Optimal Third-Harmonic Current Injection for an Asymmetrical Nine-phase PMSM with Non-Sinusoidal back-EMF

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Abstract – **The paper investigates an optimal strategy to exploit the third harmonic current injection for the torque enhancement in a nine-phase permanent magnet synchronous machine (PMSM). The machine is with asymmetrical winding configuration and has a single isolated neutral point. The optimization follows the minimization of the average power losses for a given reference torque or, equivalently, the maximization of the developed torque for a given current RMS. It is shown that, in contrast to the situation for a symmetrical configuration, the optimal ratio between the fundamental and the third harmonic components does not correspond to the ratio between the corresponding back-EMF components. It is demonstrated that this is due to the fact that the phase currents have to sum to zero; consequently, the third harmonic current injection in different three-phase sets has to be different with regard to the magnitude and phase shift. The strategy is introduced using an entirely analytical approach and its effectiveness has been successfully validated through numerical simulations.**

Keywords – **Multiphase drives, surface PMSM, non-sinusoidal back-EMF, asymmetrical nine-phase machines, third-harmonic current injection, power losses minimization.**

I. Introduction

Multiphase machines are nowadays a promising alternative to three-phases ones, especially for wind energy harvesting and for traction, including electric/hybrid vehicles, more electric aircraft and ship propulsion [1-3]. This is due to the several benefits they offer for the high-power and high-reliability applications. First, multiphase machines can continue to operate even after a fault on one or more phases (albeit with reduced power), as long as it is possible to generate a rotating field in the machine's air-gap by means of the stator currents [1-3]. Moreover, for a given mechanical load, the input power is split among more phases, thus making it possible to supply the machine with converters whose switching devices require reduced current capabilities, hence resulting in a more reliable overall system [1]. Finally, the existence of multiple phases leads to the presence of additional degrees of freedom which can be exploited for different control purposes [1-6].

A possible way to take advantage of these additional degrees of freedom for permanent magnet synchronous machines is related to the generation of non-sinusoidal magnetic flux densities in the machine's air-gap [1-6]. Indeed, assuming linearity, the flux density can be studied as a superposition of several sinusoidal spatial harmonics of the flux density and, by properly coupling the stator's and rotor's contributions, it is possible to enhance the output torque for given current limits or, equivalently, to reduce the currents for a given torque [1-3].

These strategies lead, in steady state conditions, to non-sinusoidal stator currents and therefore are usually referred to as higher order current harmonic injection. The simplest enhancement can be obtained by exploiting the third-order spatial harmonic of the magnetic flux density, which corresponds to a third harmonic injection (THI) in the currents [4-6]. With this strategy, the developed electromagnetic torque is produced by the separate contribution of the fundamental and of the third harmonic components, and a key problem is to choose the weight of each contribution to the total torque.

By following different approaches, usually related to the minimization of the current RMS and the corresponding power losses, it has been shown that, for an odd number of phases and a symmetrical winding distribution, the optimal ratio between the magnitude of the fundamental and the third harmonic of the stator currents is the same as the ratio between the corresponding magnets' induced back-EMFs [4-10]. These strategies have been effectively studied and tested for five-phases machines in [11-16]. On the other hand, there are very few examples of the application of the third harmonic injection for asymmetrical machines [17]. The main reason for this situation is that, if the third harmonic currents were to sum to zero in the single isolated neutral case, the third harmonic distribution in different three-phase systems has to be different.

The present paper focuses on a nine-phase machine with an asymmetrical winding configuration. It is shown that the third harmonic injection can be effectively exploited for torque enhancement even with an isolated neutral point, making nine the minimum number of phases with this capability among the multiple three-phases configurations (a six-phase topology requires the seventh inverter leg, [18]). However, in contrast to the symmetrical configuration, the optimal third harmonic injection ratio with respect to the fundamental component has to be chosen differently. The approach to evaluate this optimal ratio is completely analytical and based on a proper vector space decomposition (VSD) and rotational transformation. The theoretical results are validated numerically using the parameters of the machine analysed in [19].

