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# Quantum effects in Friedmann–Robertson–Walker cosmologies

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**Abstract.** Electrodynamics for self-interacting scalar fields in spatially flat Friedmann–Robertson–Walker spacetimes is studied. The corresponding 1-loop field equation for the expectation value of the complex scalar field in the conformal vacuum is derived. For exponentially expanding universes, the equations for the Bogoliubov coefficients describing the coupling of the scalar field to gravity are solved numerically. They yield a non-local correction to the Coleman–Weinberg effective potential which does not modify the pattern of minima found in static de Sitter space. Such a correction contains a dissipative term which, accounting for the decay of the classical configuration in scalar field quanta, may be relevant for the reheating stage. The physical meaning of the non-local term in the semiclassical field equation is investigated by evaluating this contribution for various background field configurations.

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## 1. Introduction

In a recent series of papers [1–3], some of the authors have studied the 1-loop effective potential for grand unified theories in de Sitter space. Our main results were a better understanding of the symmetry-breaking pattern first found in [4], a numerical approach to small perturbations of de Sitter cosmologies [2], and the analysis of SO(10) GUT theories in de Sitter cosmologies [3]. However, a constant Higgs field, with de Sitter 4-space as a background in the corresponding 1-loop effective potential, is only a mathematical idealization. A more realistic description of the early universe is instead obtained on considering a dynamical spacetime such as the one occurring in Friedmann–Robertson–Walker (FRW) models. Indeed, our early work [2] tried to study the case of a varying Higgs field by introducing a function of the Euclidean-time coordinate which reduces to the 4-sphere radius of de Sitter in the limit of constant Higgs field. Although the approximations made in [2] were legitimate for numerical purposes, the gravitational part of the action, and the 1-loop effective potential, were not actually appropriate for studying a dynamical cosmological model.

Thus, relying on the work in [5, 6], this paper studies the first step towards the completion of our programme, i.e. scalar electrodynamics with a self-interaction term for the complex scalar field. The semiclassical field equations in time-variable backgrounds contain non-local terms, which describe the coupling of the scalar field to the gravitational background. The analysis of these equations is relevant for the reheating mechanism in inflationary

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cosmology and for the dynamics of dissipation via particle production [7]. Our analysis deals with Lorentzian spacetime manifolds with FRW symmetries, and does not rely on zeta-function regularization. Hence our geometric framework is substantially different from the Riemannian 4-manifolds studied in [1–4], where the metric was positive-definite. Section 2 derives the field equations for a class of cosmological models where scalar electrodynamics is studied in FRW universes. The Coleman–Weinberg potential, and its correction, derived from the non-local term involving the Bogoliubov coefficients for the coupling of the scalar field to gravity, are obtained numerically in section 3. The semiclassical field equation and the physical meaning of such non-local terms are studied in detail in section 4. Concluding remarks are presented in section 5, and relevant details are given in the appendix.

## 2. Model and field equations

For the reasons described in the introduction, we consider a complex scalar field $\phi$ with a mass and a self-interaction term, coupled to the electromagnetic potential $A_\mu$ in curved spacetime. Hence the action functional is

$$I \equiv \int \mathcal{L}\sqrt{-\det g}\ \mathrm{d}^4x + \text{boundary terms} \tag{2.1}$$

where (cf [6])

$$\mathcal{L} \equiv \left[(\nabla_\mu + \mathrm{i}eA_\mu)\phi^\dagger\right]\left[(\nabla^\mu - \mathrm{i}eA^\mu)\phi\right] - m^2\phi^\dagger\phi - \frac{1}{4!}\lambda(\phi^\dagger\phi)^2 - \xi R\phi^\dagger\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,. \tag{2.2}$$

With a standard notation, $\nabla$ is the Levi-Civita connection on the background spacetime, $\xi$ is a dimensionless parameter, $R$ denotes the trace of the Ricci tensor, and $F_{\mu\nu} \equiv \nabla_\nu A_\mu - \nabla_\mu A_\nu$ is the electromagnetic-field tensor. Boundary terms are necessary to obtain a well-posed variational problem, and their form is obtained after integration by parts in the volume integral in (2.1). Covariant derivatives of the scalar field are here used to achieve a uniform notation (cf [6] and [8]).

