

Polarized Quarks, Gluons and Sea in Nucleon Structure Functions

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We perform a NLO analysis of polarized deep inelastic scattering data to test two different solutions to the so-called *spin crisis*, one of them based on the axial gluon anomaly and consistent with the Bjorken sum rule, and another in which the defects in the spin sum rules and in the Gottfried sum rule are related. In this case a defect is also expected for the Bjorken sum rule. The first solution is slightly favoured by the SLAC E154 results, but both options seem to be consistent with the CERN SMC data.

§1. Introduction

The EMC experiment on $g_1^p(x)$ ¹⁾ has given for the first moment of the spin dependent proton structure function $g_1^p(x)$ at $\langle Q^2 \rangle = 10 \text{ GeV}^2$

$$\Gamma_1^p = \int_0^1 g_1^p(x) dx = 0.126 \pm 0.010 \pm 0.015, \quad (1)$$

a value much smaller than the prediction of the Ellis-Jaffe sum rule²⁾ for the proton (we use $a_3 = G_A/G_V = 1.2573 \pm 0.0028$ and $a_8 = 3F - D = 0.579 \pm 0.025$ ³⁾),

$$\Gamma_{1EJ}^p = \frac{F}{2} - \frac{D}{18} = 0.185 \pm 0.003. \quad (2)$$

This has stimulated considerable experimental activity in measuring the polarized structure functions (SF) $g_1^p(x)$, $g_1^n(x)$ ($g_1^{3\text{He}}$) and $g_1^d(x)$.^{4)~6)}

From the theoretical point of view, the hypothesis has been formulated⁷⁾ that a relevant isoscalar contribution, related, in the chiral-invariant factorization scheme,⁸⁾ to a large gluon polarization, contributes to Eq. (2) at $O(\alpha_s)$ as

$$\Gamma_1^p(Q^2) = \frac{1}{12} \left(a_3 + \frac{a_8}{3} + \frac{4}{3} a_0(Q^2) \right) \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right) - \frac{\alpha_s(Q^2)}{6\pi} \Delta G(Q^2). \quad (3)$$

This would explain the defect in the Ellis-Jaffe sum rule for the proton, without affecting the more theoretically founded Bjorken sum rule,⁹⁾

$$(\Gamma_1^p - \Gamma_1^n)_{\text{LO}} = \frac{a_3}{6} = \frac{1}{6} \frac{G_A}{G_V}. \quad (4)$$

QCD corrections play an important role in modifying the r.h.s. of Eq. (4) which is, strictly speaking, valid in the limit $Q^2 \rightarrow 0$, and reads

$$(\Gamma_1^p - \Gamma_1^n)(Q^2) = \frac{1}{6} \frac{G_A}{G_V} \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right), \quad (5)$$

where only the $O(\alpha_s)$ has been retained. The validity of the Bjorken sum rule is so universally accepted that, in the analysis of the experimental data on polarized SF,^{10)–13)} it is almost always assumed, rather than tested. There are few exceptions to this manner of thinking, as for example the paper by Altarelli et al.,¹¹⁾ where a_3 is evaluated to be 1.19 ± 0.09 , and the l.h.s. of Eq. (5) at $Q^2 = 4 \text{ GeV}^2$ is 0.177 ± 0.014 , in agreement with the predicted value at $O(\alpha_s)$, 0.189 ± 0.002 .

An alternative interpretation of the defect shown by Eq. (1) has been given¹⁴⁾ by relating the defects in the Ellis-Jaffe sum rule for the proton and the Gottfried sum rule¹⁵⁾ for the isovector unpolarized SF of the nucleons,

$$\int_0^1 \frac{F_2^p(x) - F_2^n(x)}{x} dx = \frac{1}{3} + \frac{2}{3} \int_0^1 [\bar{u}(x) - \bar{d}(x)] dx = \frac{1}{3}, \quad (6)$$

to be compared with the result the NMC experiment,¹⁶⁾ namely 0.235 ± 0.026 . The defect in the Gottfried sum rule may be ascribed to a flavor asymmetry in the sea of the proton, which does not imply the breaking of the isospin symmetry, since the proton is not an isoscalar. A flavor asymmetry of the parton sea in the proton was advocated many years ago,¹⁷⁾ in connection with the Pauli principle and an essential asymmetry in the proton, which contains two valence u and one valence d quark. The sign of the asymmetry is just the right one to account for the defect in the Gottfried sum rule, $\bar{d} - \bar{u} > 0$, while to reproduce the value given by NMC, one should have

$$\bar{d} - \bar{u} = \int_0^1 [\bar{d}(x) - \bar{u}(x)] dx = 0.15 \pm 0.04. \quad (7)$$

The conservation of the $SU(3)$ non-singlet weak currents gives

$$\Delta u_{\text{val}} = 2F, \quad \Delta d_{\text{val}} = F - D. \quad (8)$$

Since in the proton u^\uparrow is the valence parton with the highest first moment (the quark model sum rules would imply $u^\uparrow = 1 + F = 1.459 \pm 0.006$), it is reasonable to assume, inspired by the Pauli principle and Eq. (7), that it is just this parton which receives less contribution from the sea and to write

$$\Delta u_{\text{val}} = 2F + \bar{u} - \bar{d}. \quad (9)$$

This implies a defect of about 0.033 (0.008) in the Ellis-Jaffe sum rule for the proton (neutron) and — what would be very relevant — a defect of about 0.025 in the

Bjorken sum rule. In this framework one should expect

$$\begin{aligned}\Gamma_1^p &= \frac{F}{2} - \frac{D}{18} + \frac{2}{9}(\bar{u} - \bar{d}) = 0.152 \pm 0.010, \\ \Gamma_1^n &= \frac{F}{3} - \frac{2D}{9} + \frac{1}{18}(\bar{u} - \bar{d}) = -0.033 \pm 0.004,\end{aligned}\quad (10)$$

in reasonable agreement with experiment.