The paper is structured as follows. Section II gives the model of the machine with emphasis on the choice of the VSD and rotational transformation matrices. Section III develops the analytical computation of the optimal third harmonic current injection coefficient, while Section IV presents the numerical validation of the approach. Section V concludes the paper.

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II. MATHEMATICAL MACHINE MODEL

The studied machine is an asymmetrical nine-phase PMSM with P_p pole pairs. The stator windings (geometrically identical) can be grouped into three symmetrical three-phase sets $\{a_1,b_1,c_1\}$, $\{a_2,b_2,c_2\}$, $\{a_3,b_3,c_3\}$ which are mutually spatially shifted by $\pi/9$. As a result, the magnetic axes of the machine can be represented through the electrical angle set:

$$[\alpha]=\left[0\ \ \frac{2\pi}{3}\ \ \frac{4\pi}{3}\ \middle|\ \frac{\pi}{9}\ \ \frac{7\pi}{9}\ \ \frac{13\pi}{9}\ \middle|\ \frac{2\pi}{9}\ \ \frac{8\pi}{9}\ \ \frac{14\pi}{9}\right] \quad (1)$$

The windings are star-connected with a single isolated neutral point, and supplied through a voltage source inverter (VSI). The system architecture is depicted in Fig. 1a, while the angular phase relationships are schematically represented in Fig. 1b.

A. Per-phase Electrical Equations

Assuming linearity, the electrical equations in terms of the phase variables are [1]:

$$\begin{cases}[u_{\text{ph}}]+v_{ON}\cdot[1]=[v_{\text{ph}}]=R\cdot[i_{\text{ph}}]+[L]\cdot\frac{\mathrm{d}}{\mathrm{d}t}[i_{\text{ph}}]+[e_{\text{ph}}]\\ [1]^{\mathrm{T}}\cdot[i_{\text{ph}}]=\sum_{k=1}^{9}i_k=0\end{cases} \quad (2)$$

where:

- $[u_{\text{ph}}]$ is the set of the inverter's leg voltages,
- v_{ON} is the voltage between the inverter's dc link mid-point O and the machine's neutral point N,
- $[v_{\text{ph}}]$ is the set of the stator winding voltages,
- $[i_{\text{ph}}]$ is the set of the stator winding currents,
- $[e_{\text{ph}}] = \mathrm{d}[\lambda_{m,\text{ph}}]/\mathrm{d}t$ is the set of the PM induced back-EMFs,
- R is the equivalent winding resistance,
- $[L]$ is the stator winding inductance matrix (which includes both the mutual and the leakage contributions), and
- $[1] = [1,1,\dots,1]^{\mathrm{T}}$ is the unitary 9×1 column vector.

The magnetic flux density in the air-gap, created by both the stator currents and by the rotor's permanent magnets, is non-sinusoidal with respect to the azimuthal electrical angle. Under the assumption of linearity, this can be studied through a Fourier series expansion into an infinite number of spatial harmonics. Assuming that the even-order harmonics are absent and the odd-order ones with the order $h \geq 9$ are negligible, the mathematical model can be further simplified, especially with respect to the electromagnetic torque generation.

By superposition principle, the flux contribution to the k-th stator winding (with $k = 1, \dots, 9$) from the rotor's permanent magnets can be expressed as:

$$\begin{aligned}\lambda_{m,k}(\theta)&=\sum_h \lambda_{Mh}\cos\left(h(\theta-\alpha_k)+\varphi_h\right)=\\ &=\lambda_{M1}\cos(\theta-\alpha_k)+\lambda_{M3}\cos\left(3(\theta-\alpha_k)+\varphi_3\right)+\dots\\ &\dots+\lambda_{M5}\cos\left(5(\theta-\alpha_k)+\varphi_5\right)+\lambda_{M7}\cos\left(7(\theta-\alpha_k)+\varphi_7\right)\end{aligned} \quad (3)$$

where λ_{Mh} and φ_h denote the magnitude and phase displacement of the h-th spatial harmonic contribution, α_k is the k-th element of (1) and denotes the k-th phase winding magnetic axis position, while θ represents the electrical angle between the rotor's and the stator's reference axes.

