Integration of Technological and Statistical Knowledge for Reliability Control

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1. Introduction

The frequent scarcity of life data (caused by the high cost of the items, the very high level of their reliability, etc.) makes it impossible to provide reliability estimates with a confidence interval of practical value. To overcome this difficulty we can analytically *integrate* the designers' and statisticians' knowledge, using reliability estimators based on the application of Bayes theorem that substantially says:

$$(1) \quad \begin{pmatrix}\text{posterior joint probability density}\\ \text{of unknown parameters}\end{pmatrix} \propto \begin{pmatrix}\text{their joint prior}\\ \text{probability density}\end{pmatrix} \times \begin{pmatrix}\text{likelihood}\\ \text{function}\end{pmatrix}$$

The technological prior knowledge is summarized into joint prior and the statistical information (life data and shape of the reliability model) is included into likelihood.

Using prior technological knowledge improves efficiency since it immediately indicates the statistical method *where* reasonable estimates are located. By means of three applicative examples, this paper shows the use of some specific bayesian reliability estimators. They are known as "Practical Bayes Estimators" (PBE) since they were developed from an Engineers' point of view and applied in several technological contexts during past years [1][2][3][4][5]. They to reduce the needed number of failure data reinforcing the statistical experimental information by incorporating the available "a priori" Engineers' knowledge in the estimation procedure.

2. Applicative example 1: pseudo random reliability data

Consider the following pseudo-random sample generated according to the Weibull distribution with scale parameter $\alpha = 1$ and shape parameter $\beta = 3$:

(2) 0.69, 0.97, 1.07.

Suppose that technological consideration suggest that the value of the unknown parameter β must stay in the interval (1, 3) being the involved failure mechanism neither of early type $(\beta < 1)$ nor of late wear-out type $(\beta > 2.5 \div 3)$. Besides, suppose that on the basis of routine design evaluations and past experience a reliable life $x_{0.98} = 0.59$ is expected, that is 98% of the produced items is expected to overcome the running time equal to 0.59 without any failure.

Table 1. Bayes and Maximum Likelihood parameter estimates based on the sample data (2).

estimates	$\hat{x}_{0.98}$	$\hat{\beta}$
PBE	0.27	2.61
MLE	0.57	7.34

This prior information is not very precise, since it barely includes the true value of β and anticipates an 84% biased percentile, being the true value equal to 0.27 and not 0.59. However, the PBE give the estimates of Table 1 compared with those obtained using the classical Maximum Likelihood method.

3. Applicative example 2: field reliability data

The design team of a new component, for a heavy duty truck, collected the following experimental pieces of information: *a)* only 2 failure data, 2.3 and 8.0 hundreds of thousands of cycles; *b)* the whole team is confident that 99.9% of the items will overcome 2 hundred thousand cycles without any problems; *c)* the team is in a position to support the Weibull distribution hypothesis and anticipate the interval (1, 3) for the shape parameter.

However, the team believes that the most critical piece of prior information is the fraction $(R = 0.999)$ of the items that will overcome 2 hundred thousand cycles without any problems. So they decide to test the sensitiveness of the estimates to the anticipated value for R, considering also the alternative values $R = 0.98$ and $R = 0.95$. Using the PBE, the estimates reported in Table 2 are obtained.

Table 2. PBE based on the above information supplied by a design team, considering three alternative values for R.

adopted value for R	$\hat{x}_R$	$\hat{\beta}$
0.999	0.878	2.654
0.980	1.204	2.170
0.950	1.437	1.936

4. Applicative example 3: breaking strength control data

In [7] the control of the first percentile (equivalent to the reliable life x_R, $R = 0.99$) of the breaking strength distribution of a certain type of carbon fibers is considered. This kind of carbon fiber is produced to manufacture composite materials which need fibers with a breaking strength greater than 1.22 GPa (giga-Pascal) with 99% probability.

In order to control that this specification $(x_R = 1.22,\ R = 0.99)$ is met, a sample of $N = 5$ fibers, each 50 mm long, is selected from the manufacturing process periodically, and the breaking stress of each fiber is measured.

When the process was certainly in-control, ten samples of size $N = 5$ of breaking stress of these carbon fibers were sampled (see Table 3, [7]). On the basis of some past knowledge and experience, the Weibull distribution is considered to closely fit such breaking stress and is assumed to be the underlying distribution of the control chart.

Table 3. Breaking stresses (GPa) of carbon fibers (in control state, $x_R = 1.22$, $R = 0.99$).

j-th sample	Stress				
1	3.70	2.74	2.73	2.50	3.60
2	3.11	3.27	2.87	1.47	3.11
3	4.42	2.41	3.19	3.22	1.69
4	3.28	3.09	1.87	3.15	4.90
5	3.75	2.43	2.95	2.97	3.39
6	2.96	2.53	2.67	2.93	3.22
7	3.39	2.81	4.20	3.33	2.55
8	3.31	3.31	2.85	2.56	3.56
9	3.15	2.35	2.55	2.59	2.38
10	2.81	2.77	2.17	2.83	1.92

Differently from previous samples, the ten samples reported in Table 4 [7] are representative of an out-of-control state, characterized by the first percentile shifted to 0.26 GPa from the original value of 1.22 GPa.