Following [5], we now split the complex scalar field $\phi$ as the sum of a variable real-valued background field $\phi_c$, and of a complex-valued fluctuation $\varphi$, i.e.

$$\phi = \phi_c + \varphi\,. \tag{2.3}$$

The *conformal vacuum* [9] is chosen here, and the quantum fluctuation $\varphi$ has vanishing expectation value in such a state, $\langle\varphi\rangle = 0$, so that $\langle\phi\rangle = \phi_c$. The field equations for $A_\mu, \phi_c$ and $\varphi$ are obtained by setting to zero the corresponding functional derivatives of the action. As far as the gauge potential $A_\mu$ is concerned, it is convenient to impose the Lorentz gauge $\nabla^\mu A_\mu = 0$. At this stage, to quantize the theory, one can follow the Gupta–Bleuler method, or the Faddeev–Popov procedure, or to eliminate the residual gauge freedom by imposing the relativistic gauge condition proposed by Ford [6], i.e. $X^\mu A_\mu = 0$, where $X$ is a timelike vector field. Such a field admits a *natural* form in FRW spacetimes (see below). Thus, on defining the operator $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$, the resulting form of the field equations is (cf [6])

$$\left(g^{\mu\nu}\Box - R^{\mu\nu}\right)A_\nu = -2e^2A^\mu\phi^\dagger\phi - \mathrm{i}e\left(\phi^\dagger\nabla^\mu\phi - \phi\nabla^\mu\phi^\dagger\right) \tag{2.4}$$

$$\left[\Box + m^2 + \xi R + \tfrac{1}{12}\lambda\phi_c^2 - e^2\langle A_\mu A^\mu\rangle + \tfrac{1}{24}\lambda\langle\varphi^2 + (\varphi^\dagger)^2 + 4\varphi\varphi^\dagger\rangle\right]\phi_c = \mathrm{i}e\langle A^\mu\nabla_\mu(\varphi - \varphi^\dagger)\rangle \tag{2.5}$$

$$\left[\Box + m^2 + \xi R + \tfrac{1}{6}\lambda\phi_c^2\right]\varphi + \tfrac{1}{12}\lambda\phi_c^2\,\varphi^\dagger = 0\,. \tag{2.6}$$

Note that equations (2.4)–(2.6) have been obtained by retaining in the action (2.1) only terms quadratic in the fluctuations $\varphi$ and $A_\mu$, and setting to zero all terms involving $\nabla^\nu A_\nu$ and its covariant derivatives in the field equations. The latter condition is sufficient to derive (2.4). Moreover, we require that $\nabla^\mu \phi_c$ should be proportional to $X^\mu$ [6]. Note also that the contribution of

$$\left[\Box + m^2 + \xi R + \tfrac{1}{12}\lambda\phi_c^2\right]\phi_c$$

has been neglected in the course of deriving (2.6), since equation (2.5) implies that such a contribution is of second order in the quantum fluctuations. By taking the complex conjugate of equation (2.6), and defining $\varphi \equiv (\varphi_1 + i\varphi_2)/\sqrt{2}$, $\lambda_1 \equiv \lambda/2$ and $\lambda_2 \equiv \lambda/6$, the addition and subtraction of the resulting equations leads to decoupled equations for the real and imaginary parts of $\varphi$, i.e.

$$\left[\Box + m^2 + \xi R + \tfrac{1}{2}\lambda_j\phi_c^2\right]\varphi_j = 0 \qquad \text{for all } j = 1, 2\,. \tag{2.7}$$

Moreover, by virtue of our particular gauge conditions, equation (2.5) takes the form (see the appendix)

$$\left[\Box + m^2 + \xi R + \tfrac{1}{12}\lambda\phi_c^2 - e^2\langle A_\mu A^\mu\rangle + \tfrac{1}{4}\lambda_1\langle\varphi_1^2\rangle + \tfrac{1}{4}\lambda_2\langle\varphi_2^2\rangle\right]\phi_c = 0\,. \tag{2.8}$$

Interestingly, the effects of quantum fluctuations in (2.8) reduce to a linear superposition of the self-interaction term studied in [5] and of the electromagnetic term studied in [6], without any coupling. The term $\langle A_\mu A^\mu\rangle$ is evaluated on considering the integral equation equivalent to (2.4), as shown in [6] and in the appendix. Such an analysis proves that, in the case of a spatially flat FRW background, the renormalized expectation value of $A_\mu A^\mu$ in the conformal vacuum can be written as [6]

$$\langle A_\mu A^\mu\rangle = B\ X^\mu X^\nu R_{\mu\nu} \tag{2.9}$$

where $B = 9.682 \times 10^{-3}$. In particular, in a de Sitter universe, $\langle A_\mu A^\mu\rangle = 12BH^2$, where $H$ is the Hubble parameter [6].