Our strategy, to make a test for these two different interpretations, will be to compare experimental data with the predictions of parton distributions, whose properties will be dictated by these two assumptions. Since the data extend on a wide Q^2 range, we need a method to solve the integro-differential evolution equations¹⁸⁾ at the next-to-leading order (NLO). For this purpose, we choose to reconstruct the SF by using a truncated Jacobi polynomial expansion.^{19), 20)} The main advantage of this method is that it is fast, since one can analytically calculate the moments (Mellin transforms) of the SF which enter the expansion. Indeed, this was already used in Ref. 21) for a NLO fit of the unpolarized distributions and in Ref. 13) for the polarized ones.

This paper is organized as follows. In the next section we present the parameterization used to describe the SLAC data, with different options inspired by the two interpretations previously considered. Then we present the Jacobi reconstruction method and recall the QCD evolution of the moments. Finally, we report the results of our analysis for the SLAC data and the QCD evolution to the SMC higher Q^2 value, before giving our conclusions.

§2. Description of the SLAC data

With respect to our previous paper,²²⁾ where the analysis of the data was performed at leading order in α_s and in the chiral-invariant factorization scheme, here we take advantage of the knowledge of the NLO anomalous dimensions and coefficient functions within the \overline{MS} renormalization scheme, which is the most usual one chosen in the gauge-invariant factorization scheme. Therefore, instead of Eq. (13) of our previous paper, we write (we assume a flavor symmetric sea, $\Delta u_{\text{sea}} = \Delta \bar{u}_{\text{sea}} = \Delta d_{\text{sea}} = \Delta \bar{d}_{\text{sea}} = \Delta s_{\text{sea}} = \Delta \bar{s}_{\text{sea}} \equiv \Delta q_{\text{sea}}$):

$$\begin{aligned}x\Delta u_{\text{val}}(x, Q_0^2) &= \eta_u A_u x^{a_u} (1-x)^{b_u} (1+\gamma_u x), \\ x\Delta d_{\text{val}}(x, Q_0^2) &= \eta_d A_d x^{a_d} (1-x)^{b_d} (1+\gamma_d x), \\ x\Delta q_{\text{sea}}(x, Q_0^2) &= x\Delta \bar{q}_{\text{sea}}(x, Q_0^2) = \eta_s A_s x^{a_s} (1-x)^{b_s} (1+\gamma_s x), \\ x\Delta G(x, Q_0^2) &= \eta_G A_G x^{a_G} (1-x)^{b_G} (1+\gamma_G x),\end{aligned}\quad (11)$$

at $Q_0^2 = 4 \text{ GeV}^2$, where η_q ($q = u, d, s, G$) are the first moments of the distributions, and $A_q = A_q(a_q, b_q, \gamma_q)$ is given by

$$A_q^{-1} = \int_0^1 dx x^{a_q-1} (1-x)^{b_q} (1+\gamma_q x) = \left(1 + \gamma_q \frac{a_q}{a_q + b_q + 1}\right) \frac{\Gamma(a_q)\Gamma(b_q + 1)}{\Gamma(a_q + b_q + 1)}, \quad (12)$$

in such a way that

$$\int_0^1 dx A_q x^{a_q-1} (1-x)^{b_q} (1+\gamma_q x) = 1. \quad (13)$$

We fix $\gamma_s = \gamma_G = 0$, since it is not possible to fit the values of these parameters with sufficient accuracy.

We fit the asymmetries

$$A_1^{p(n)}(x, Q^2) = \frac{g_1^{p(n)}(x, Q^2)}{F_1^{p(n)}(x, Q^2)} = \frac{g_1^{p(n)}(x, Q^2)}{F_2^{p(n)}(x, Q^2)} 2x(1 + R^{p(n)}(x, Q^2)), \quad (14)$$

which are just the quantities experimentally measured. The choice of A_1 would also allow one to minimize the higher twist contribution (that should partly cancel in the ratio in Eq. (14)), instead of cutting the data at low Q^2 . In constructing the ratio in Eq. (14), we use the MRS parameterization at $Q_0^2 = 4 \text{ GeV}^2$ ²³ for the unpolarized distributions. The deuteron asymmetry, $A_1^d(x, Q^2)$, is given by

$$A_1^d(x, Q^2) = \frac{g_1^d(x, Q^2)}{F_1^d(x, Q^2)}, \quad (15)$$

with

$$g_1^d(x, Q^2) = \frac{1}{2} \left(1 - \frac{3}{2} \omega_D \right) (g_1^p(x, Q^2) + g_1^n(x, Q^2)), \quad (16)$$

where $\omega_D = 0.058$ ²⁴ takes into account the small D-wave component in the deuteron ground state. We only include in the fit the data obtained by the SLAC experiments.⁴ The HERMES data⁶ are not used since their lower precision does not change the results. We will compare the results with the SMC proton and deuteron data,⁵ by evolving the distributions to the Q^2 of the CERN experiment.

