. Deshpande, P. Lo, J. Trampetic, G. Eilam, and P. Singer, Phys. Rev. Lett. **59**, 183 (1987); B. Grinstein, R. Springer, and M.B. Wise, Nucl. Phys. **B339**, 269 (1990); G. Cella, G. Curci, G. Ricciardi, and A. Viceré, Phys. Lett. B **248**, 181 (1990); M. Misiak, *ibid.* **269**, 161 (1991); M. Ciuchini, E. Franco, G. Martinelli, L. Reina, and L. Silvestrini, *ibid.* **316**, 127 (1993); **334**, 137 (1994).

[32] N. Isgur and M.B. Wise, Phys. Rev. D **42**, 2388 (1990).

[33] P. Colangelo, C.A. Dominguez, G. Nardulli and N. Paver,

Phys. Lett. B **317**, 183 (1993).
[34] A. Ali, V.M. Braun, and H. Simma, Z. Phys. C **63**, 437 (1994).
[35] CLEO Collaboration, R. Ammar *et al.*, Phys. Rev. Lett. **71**, 674 (1993)
[36] P. Santorelli, Z. Phys. C **61**, 449 (1994).
[37] N. Isgur, D. Scora, B. Grinstein, and M.B. Wise, Phys. Rev. D **39**, 799 (1989); P.J. O'Donnell and H.K.K. Tung, *ibid.* **44**, 741 (1991).