Table 4. Breaking stresses (GPa) of carbon fibers (out-of-control state, $x_R = 0.26,\ R = 0.99$).

j-th sample	Stress				
1	1.41	3.68	2.97	1.36	0.98
2	2.76	4.91	3.68	1.84	1.59
3	3.19	1.57	0.81	5.56	1.73
4	1.59	2.00	1.22	1.12	1.71
5	2.17	1.17	5.08	2.48	1.18
6	3.51	2.17	1.69	1.25	4.38
7	1.84	0.39	3.68	2.48	0.85
8	1.61	2.79	4.70	2.03	1.80
9	1.57	1.08	2.03	1.61	2.12
10	1.89	2.88	2.82	2.05	3.65

Figure 1 shows the control chart of the first percentile estimated by means of the PBE. The chart is drawn using the data of Table 3. On the basis of the information provided in [7], the prior interval (2.8, 6.8) for β and the anticipated (mean) value 1.22 for x_R are adopted. Applying the PBE to the whole sample (50 data) the estimates $\hat{\beta} = 4.69$ and $\hat{x}_R = 1.20$ are obtained. So, the central line (CL) is $\hat{x}_R = 1.20$; the upper control limits (UCL) and the lower control limits (LCL) (for both $\alpha = 5\%$ and $\alpha = 0.1\%$) are obtained calculating the needed percentiles from those of the Standard Gamma distribution, thanks to a peculiar property of the PBE discussed in [10]. The chart is then applied to the data reported in Table 4. On the basis of the above estimates, the prior interval $(2.69 = \hat{\beta} - 2;$ $6.69 = \hat{\beta} + 2)$ for β and the anticipated (mean) value $1.20 = \hat{x}_R$ for x_R are used in order to calculate the estimates from all the ten samples of size $N = 5$ (Table 4).

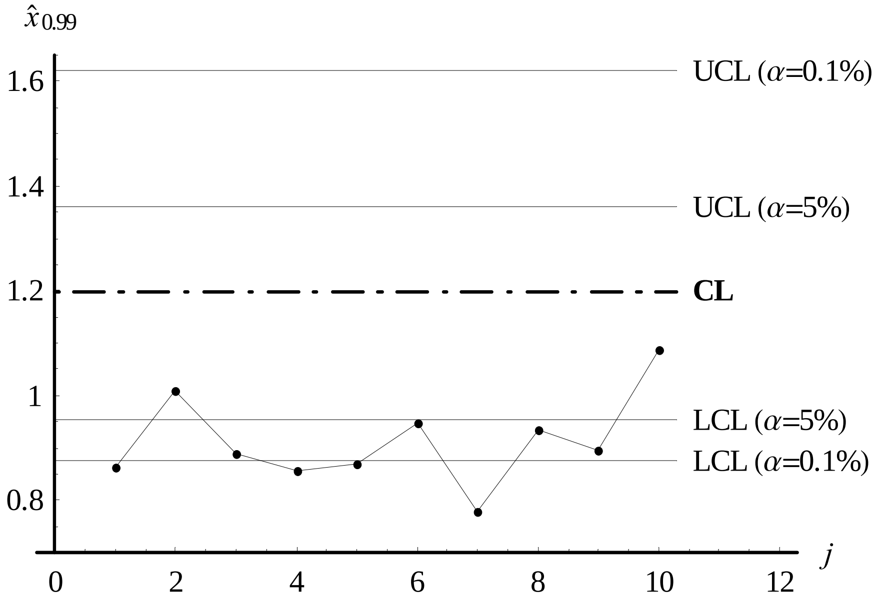

Figure 1 Weibull control chart of the first percentile $(x_R,\ R = 0.99)$ for the out-of-control process data (Table 4).

5. Concluding remarks

When the experimental data are very few (say 2, 3) the PBE can still be used as a simple tool to improve the technological reliability predictions. In fact, in these case, the PBE work as a filter that always improves — in mean — the prior pieces of information if these are poor, or substantially confirms them if they are good. So, we can look at the Bayes theorem as a tool that allows Statistics to help Engineering and not vice versa.

In this case, the controversy about whether to use Bayesian or non-Bayesian methods appears surmounted since alternative classical estimators, like the Maximum Likelihood ones, give estimates that are very often worse than and/or in contrast with elementary technological knowledge.

Finally, the PBE provide a new satisfactory approach to Shewhart-type chart for Weibull percentiles, where very few alternative methods are available [6][7][8][9]. Moreover, the approach can be also used for individual control charts, that is for data which are not collected in subgroups.

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RÉSUMÉ

Ce travail illustre l'utilisation des estimateurs bayesian connues avec le nom "Estimateurs Bayesian Pratiques" car il furent idées du point de vue des ingénieurs. Les estimateurs fondent de façon analytique les informations technologiques avec celles statistique, utilisant le théorème de Bayes comme si il fût un récipient de fusion (mathématique). De tel façon ils donnent plus efficience au proches d'évaluation statistique sans abîmer aucunes informations et aidant l'amélioration de l'estime e le contrôle de la fiabilité. Des exemples concernant l'évaluation de la vie fiable et une nouvelle carte de contrôle de un percentile sont développés à la base de donnés Weibull