In all the cases we fix

$$\eta_d = F - D = -0.339 \pm 0.013, \quad (17)$$

as expected from the conservation of the $SU(3)$ non-singlet weak currents. The two different interpretations mentioned above are characterized, for the first option (see fits **A**, **B** and **C** below), by the additional constraint

$$\eta_u = 2F = 0.918 \pm 0.013, \quad (18)$$

in such a way to obey the Bjorken sum rule with the first order correction, Eq. (5), consistently with the NLO approximation. For the second option (see fits **D**, **E**, **F** and **G** below) η_u is a free parameter, and the Bjorken sum rule departs from the theoretical expectation. The sea and gluon parameters, η_s and η_G , are left free in both approaches (we impose $\eta_G \leq 3$ since, for high values of this parameter, the χ^2 exhibits a weak dependence on it), and we will also consider cases with vanishing η_s or (and) η_G to evaluate from the data how much their contributions are needed in the two cases. We also consider a case (fit **A**) with η_s fixed by requiring that the NLO value of F_1^p at $\langle Q^2 \rangle = 3 \text{ GeV}^2$ should be that reported by SLAC E143, namely

0.127 ± 0.011 . Further limitations on the parameters are

$$\begin{aligned}
 & a_v > a_s > a_G \\
 b_u > 3.96, \quad b_d > 4.41, \quad b_s > 10.1, \quad b_G > 6.06,
 \end{aligned}
 \tag{19}$$

in order to satisfy the positivity constraints with respect to the unpolarized distributions.

§3. The Jacobi reconstruction method

There are different methods to solve the evolution equations¹⁸⁾ for the distributions (or the SF). One possibility is to numerically solve them in x -space.^{25), 26)} A faster choice is based on the use of the Mellin moments,

$$F_n(Q^2) \equiv \int_0^1 x^{n-1} F(x, Q^2) dx,
 \tag{20}$$

since they are analytically calculable for distributions like the ones in Eq. (11). After transforming the integro-differential equations to the n -space, they become ordinary differential equations in the Q^2 variable, which are analytically solved, and the solution is then numerically inverted to give the distributions in the x -space.

An alternative choice, which avoids the numerical inversion, is to express the SF by means of orthogonal polynomials.²⁷⁾ On the one hand, this method is more favorable, because it allows one to use the analytical expressions of the Mellin moments; on the other hand, a positive feature is also their weak dependence on the values of the reconstructed function outside the region where data are collected.

The expansion of the SF in terms of Jacobi polynomials, $\Theta_k^{\alpha\beta}(x)$, takes the form

$$F(x, Q^2) = x^\alpha (1-x)^\beta \sum_{k=0}^{\infty} a_k^{(\alpha\beta)}(Q^2) \Theta_k^{\alpha\beta}(x),
 \tag{21}$$

where the Jacobi polynomials satisfy the orthogonality condition

$$\int_0^1 x^\alpha (1-x)^\beta \Theta_k^{\alpha\beta} \Theta_l^{\alpha\beta} dx = \delta_{kl},
 \tag{22}$$

and the coefficients $a_k^{(\alpha\beta)}(Q^2)$ are defined by

$$a_k^{(\alpha\beta)}(Q^2) = \int_0^1 F(x, Q^2) \Theta_k^{\alpha\beta}(x) dx.
 \tag{23}$$

The expansion in Eq. (21) becomes useful if we consider the power expansion of Jacobi polynomials,

$$\Theta_k^{\alpha\beta}(x) = \sum_{j=0}^k c_j^{(k)}(\alpha, \beta) x^j,
 \tag{24}$$

with $((\alpha)_k \equiv \Gamma(\alpha + k)/\Gamma(\alpha))$ is the Pochhammer symbol)

$$\begin{aligned}
 c_j^{(k)}(\alpha, \beta) &= (-1)^j \binom{k}{j} \\
 &\times \frac{(\alpha + 1)_k (\alpha + \beta + k + 1)_j}{(\alpha + 1)_j} \\
 &\times \sqrt{\frac{(\alpha + \beta + 2k + 1) \Gamma(\alpha + \beta + k + 1)}{\Gamma(\alpha + k + 1) \Gamma(\beta + k + 1) k!}}.
 \end{aligned} \tag{25}$$

By inserting this into Eq. (23), we have

$$a_k^{(\alpha\beta)}(Q^2) = \sum_{j=0}^k c_j^{(k)}(\alpha, \beta) F_{j+1}(Q^2), \tag{26}$$

and for $F(x, Q^2)$ we obtain

$$F(x, Q^2) = x^\alpha (1-x)^\beta \sum_{k=0}^{\infty} \Theta_k^{\alpha\beta}(x) \sum_{j=0}^k c_j^{(k)}(\alpha, \beta) F_{j+1}(Q^2). \tag{27}$$

This formula is very useful because the Q^2 dependence of $F(x, Q^2)$ is contained in its moments, for which the solution of the evolution equations up to the NLO is well known. Evidently, one must approximate Eq. (27) by truncating the infinite series at a finite number of terms:

$$F^{(N)}(x) = x^\alpha (1-x)^\beta \sum_{k=0}^N J_k^{\alpha\beta}(x) \sum_{j=0}^k c_j^{(k)}(\alpha, \beta) F_{j+1}. \tag{28}$$

This truncation implies that outside the range where data have their values, the approximation can fail, and it is possible that oscillations appear; however, by choosing a suitable value of N , one can reconstruct the function with sufficient precision. In our analysis, we reconstructed the unpolarized SF with $N = 16$, $\alpha = -0.99$ and $\beta = 4.03$, while for the polarized ones we used $N = 8$ and allowed α and β to vary. The values found in the different cases considered below are reported in Table I.