Following [5], we can now write the set of equations leading to the solution of (2.7) and (2.8) in spatially flat FRW backgrounds. Without making any approximation, if one defines ($a$ being the cosmic scale factor)

$$\tau \equiv \int_{t_0}^{t} \frac{\mathrm{d}y}{a(y)} \tag{2.10}$$

and, for all $j = 1, 2$

$$\Omega_{j,k}^2 \equiv k^2 + a^2(\tau)\left[m^2 + \left(\xi - \tfrac{1}{6}\right)R(\tau) + \tfrac{1}{2}\lambda_j\phi_c^2(\tau)\right] \tag{2.11}$$

the 1-loop field equations resulting from (2.7) and (2.8) are (cf (3.32) and (3.33) in [5])

$$\frac{1}{a^2}\frac{\mathrm{d}^2\phi_c}{\mathrm{d}\tau^2} + \frac{2}{a^3}\frac{\mathrm{d}a}{\mathrm{d}\tau}\frac{\mathrm{d}\phi_c}{\mathrm{d}\tau} - e^2\langle A_\mu A^\mu\rangle\phi_c + \frac{\partial V_{\mathrm{eff}}}{\partial\phi_c} + \frac{\phi_c}{4\pi^2 a^2}\sum_{j=1}^{2}\frac{1}{2}\lambda_j\int_0^\infty \mathrm{d}k\, k^2\Omega_{j,k}^{-1}\left[s_{j,k} + \mathrm{Re}\, z_{j,k}\right] = 0 \tag{2.12}$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}s_{j,k} = \left(\frac{\mathrm{d}}{\mathrm{d}\tau}\log\Omega_{j,k}\right)\mathrm{Re}\, z_{j,k} \tag{2.13}$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}z_{j,k} = \left(\frac{\mathrm{d}}{\mathrm{d}\tau}\log\Omega_{j,k}\right)\left(s_{j,k} + \frac{1}{2}\right) - 2\mathrm{i}\Omega_{j,k}z_{j,k}\,. \tag{2.14}$$

The form of $\partial V_{\rm eff}/\partial \phi_c$ is given in the appendix. Equations (2.13) and (2.14) are necessary to find solutions of (2.7) by using Fourier-transform techniques and a suitable change of coordinates, as shown in [5]. The initial conditions for $s_{j,k}$ and $z_{j,k}$ are the ones appropriate for the choice of a conformal vacuum, i.e. [5]

$$s_{j,k}(\tau = 0) = 0 \qquad z_{j,k}(\tau = 0) = 0\,. \tag{2.15}$$

Of course, the values of $m, \xi$ and $\lambda_j$ should now be regarded as the renormalized values of such parameters [5].

Note that the second line of equation (2.12) is a typical non-local term, resulting from the self-interaction of the scalar field (see equation (2.7)) and from its coupling to the geometric background. Moreover, a dissipative term exists which is part of the non-local correction to the Coleman–Weinberg potential, and is due to the decay processes of $\phi_c$ in scalar field quanta. By means of such decays, energy is transferred from $\phi_c$ to the relativistic degrees of freedom $\varphi$ (i.e. radiation). As is well known, if this release of energy is sufficiently strong, radiation becomes dominant and hence the inflationary phase ends. This leads to the reheating stage, which is as important as the exponential expansion for the dynamics of the early universe.

## 3. Numerical evaluation of the 1-loop effective potential

In this section, for the physically relevant case of an exponentially expanding FRW universe, we compute the non-local term on the second line of (2.12). After integration of such a term with respect to $\phi_c$, the resulting expression is compared with the Coleman–Weinberg potential $V_{\rm eff}$.

The numerical analysis has been performed for a conformally invariant scalar field (hence $\xi = \frac{1}{6}$ and $m = 0$), by solving equations (2.13) and (2.14) with the help of the NAG routine D02BAF, with initial conditions (2.15). We have fixed $\lambda = 10^{-2}$, which ensures the reliability of the perturbative approach, and the Hubble parameter $H = 10^{-1}\ M_{\rm PL}$, which is an intermediate choice between a chaotic model ($H \simeq M_{\rm PL}$) and a GUT inflationary phase ($H \simeq 10^{-4}\ M_{\rm PL}$).

In the case of exponential expansion in the time variable $t$, which implies (see equation (2.10))

$$a(\tau) = \frac{a(0)}{(1 - a(0)H\tau)} \tag{3.1}$$

and for a classical field configuration independent of $\tau$, $\phi_c = \phi_{c0}$, we express the second line of (2.12) as a function of $\tau$ and $\phi_{c0}$. In figure 1 we plot the integral with respect to $\phi_{c0}$ of the above quantity when the conformal time $\tau$ varies between 0 and $1/H$, which corresponds to a large $e$-fold number, $\phi_{c0} \in [0, 10\ M_{\rm PL}]$, and $a(0)$ has been set to 1. This configuration for $a(\tau)$ and fixed $\phi_{c0}$ corresponds to an exact de Sitter space, but unlike [1–4], with a Lorentzian signature. The non-local term describes an energy exchange between the gravitational field and the field $\phi$ (see equation (2.3)). Such an exchange may lead to dissipative or non-dissipative processes in the early universe, depending on the sign of the non-local term, as will be clear from the analysis in section 4. For a constant background field $\phi_c$, only a scale factor varying in time yields the above non-local corrections to the Coleman–Weinberg potential $V_{\rm eff}$.