B. Choice of the VSD Transformation

Each phase variable set $[f_{\text{ph}}]$ of the model (2) can be transformed into a corresponding VSD set $[f_{\text{VSD}}]$:

$$[f_{\text{VSD}}]=[\mathrm{C}]\cdot[f_{\text{ph}}] \quad\Leftrightarrow\quad [f_{\text{ph}}]=[\mathrm{C}]^{-1}\cdot[f_{\text{VSD}}]=[\mathrm{T}]\cdot[f_{\text{VSD}}] \quad (4)$$

In order to highlight both the control of the spatial harmonics for the torque enhancement and the algebraic constraint due to the single isolated neutral point, the chosen transformation is:

$$[\mathrm{C}]=\sqrt{\frac{2}{9}}\cdot\begin{bmatrix}\cos([\alpha])\\ \sin([\alpha])\\ \cos(3\cdot[\alpha])\\ \sin(3\cdot[\alpha])\\ \cos(5\cdot[\alpha])\\ \sin(5\cdot[\alpha])\\ \cos(7\cdot[\alpha])\\ \sin(7\cdot[\alpha])\\ [1]^{\mathrm{T}}/\sqrt{2}\end{bmatrix};\quad [f_{\text{VSD}}]=\begin{bmatrix}f_\alpha\\ f_\beta\\ f_{x3}\\ f_{y3}\\ f_{x5}\\ f_{y5}\\ f_{x7}\\ f_{y7}\\ f_0\end{bmatrix} \quad (5)$$

It can be easily verified that [C], defined as per (5) is a full-rank matrix and guarantees a one-to-one correspondence between the phase variables and the VSD ones.

The chosen transformation matrix resembles a standard power invariant one used for symmetrical machines but, given the angle dispositions (1), in this case it does not guarantee the orthogonality condition, meaning that $[\mathrm{T}]=[\mathrm{C}]^{-1} \neq [\mathrm{C}]^{\mathrm{T}}$.

C. Choice of the Rotational Transformation

Each of the subsets α-β, $x3$-$y3$, $x5$-$y5$ and $x7$-$y7$ of the VSD vector $[f_{\text{VSD}}]$ defined with the transformation matrix (5) can be further transformed into a corresponding rotating set synchronous with the corresponding h-th spatial harmonic of

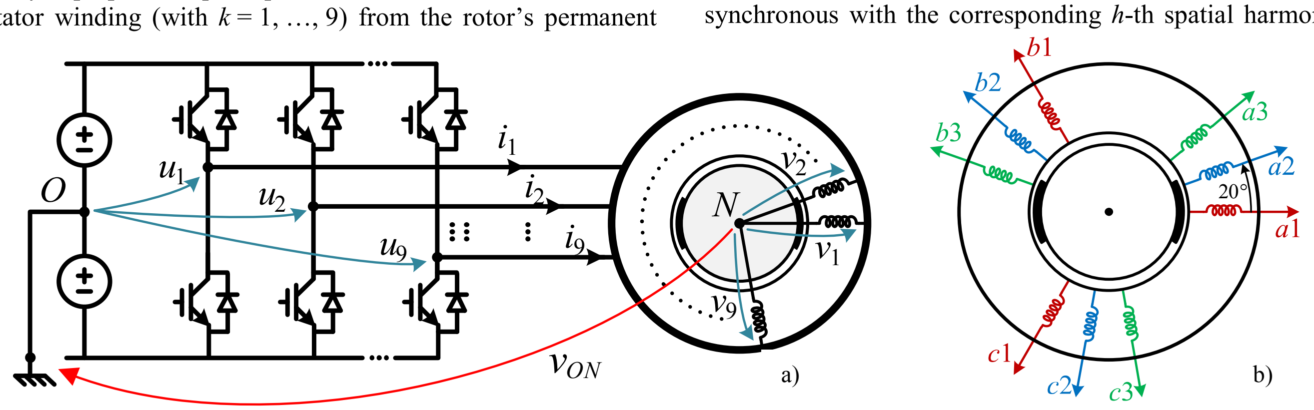

Fig. 1. Nine-phase PM synchronous machine system architecture, and asymmetrical magnetic axis dispositions b).