§4. Evolution of the QCD moments

Given the Mellin moments of the charge-conjugation even (odd) non-singlet, singlet and gluon unpolarized distribution, at Q_0^2 we have

$$\begin{aligned}
 Q_{Ns}^\eta(Q_0^2) &\equiv \int_0^1 x^{n-1} Q_{Ns}^\eta(x, Q_0^2) dx, & (\eta = \pm 1) \\
 \Sigma_n(Q_0^2) &\equiv \int_0^1 x^{n-1} \Sigma(x, Q_0^2) dx, \\
 G_n(Q_0^2) &\equiv \int_0^1 x^{n-1} G(x, Q_0^2) dx,
 \end{aligned} \tag{29}$$

Table I. The values of the parameters α and β , defined in Eq. (21), for the polarized structure functions corresponding to the different fits (see §3).

	α	β
A	0.173	3.03
B	1.14	4.20
C	0.265	2.27
D	1.37	4.32
E	1.34	4.21
F	0.247	3.45
G	1.36	0.157

their evolution with Q^2 is given by the renormalization group equations. In the \overline{MS} scheme one obtains, for example for the non-singlet distribution,

$$Q_{NSn}^\eta(Q^2) = Q_{NSn}^\eta(Q_0^2) \left(\frac{\alpha_s^{\text{NLO}}(Q^2)}{\alpha_s^{\text{NLO}}(Q_0^2)} \right)^{\frac{\gamma_{NS}^{(0)n}}{2\beta_0}} \times \left[1 + \frac{\alpha_s^{\text{LO}}(Q^2) - \alpha_s^{\text{LO}}(Q_0^2)}{4\pi} \left(\frac{\gamma_{NS}^{(1)n\eta}}{2\beta_0} - \frac{\gamma_{NS}^{(0)n}\beta_1}{2\beta_0^2} \right) \right]. \quad (30)$$

The complete expressions for the evolution of the distributions can be found in the Appendix. $\gamma_{NS}^{(i)n}$ and $\gamma_{\psi\psi}^{(i)n}$ (the latter appears in Eq. (A.1)) enter the unpolarized anomalous dimensions for the non-singlet and singlet operators at NLO in the perturbative expansion

$$\gamma_a^{n\eta} = \frac{\alpha_s}{4\pi} \left(\gamma_a^{(0)n} + \frac{\alpha_s}{4\pi} \gamma_a^{(1)n\eta} \right), \quad (31)$$

($\eta = \pm 1$ and $\eta = 1$ for the non-singlet and the singlet respectively), whose expressions were calculated in Refs. 28) ~ 30). We also have

$$\frac{\alpha_s^{\text{NLO}}(Q^2)}{4\pi} = \frac{1}{\beta_0 \ln \frac{Q^2}{\Lambda^2}} - \frac{\beta_1 \ln \frac{Q^2}{\Lambda^2}}{\beta_0^3 \ln^2 \frac{Q^2}{\Lambda^2}},$$

$$\beta_0 = 11 - \frac{2}{3}n_f, \quad \beta_1 = 102 - \frac{38}{3}n_f, \quad (32)$$

which is the QCD coupling constant at NLO (the LO expression is obtained by setting $\beta_1 = 0$ in Eq. (32)). According to the number of active flavors n_f , the value of $\Lambda_{\text{NLO}}^{(n_f)}$ is modified. We use $\Lambda_{\text{NLO}}^{(5)} = 0.2263$, so to have $\alpha_s(M_Z^2) = 0.118$.³¹⁾ From the matching of α_s at the quark thresholds, we obtain, for $m_b = 4.5$ GeV, $\Lambda_{\text{NLO}}^{(4)} = 0.326$ and $\Lambda_{\text{NLO}}^{(3)} = 0.3797$, for $m_c = 1.5$ GeV.

Expressions analogous to Eqs. (29) and (A.1) can be written for the polarized distributions, where the polarized anomalous dimensions can be found in Ref. 10). The polarized non-singlet and singlet distributions are defined as

$$Q_{NS}^p \equiv a_3 + \frac{a_8}{3} = \frac{4}{3}\Delta u_{\text{val}} - \frac{2}{3}\Delta d_{\text{val}},$$

$$Q_{NS}^n \equiv -a_3 + \frac{a_8}{3} = -\frac{2}{3}\Delta u_{\text{val}} + \frac{4}{3}\Delta d_{\text{val}},$$

$$\Sigma \equiv a_0 = \Delta u_{\text{val}} + \Delta d_{\text{val}} + 6\Delta q_{\text{sea}}. \quad (33)$$

For the evolution of the polarized moments we used the fixed-flavor scheme^{32), 10)} with $n_f = 3$.

Since the coefficients $c_j^{(k)}$ in Eq. (25) contain in the denominator higher and higher factorials as N becomes large, we must take into account the corrections to the anomalous dimensions described in Ref. 20). They depend on the type of distribution we consider, even or odd (+ or - prescription, respectively), and it amounts to the substitutions in the anomalous dimensions reported in the Appendix.