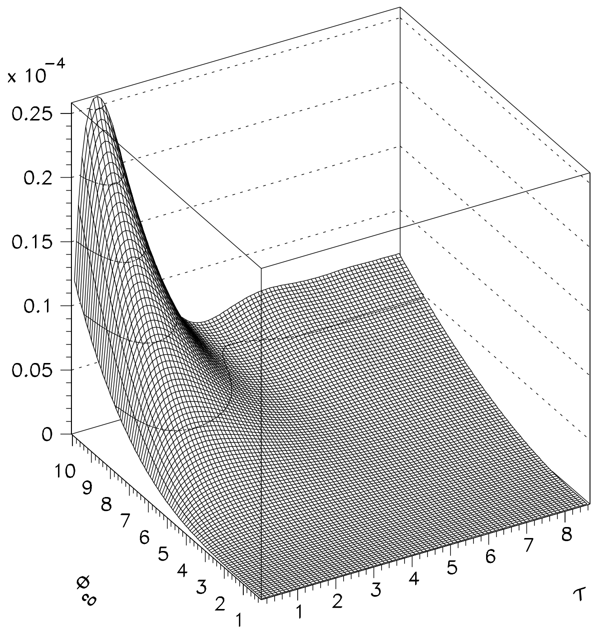


**Figure 1.** In Planck units (used also in figures 2, 3, 4 and 5), the correction to the Coleman–Weinberg potential resulting from the Bogoliubov coefficients in (2.12)–(2.14) is plotted against $\tau$ and $\phi_{c0}$.

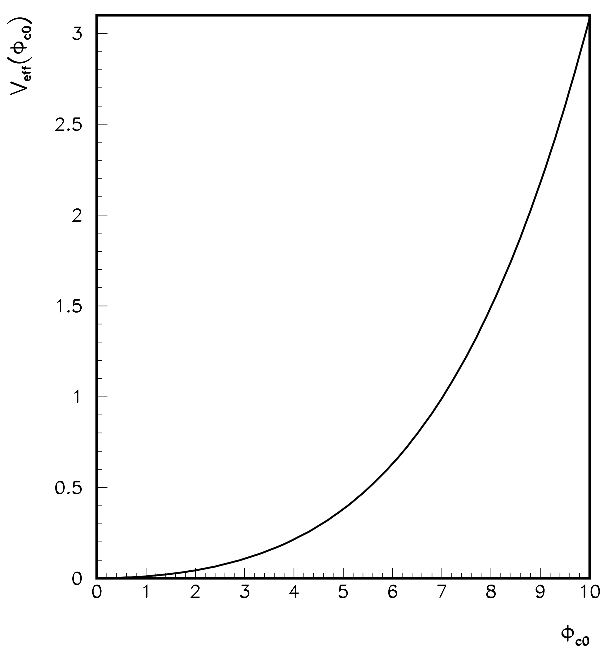


**Figure 2.** The Coleman–Weinberg 1-loop effective potential is plotted against $\phi_{c0}$, in the case of a de Sitter background.

In figure 2, we plot the Coleman–Weinberg potential $V_{\text{eff}}(\phi_c)$ (resulting from the integration of (A.7) for $\mu_1 = \mu_2 = M_{\text{PL}}$ and $V_{\text{eff}}(0) = V_0 = 3M_{\text{PL}}^2 H^2/(8\pi)$) for the same choice of parameters. As one can see from figures 1 and 2, the contribution of non-local terms to the 1-loop effective potential is very small, and hence its effect on the semiclassical equation of motion is negligible in a first approximation (see section 4).

## 4. Semiclassical field equation and non-local effects

The analysis of the previous section has shown that the effect of the Bogoliubov coefficients on the Coleman–Weinberg potential does not modify the pattern of minima found in static de Sitter space [8]. It is therefore legitimate to study the equation (2.12) when, in a first approximation, its second line is neglected, and then to use the resulting solution to evaluate the non-local correction. Here, the value of $V_{\rm eff}(0)$ is set equal to $V_0$ as above, and it dominates the energy, as it occurs in a de Sitter universe, for $\phi_c < M_{\rm PL}$.