§5. Data analysis

By considering the SLAC data of the E142, E143 and E154 experiments,⁴⁾ we obtain the results reported in Tables II and III.

A quick study of Tables II and III is very instructive. It shows that one is able to obtain satisfactory fits to the data with assumptions inspired from both the two different options we have considered. First η_u and η_d fixed to the values expected by $SU(3)$ non-singlet current conservation, with the Bjorken sum rule obeyed and η_s given by the defect in the EJ sum rule (fit **A**). Second η_d fixed to that value and without sea and gluon contributions for the option inspired by the role of the Pauli principle (fit **D**). The first option is more successful in describing the neutron (^3He) data (a total contribution to the χ^2 of 7 instead of 20 for the 19 data points of E142 and E154) and deuteron data (a contribution to the χ^2 of 55 instead of 63 for 56 data points), while the second one is slightly preferable for the proton data (a contribution to the χ^2 of 55 instead of 74 for 63 data points). We also note that, with the first option, by taking $\eta_s = 0$ we obtain a rather bad fit (fit **C**).

Fits **A** and **D** differ in the x dependence of the valence parton distribution which is more singular at $x \rightarrow 0$ ($x^{-0.427}$) for fit **A** than for fit **D** ($x^{-0.119}$). They also differ in the behavior for $x \rightarrow 1$, where one has a faster power decreasing of the d partons for fit **D** ($(1-x)^{6.50}$) than for fit **A** ($(1-x)^{4.66}$). It is appealing, from the point of view of the interpretation inspired by the Pauli principle, to find in the case of fit **D**, $\Delta u_{\text{val}} = 0.741 \simeq 0.918 - 0.150 = 0.768$, as expected by the connection to the defect in the Gottfried sum rule, but it is worth remarking that the better description of the ^3He data by fit **A** seems more relevant than the better description of the proton by fit **D**. When we leave free η_u and η_s , with or without gluons (fits **F** and **G**), we

Table II. The values of the parameters found with the parameterization in Eq. (11) for the different fits corresponding to the first option (see the text for more information). The results for the r.h.s. of Ellis-Jaffe and Bjorken sum rules are also shown. The asterisk (*) indicates fixed values.

	A	B	C
η_u	$0.918 \pm 0.013^*$	$0.918 \pm 0.013^*$	$0.918 \pm 0.013^*$
η_d	$-0.339 \pm 0.013^*$	$-0.339 \pm 0.013^*$	$-0.339 \pm 0.013^*$
η_s	$-0.065 \pm 0.019^*$	-0.050 ± 0.015	0^*
η_G	1.69 ± 0.64	0^*	3.00 ± 0.08
a_v	0.573 ± 0.057	0.587 ± 0.057	0.320 ± 0.046
a_s	0.338 ± 0.080	0.587 ± 0.063	-
a_G	0.338 ± 0.082	-	0.312 ± 0.106
b_u	3.96 ± 0.02	3.96 ± 0.02	3.960 ± 0.008
b_d	4.66 ± 0.35	4.71 ± 0.35	5.79 ± 0.37
b_s	10.1 ± 0.8	10.1 ± 0.3	-
b_G	6.06 ± 0.22	-	8.54 ± 1.08
γ_v	7.53 ± 2.06	7.38 ± 1.93	28.3 ± 7.7
EJ_p	0.128 ± 0.012	0.137 ± 0.009	0.167 ± 0.003
EJ_n	-0.061 ± 0.012	-0.052 ± 0.009	-0.022 ± 0.003
Bj	$0.189 \pm 0.003^*$	$0.189 \pm 0.003^*$	$0.189 \pm 0.003^*$
χ^2/NDF	1.07	1.07	1.64

Table III. Same as Table II for fits corresponding to the second option.

	D	E	F	G
η_u	0.741 ± 0.016	0.754 ± 0.021	0.822 ± 0.024	0.820 ± 0.038
η_d	$-0.339 \pm 0.013^*$	$-0.339 \pm 0.013^*$	$-0.339 \pm 0.013^*$	$-0.339 \pm 0.013^*$
η_s	0^*	0^*	-0.040 ± 0.020	-0.028 ± 0.011
η_G	0^*	3.00 ± 3.00	1.35 ± 0.23	0^*
a_v	0.881 ± 0.087	0.853 ± 0.091	0.730 ± 0.075	0.682 ± 0.090
a_s	-	-	0.730 ± 0.163	0.522 ± 0.220
a_G	-	0.053 ± 0.152	0.730 ± 0.185	-
b_u	3.960 ± 0.003	3.96 ± 0.06	3.96 ± 0.04	3.960 ± 0.003
b_d	6.50 ± 0.46	6.25 ± 0.47	5.36 ± 0.44	5.41 ± 0.50
b_s	-	-	41.5 ± 11.1	10.1 ± 0.4
b_G	-	6.06 ± 3.91	6.06 ± 2.48	-
γ_v	3.70 ± 1.56	3.97 ± 1.66	5.64 ± 1.89	6.65 ± 2.50
EJ_p	0.132 ± 0.003	0.134 ± 0.004	0.124 ± 0.013	0.131 ± 0.010
EJ_n	-0.031 ± 0.003	-0.030 ± 0.003	-0.051 ± 0.012	-0.044 ± 0.007
Bj	0.162 ± 0.003	0.164 ± 0.004	0.175 ± 0.004	0.174 ± 0.006
χ^2/NDF	1.05	1.07	0.97	1.01

find slightly better fits, with a value for η_u in fit **F**, almost the **A** and **D** average:

$$\begin{aligned}
 \eta_u^{\text{F}} &= 0.822 \simeq \frac{\eta_u^{\text{A}} + \eta_u^{\text{D}}}{2} = 0.830, \\
 \eta_s^{\text{F}} &= -0.040 \simeq \frac{3}{5} \eta_s^{\text{A}}, \\
 \eta_G^{\text{F}} &= 1.35 \simeq \eta_G^{\text{A}}.
 \end{aligned} \tag{34}$$

In Fig. 1 the polarized parton distributions are plotted for fits **A** and **D**, and here we note that fit **A** is very similar to a solution obtained in Ref. 13) (see Fig. 5). In Fig. 2 we compare the predictions of the same fits with the SLAC data. In Fig. 3 we show the evolution of fits **A** and **D** to $\langle Q^2 \rangle = 10 \text{ GeV}^2$, compared with the SMC data, which are consistent with both of them. For this evolution we used the FORTRAN code by Fasching,^{25),*)} because the SMC data extend on a different range of x and Q^2 , and the previously calculated α and β in the Jacobi reconstruction of the structure functions would not allow one to describe them well.

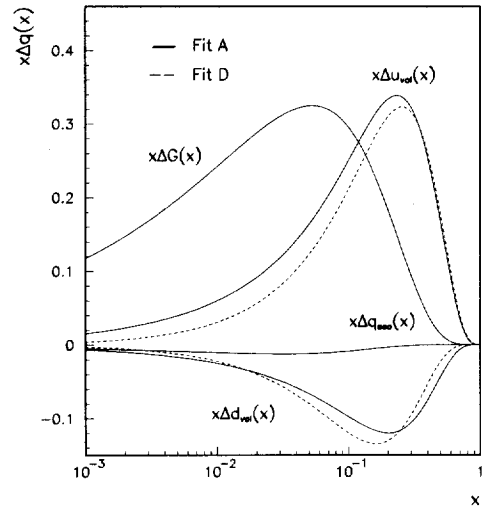


Fig. 1. The polarized parton distributions for fits **A** and **D** at $Q_0^2 = 4 \text{ GeV}^2$. Note that, in the case of fit **D**, sea and gluons are vanishing.

*) We thank the author for providing us his code.

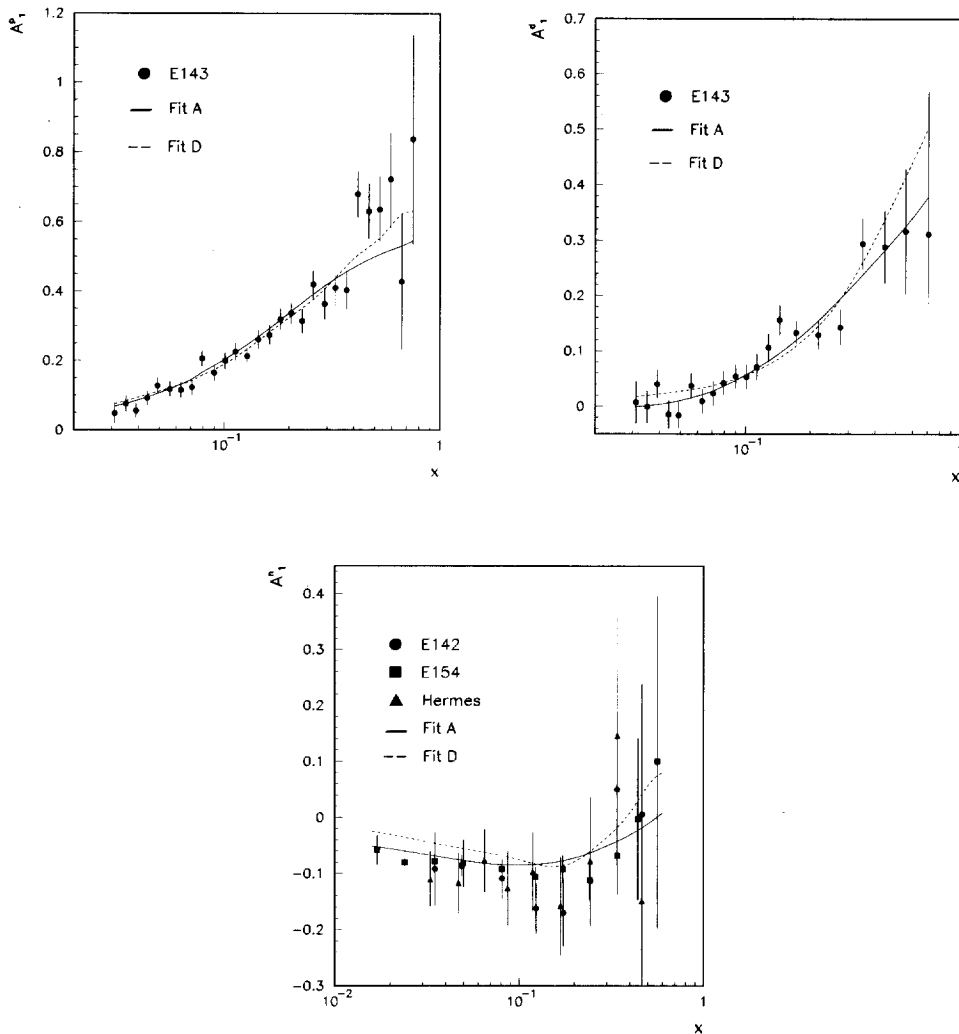


Fig. 2. The prediction of fits **A** and **D** with data are shown for proton, deuteron and neutron asymmetries. The curves correspond to the fits at the Q^2 of the data points for proton and deuteron, and at $Q_0^2 = 4 \text{ GeV}^2$ for neutron. The HERMES data⁶⁾ are shown, but they are not used in the fits.