On studying the limiting form of equation (2.12) when the second line is neglected, it can be easily seen that the self-interaction term (see equation (A.7)) plays a key role in obtaining a sensible physical model. In other words, for vanishing $\lambda$, equation (2.12) admits a runaway exact solution for $\phi_c$ in the form (denoting by $D_1$ and $D_2$ two integration constants)

$$\phi_c = D_1 \exp(-\beta_1 H t) + D_2 \exp(-\beta_2 H t) \tag{4.1}$$

where $\beta_1 \equiv \frac{1}{2}\left(3 + \sqrt{9 + 48Be^2}\right)$ and $\beta_2 \equiv \frac{1}{2}\left(3 - \sqrt{9 + 48Be^2}\right)$, and we have re-expressed $\tau$ in terms of $t$ by means of (2.10). This behaviour results from the particular form taken by the potential in (2.12), i.e. $-6BH^2e^2\phi_c^2$, which is unbounded from below.

We thus study the equation given by the first line of (2.12) with non-vanishing $\lambda$, i.e.

$$\frac{1}{a^2}\frac{\mathrm{d}^2\phi_c}{\mathrm{d}\tau^2} + \frac{2}{a^3}\frac{\mathrm{d}a}{\mathrm{d}\tau}\frac{\mathrm{d}\phi_c}{\mathrm{d}\tau} - e^2\langle A_\mu A^\mu\rangle\phi_c + \frac{\partial V_{\rm eff}}{\partial\phi_c} = 0 \tag{4.2}$$

subject to the initial conditions $\phi_c(0) = M_{\rm PL}$ and $\mathrm{d}\phi_c/\mathrm{d}\tau(\tau = 0) = 0$. In figures 3 and 4, $\phi_c(\tau)$ and $\mathrm{d}\phi_c/\mathrm{d}\tau$ are plotted. As one might expect, $\phi_c(\tau)$ decreases until it reaches the zero value, by virtue of the nature of the potential (see figure 2), whilst the kinetic energy increases.

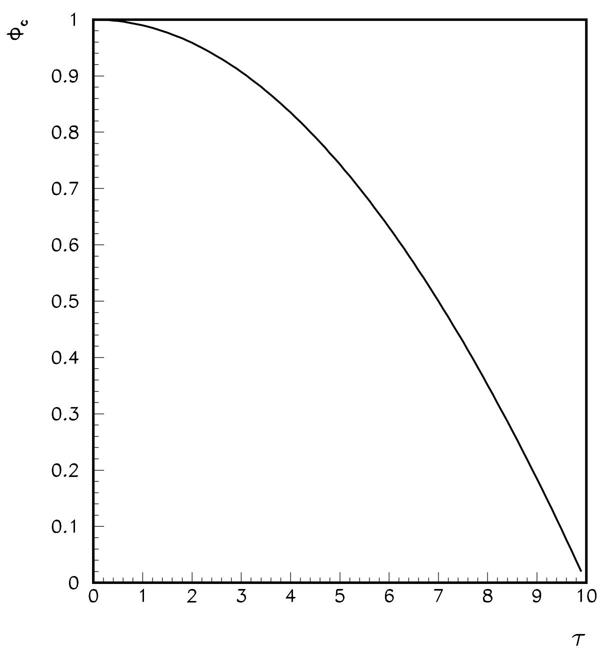


**Figure 3.** The solution of the semiclassical field equation (4.2) for the background field configuration is plotted against $\tau$.

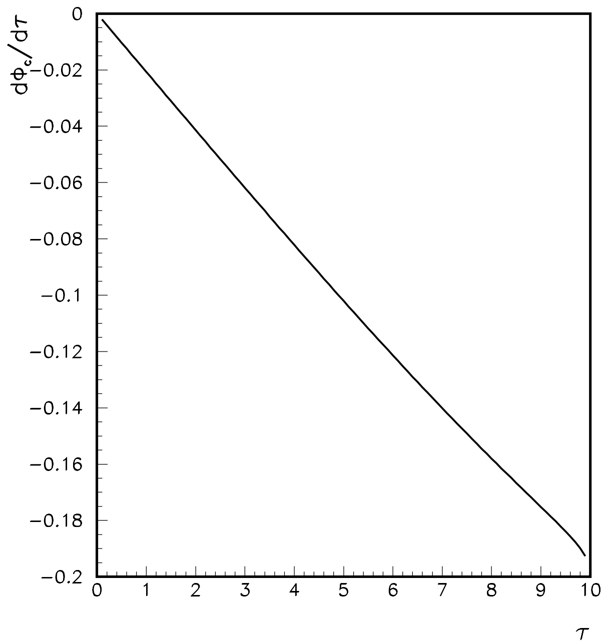


**Figure 4.** The derivative with respect to $\tau$, for the numerical solution plotted in figure 3.

We have then used the solution of (4.2) to evaluate the non-local effects in our model. For this purpose, starting from the definitions of energy density $\rho_{\phi_c}$ and pressure $p_{\phi_c}$ of the background scalar field in a de Sitter universe, i.e.