§6. Conclusions

We have studied the most accurate available data on polarized deep inelastic scattering with the purpose of testing two possible theoretical interpretations of the *spin crisis*. As a result of our analysis, we can conclude that the interpretation related to the Pauli principle is not contradicted by the rather precise SLAC data which, however, slightly favor the generally accepted solution with the Bjorken sum rule obeyed and a sea contribution to the proton spin, in the gauge-invariant factorization scheme. Obviously, a better determination of the small x region could help to further

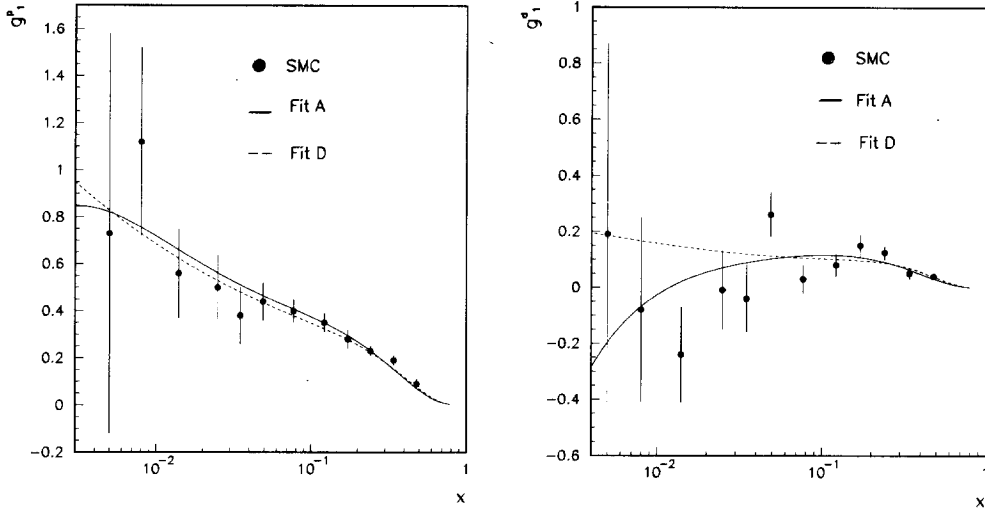


Fig. 3. The predictions of fits **A** and **D** for the proton and deuteron g_1 structure functions at $(Q^2) = 10 \text{ GeV}^2$ are compared with the most recent data obtained by SMC (see Ref. 5)).

clarify the present situation.

Appendix

The complete solutions of the evolution equations for the non-singlet, singlet and gluon distributions, in the \overline{MS} scheme, are as follows:

$$\begin{aligned}
 Q_{NSn}^n(Q^2) &= Q_{NSn}^n(Q_0^2) \left(\frac{\alpha_s^{\text{NLO}}(Q^2)}{\alpha_s^{\text{NLO}}(Q_0^2)} \right)^{\frac{\gamma_{NS}^{(0)n}}{2\beta_0}} \\
 &\quad \cdot \left[1 + \frac{\alpha_s^{\text{LO}}(Q^2) - \alpha_s^{\text{LO}}(Q_0^2)}{4\pi} \left(\frac{\gamma_{NS}^{(1)n\eta}}{2\beta_0} - \frac{\gamma_{NS}^{(0)n}\beta_1}{2\beta_0^2} \right) \right], \\
 \Sigma_n(Q^2) &= [(1 - \alpha_n) \Sigma_n(Q_0^2) - \tilde{\alpha}_n G_n(Q_0^2)] \left(\frac{\alpha_s^{\text{NLO}}(Q^2)}{\alpha_s^{\text{NLO}}(Q_0^2)} \right)^{\frac{\lambda_n^+}{2\beta_0}} \\
 &\quad \cdot \left\{ 1 + \frac{\alpha_s^{\text{LO}}(Q^2) - \alpha_s^{\text{LO}}(Q_0^2)}{4\pi} \left(\frac{\gamma_{++}^{(1)n}}{2\beta_0} - \frac{\lambda_n^+\beta_1}{2\beta_0^2} \right) + \left[\frac{\alpha_s^{\text{LO}}(Q_0^2)}{4\pi} \left(\frac{\alpha_s^{\text{LO}}(Q^2)}{\alpha_s^{\text{LO}}(Q_0^2)} \right)^{\frac{\lambda_n^+ - \lambda_n^-}{2\beta_0}} \right. \right. \\
 &\quad \left. \left. - \frac{\alpha_s^{\text{LO}}(Q^2)}{4\pi} \right] \frac{\gamma_{+-}^{(1)n}}{2\beta_0 + \lambda_n^+ - \lambda_n^-} \right\} \\
 &\quad + [\alpha_n \Sigma_n(Q_0^2) - \tilde{\alpha}_n G_n(Q_0^2)] \left(\frac{\alpha_s^{\text{NLO}}(Q^2)}{\alpha_s^{\text{NLO}}(Q_0^2)} \right)^{\frac{\lambda_n^-}{2\beta_0}}
 \end{aligned}$$