$$\rho_{\phi_c} \equiv \tfrac{1}{2}\dot{\phi}_c^2 + V_{\rm eff} - 6Be^2H^2\phi_c^2 \tag{4.3}$$

$$p_{\phi_c} \equiv \tfrac{1}{2}\dot{\phi}_c^2 - V_{\rm eff} + 6Be^2H^2\phi_c^2 \tag{4.4}$$

one gets from (2.12) the equation

$$\frac{\mathrm{d}\rho_{\phi_c}}{\mathrm{d}\tau} + 3Ha(\rho_{\phi_c} + p_{\phi_c}) = -\Theta(\phi_c(\tau), \tau)\frac{\mathrm{d}\phi_c}{\mathrm{d}\tau}\,. \tag{4.5}$$

With our notation, $\Theta$ corresponds to the whole second line of (2.12),

$$\Theta(\phi_c(\tau), \tau) \equiv \frac{\phi_c}{4\pi^2a^2}\sum_{j=1}^{2}\frac{1}{2}\lambda_j\int_0^\infty \mathrm{d}k\; k^2\Omega_{j,k}^{-1}[s_{j,k} + \mathrm{Re}\, z_{j,k}]\,. \tag{4.6}$$

If the right-hand side of (4.5) can be viewed as a dissipative term, which implies that it always takes negative values and depends quadratically on $\mathrm{d}\phi_c/\mathrm{d}\tau$, the Bianchi identity leads to the following equation for the energy density $\rho_R$ of radiation:

$$\frac{\mathrm{d}\rho_R}{\mathrm{d}\tau} + 4Ha\;\rho_R = \Theta(\phi_c(\tau), \tau)\frac{\mathrm{d}\phi_c}{\mathrm{d}\tau}\,. \tag{4.7}$$

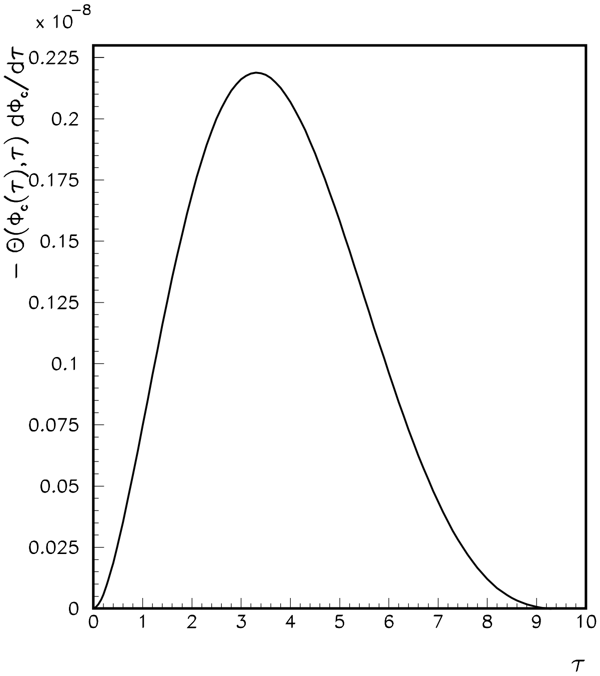


**Figure 5.** The right-hand side of equation (4.5) is plotted against $\tau$.

In figure 5 we plot the right-hand side of (4.5) corresponding to the solution of equation (4.2). Note that its sign turns out to be positive for the largest part of the $\tau$-interval, and hence cannot actually lead to dissipative effects. A naturally occurring question is how to interpret this lack of dissipation. Indeed, in the adiabatic approximation studied in [5], the coupling of the scalar field to a fermionic field by a Yukawa term produces a vacuum energy loss rate proportional to $(Ha\phi_c + \mathrm{d}\phi_c/\mathrm{d}\tau)\mathrm{d}\phi_c/\mathrm{d}\tau$. In this last expression the second term is clearly a dissipative effect, since it does not depend on the sign of the velocity and in the formula reported in [5] it occurs with the correct sign to represent an energy loss. By contrast, the first term does depend on the sign of $\mathrm{d}\phi_c/\mathrm{d}\tau$ and in de Sitter, where $H$ is constant, it only represents a further quantum correction to the energy and pressure of the $\phi_c$ fluid.

These considerations seem to suggest that also in our case, where one deals with decays of the classical field configuration into its quanta, the non-local term in the semiclassical field equation may be essentially a linear combination of $\phi_c$ and $\mathrm{d}\phi_c/\mathrm{d}\tau$. If this property holds, one can expect that the evaluation of $(Ha\phi_c + \mathrm{d}\phi_c/\mathrm{d}\tau)\mathrm{d}\phi_c/\mathrm{d}\tau$ may indicate when dissipative effects are likely to occur, depending on whether the first or the second term of this linear combination is dominant. In other words, the sign of the right-hand side of (4.5) shown in figure 5 can be understood by pointing out that for the solution of (4.2), shown in figures 3 and 4, the ratio $(\mathrm{d}\phi_c/\mathrm{d}\tau)/Ha\phi_c$ is smaller than 1 whilst $\mathrm{d}\phi_c/\mathrm{d}\tau$ is negative, when $\tau \in [0, 9.2\ M_{\mathrm{PL}}^{-1}]$. Note that, in the neighbourhood of $\tau = 9.2\ M_{\mathrm{PL}}^{-1}$, $-\Theta\ \mathrm{d}\phi_c/\mathrm{d}\tau$ vanishes, and this corresponds to the value of $\tau$ such that the linear combination $(Ha\phi_c + \mathrm{d}\phi_c/\mathrm{d}\tau)$ vanishes.

As a further check of the conjecture about the functional dependence of the non-local term of (2.12) on $\phi_c$ and $\mathrm{d}\phi_c/\mathrm{d}\tau$, one should also analyse other situations where, unlike before, the ratio $(\mathrm{d}\phi_c/\mathrm{d}\tau)/Ha\phi_c$ is larger than 1. This can be done, for example, by taking as a trial function for $\phi_c(\tau)$ the linear combination $\phi_c(\tau) = \phi_{c0} + \alpha\tau$, where $\alpha$ and $\phi_{c0}$ are two arbitrary parameters. The resulting analysis, performed by varying $\alpha$ and $\phi_{c0}$ so as to reproduce the conditions $(\mathrm{d}\phi_c/\mathrm{d}\tau)/Ha\phi_c > 1$ or $(\mathrm{d}\phi_c/\mathrm{d}\tau)/Ha\phi_c < 1$, seems to confirm that, in de Sitter, $\Theta(\phi_c(\tau), \tau)$ is indeed dominated by a term proportional to the combination $(Ha\phi_c + \mathrm{d}\phi_c/\mathrm{d}\tau)$.

## 5. Concluding remarks

This paper has studied scalar electrodynamics in a spatially flat FRW universe, by including a self-interaction term for a conformally invariant scalar field (cf [5, 6, 10]). On imposing the Lorentz gauge and the supplementary condition (A.4), the 1-loop equation for the expectation value of the complex scalar field in the conformal vacuum shows a linear superposition of the self-interaction term of [5] and of the electromagnetic term [6]. The results of our investigation are thus as follows.

First, the numerical solution for the Bogoliubov coefficients in (2.13) and (2.14), subject to the initial conditions (2.15), has been obtained and used to find the correction to the Coleman–Weinberg effective potential by integrating with respect to $\phi_c$ the second line of (2.12). The pattern of minima in the effective potential is not modified by the non-local term in (2.12). For the particular values of parameters considered in our investigation, the non-local corrections turn out to be several orders of magnitude smaller.

Second, the limiting form of equation (2.12), i.e. equation (4.2), has been studied, and its numerical solution has been used to evaluate non-local effects in our cosmological model. Such a solution corresponds to a slow-roll dynamics. Interestingly, the approximate calculation of the function $\Theta$ defined in (4.6) shows that $\Theta$ does not lead necessarily to dissipative effects in the early universe. Nevertheless, in a de Sitter model, the right-hand side of (4.5) is very well approximated by the same combination of $\phi_c$ and $\mathrm{d}\phi_c/\mathrm{d}\tau$ which results from the adiabatic case studied in [5], where the coupling of a scalar field to a fermionic field was instead considered. More precisely, we have found that, for various forms of $\phi_c(\tau)$, the right-hand side of (4.5) is essentially given by

$$-A\left(Ha\,\phi_c + \frac{\mathrm{d}\phi_c}{\mathrm{d}\tau}\right)\frac{\mathrm{d}\phi_c}{\mathrm{d}\tau}$$

where $A$ is a positive function of $\tau$. The first term in parentheses leads to a further quantum correction to the energy density of the scalar field, at least when the function $A$ is slowly varying. The second term is purely dissipative (see the end of section 2).

One of the main motivations for studying non-local corrections to the Coleman–Weinberg potential was their possible relevance for the reheating of the early universe [7]. Our analysis confirms that they cannot be simply re-expressed by a term of the kind $\Gamma\ \mathrm{d}\phi_c/\mathrm{d}\tau$, as first found in [7], where a different approach to the effective action is used with respect to [5]. A non-trivial open problem is the numerical evaluation of the function $A$ in our de Sitter model. Moreover, it appears necessary to understand how the form of $A$ depends on the specific choice of the background field $\phi_c(\tau)$. Last, but not least, the whole analysis should be repeated for FRW models which are not in a de Sitter phase.