$$\begin{aligned}
& \left\{ 1 + \frac{\alpha_s^{\text{LO}}(Q^2) - \alpha_s^{\text{LO}}(Q_0^2)}{4\pi} \left(\frac{\gamma_{--}^{(1)n}}{2\beta_0} - \frac{\lambda_-^n \beta_1}{2\beta_0^2} \right) + \left[\frac{\alpha_s^{\text{LO}}(Q_0^2)}{4\pi} \left(\frac{\alpha_s^{\text{LO}}(Q^2)}{\alpha_s^{\text{LO}}(Q_0^2)} \right)^{\frac{\lambda_+^n - \lambda_-^n}{2\beta_0}} \right. \right. \\
& \quad \left. \left. - \frac{\alpha_s^{\text{LO}}(Q^2)}{4\pi} \right] \frac{\gamma_{-+}^{(1)n}}{2\beta_0 + \lambda_-^n - \lambda_+^n} \right\}, \\
G_n(Q^2) &= [\alpha_n G_n(Q_0^2) - \epsilon_n \Sigma_n(Q_0^2)] \left(\frac{\alpha_s^{\text{NLO}}(Q^2)}{\alpha_s^{\text{NLO}}(Q_0^2)} \right)^{\frac{\lambda_-^n}{2\beta_0}} \\
& \cdot \left\{ 1 + \frac{\alpha_s^{\text{LO}}(Q^2) - \alpha_s^{\text{LO}}(Q_0^2)}{4\pi} \left(\frac{\gamma_{++}^{(1)n}}{2\beta_0} - \frac{\lambda_+^n \beta_1}{2\beta_0^2} \right) + \left[\frac{\alpha_s^{\text{LO}}(Q_0^2)}{4\pi} \left(\frac{\alpha_s^{\text{LO}}(Q^2)}{\alpha_s^{\text{LO}}(Q_0^2)} \right)^{\frac{\lambda_+^n - \lambda_-^n}{2\beta_0}} \right. \right. \\
& \quad \left. \left. - \frac{\alpha_s^{\text{LO}}(Q^2)}{4\pi} \right] \frac{\gamma_{+-}^{(1)n}}{2\beta_0 + \lambda_+^n - \lambda_-^n} \frac{\gamma_{\psi\psi}^{(0)n} - \lambda_-^n}{\gamma_{\psi\psi}^{(0)n} - \lambda_+^n} \right\} \\
& + [(1 - \alpha_n) G_n(Q_0^2) - \epsilon_n \Sigma_n(Q_0^2)] \left(\frac{\alpha_s^{\text{NLO}}(Q^2)}{\alpha_s^{\text{NLO}}(Q_0^2)} \right)^{\frac{\lambda_-^n}{2\beta_0}} \\
& \cdot \left\{ 1 + \frac{\alpha_s^{\text{LO}}(Q^2) - \alpha_s^{\text{LO}}(Q_0^2)}{4\pi} \left(\frac{\gamma_{--}^{(1)n}}{2\beta_0} - \frac{\lambda_-^n \beta_1}{2\beta_0^2} \right) + \left[\frac{\alpha_s^{\text{LO}}(Q_0^2)}{4\pi} \left(\frac{\alpha_s^{\text{LO}}(Q^2)}{\alpha_s^{\text{LO}}(Q_0^2)} \right)^{\frac{\lambda_+^n - \lambda_-^n}{2\beta_0}} \right. \right. \\
& \quad \left. \left. - \frac{\alpha_s^{\text{LO}}(Q^2)}{4\pi} \right] \frac{\gamma_{-+}^{(1)n}}{2\beta_0 + \lambda_-^n - \lambda_+^n} \frac{\gamma_{\psi\psi}^{(0)n} - \lambda_+^n}{\gamma_{\psi\psi}^{(0)n} - \lambda_-^n} \right\}. \tag{A.1}
\end{aligned}$$

The quantities α_n , $\tilde{\alpha}_n$, ϵ_n , λ_{\pm}^n , $\gamma_{\pm\pm}$, $\gamma_{NS}^{(i)n}$ and $\gamma_{\psi\psi}^{(i)n}$ are defined and given in Ref. 33). The corrections to the anomalous dimensions referred to in §4 consist of the following substitutions:³⁴⁾

$$\begin{aligned}
(-1)^n &\rightarrow \pm 1, \\
S'_2\left(\frac{n}{2}\right) &\rightarrow (-1)^n \left\{ \pm S'_2\left(\frac{n}{2}\right) + \eta_{\pm}(n) [-2S_2(n) + \zeta(2)] \right\}, \\
S'_3\left(\frac{n}{2}\right) &\rightarrow (-1)^n \left\{ \pm S'_3\left(\frac{n}{2}\right) + \eta_{\pm}(n) [-4S_3(n) + 3\zeta(3)] \right\}, \\
\tilde{S}(n) &\rightarrow (-1)^n \left[\pm \tilde{S}(n) + \eta_{\pm}(n) \frac{5}{8} \zeta(3) \right], \tag{A.2}
\end{aligned}$$

where the series $S_k(n)$, $S'_k(n/2)$ and $\tilde{S}(n)$ are defined, for example, in Ref. 29), and ζ is the Riemann zeta function.

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