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**Appendix**

Following [6], we denote by $D_R^{\mu\nu}(x, x')$ the photon retarded Green's function, which satisfies the equation

$$\left(\delta^\mu_\rho \,\Box\, - \nabla^\mu \nabla_\rho - R^\mu_\rho\right)_x D_R^{\rho\nu}(x, x') = g^{\mu\nu}\ \delta(x, x')/\sqrt{-\det g}\,. \tag{A.1}$$

Thus, by writing $A^\mu_{\rm in}(x)$ for the photon in-field, the solution of (2.4) to order $e$, in the Lorentz gauge

$$\nabla^\mu A_\mu = 0 \tag{A.2}$$

is given by [6]

$$A^\mu \cong A^\mu_{\rm in} - {\rm i}e \int {\rm d}^4x'\ \sqrt{-\det g(x')}\ D_R^{\mu\nu}(x, x')\left(\phi^\dagger \nabla_\nu \phi - \phi \nabla_\nu \phi^\dagger\right)_{\rm in}. \tag{A.3}$$

The residual gauge freedom of the problem is dealt with by imposing the additional condition [6]

$$X^\mu A_\mu = 0 \tag{A.4}$$

where $X$ is the same timelike vector field appearing in (2.9). Equations (A.2)–(A.4) imply that, on inserting (A.3) into the right-hand side of (2.5), the only non-trivial contribution still vanishes after integration by parts, since

$$D_R^{\mu\nu}\ \nabla_\nu \phi_c = 0 \qquad \nabla_\mu\ D_R^{\mu\nu} = 0\,. \tag{A.5}$$

In equation (2.12) for the expectation value $\phi_c$ of the scalar field $\phi$, the derivative with respect to $\phi_c$ of the 1-loop effective potential is given, in the case of non-vanishing renormalized mass, by (cf equation (3.15) of [5])

$$\begin{aligned}\frac{\partial V_{\rm eff}}{\partial \phi_c} &= m^2\phi_c + \xi R\phi_c + \frac{1}{12}\lambda\phi_c^3 - \frac{1}{96\pi^2}\lambda\left(\xi - \frac{1}{6}\right)R\phi_c - \frac{5}{2304\pi^2}\lambda^2\phi_c^3 \\ &\quad + \frac{1}{384\pi^2}\lambda\left(m^2 + \left(\xi - \frac{1}{6}\right)R + \frac{1}{12}\lambda\phi_c^2\right)\phi_c \log\left|\frac{m^2 + (\xi - \frac{1}{6})R + \frac{1}{12}\lambda\phi_c^2}{m^2}\right| \\ &\quad + \frac{1}{128\pi^2}\lambda\left(m^2 + \left(\xi - \frac{1}{6}\right)R + \frac{1}{4}\lambda\phi_c^2\right)\phi_c \log\left|\frac{m^2 + (\xi - \frac{1}{6})R + \frac{1}{4}\lambda\phi_c^2}{m^2}\right|\end{aligned} \tag{A.6}$$

and for $m = 0$ by (cf equation (3.16) of [5])

$$\begin{aligned}\frac{\partial V_{\rm eff}}{\partial \phi_c} &= \xi R\phi_c + \frac{1}{12}\lambda\phi_c^3 + \frac{1}{4608\pi^2}\lambda^2\phi_c^3\left(\log\left|\frac{(\xi - \frac{1}{6})R + \frac{1}{12}\lambda\phi_c^2}{(\lambda/12)\mu_1^2}\right| - \frac{11}{3}\right) \\ &\quad + \frac{1}{512\pi^2}\lambda^2\phi_c^3\left(\log\left|\frac{(\xi - \frac{1}{6})R + \frac{1}{4}\lambda\phi_c^2}{(\lambda/4)\mu_1^2}\right| - \frac{11}{3}\right) \\ &\quad + \frac{1}{384\pi^2}\lambda\left(\xi - \frac{1}{6}\right)R\phi_c\left(\log\left|\frac{(\xi - \frac{1}{6})R + \frac{1}{12}\lambda\phi_c^2}{(\xi - \frac{1}{6})\mu_2^2}\right| - 1\right) \\ &\quad + \frac{1}{128\pi^2}\lambda\left(\xi - \frac{1}{6}\right)R\phi_c\left(\log\left|\frac{(\xi - \frac{1}{6})R + \frac{1}{4}\lambda\phi_c^2}{(\xi - \frac{1}{6})\mu_2^2}\right| - 1\right).\end{aligned} \tag{A.7}$$

In agreement with the notation of section 2, $m, \xi$ and $\lambda$ are the renormalized values of our parameters, and $\mu_1$ and $\mu_2$ are completely arbitrary renormalization points. Our equations (A.6) and (A.7) are obtained by imposing the renormalization conditions (3.12$a$)–(3.12$c$) of [5] and bearing in mind that the numerical coefficients in our equation (2.8) differ from the ones occurring in equation (2.19) of [5], since we study a complex scalar field.

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