the PM flux density (with h = 1,3,5,7) according to:

$$[f_{\mathrm{dq}}]=[\mathrm{D}](\theta)\cdot[f_{\mathrm{VSD}}] \quad \Leftrightarrow \quad [f_{\mathrm{VSD}}]=[\mathrm{D}]^{-1}(\theta)\cdot[f_{\mathrm{dq}}] \tag{6}$$

The rotational matrix $[\mathrm{D}](\theta)$ is conveniently defined as a block diagonal matrix of two-dimensional rotational submatrices as:

$$[\mathrm{D}](\theta)=\begin{bmatrix}[\mathrm{D}_1](\theta) & [0_{2\times2}] & [0_{2\times2}] & [0_{2\times2}] & [0_{2\times1}]\\ [0_{2\times2}] & [\mathrm{D}_3](\theta) & [0_{2\times2}] & [0_{2\times2}] & [0_{2\times1}]\\ [0_{2\times2}] & [0_{2\times2}] & [\mathrm{D}_5](\theta) & [0_{2\times2}] & [0_{2\times1}]\\ [0_{2\times2}] & [0_{2\times2}] & [0_{2\times2}] & [\mathrm{D}_7](\theta) & [0_{2\times1}]\\ [0_{1\times2}] & [0_{1\times2}] & [0_{1\times2}] & [0_{1\times2}] & 1\end{bmatrix}; \tag{7}$$

$$[f_{\mathrm{dq}}]=\begin{bmatrix}f_{d1} & f_{q1} & f_{d3} & f_{q3} & f_{d5} & f_{q5} & f_{d7} & f_{q7} & f_{\emptyset}\end{bmatrix}^{\mathrm{T}}$$

Each h-th submatrix is given by:

$$[\mathrm{D}_h](\theta)=\begin{bmatrix}\cos(h\theta+\varphi_h) & \sin(h\theta+\varphi_h)\\ -\sin(h\theta+\varphi_h) & \cos(h\theta+\varphi_h)\end{bmatrix} \tag{8}$$

with the harmonic phase displacements φ_h defined as per (3). It can be easily verified that it is unitary, i.e. $[\mathrm{D}]^{-1}(\theta)=[\mathrm{D}]^{\mathrm{T}}(\theta)$.

D. Electromagnetic Torque Expression

Once the synchronous current set $[i_{\mathrm{dq}}]$ is found through (4)-(6) with the matrices (5)-(7), the electromagnetic torque developed by the machine takes a particularly simple form:

$$\begin{aligned}T_{em}&=\sqrt{\frac{9}{2}}P_p\cdot\sum_h h\,\lambda_{Mh}\,i_{qh}=\sqrt{\frac{9}{2}}P_p\cdot\left[\lambda_{M1}\,i_{q1}+3\,\lambda_{M3}\,i_{q3}+\dots\right.\\ &\left.\dots+5\,\lambda_{M5}\,i_{q5}+7\,\lambda_{M7}\,i_{q7}\right]=[\kappa]^{\mathrm{T}}\cdot[i_{\mathrm{dq}}]\end{aligned} \tag{9}$$

with the equivalent torque gain vector defined as:

$$\begin{aligned}[\kappa]&=\begin{bmatrix}\kappa_{d1} & \kappa_{q1} & \kappa_{d3} & \kappa_{q3} & \kappa_{d5} & \kappa_{q5} & \kappa_{d7} & \kappa_{q7} & \kappa_{\emptyset}\end{bmatrix}^{\mathrm{T}}=\\ &=\left(\sqrt{9/2}\cdot P_p\right)\cdot\begin{bmatrix}0 & \lambda_{M1} & 0 & 3\lambda_{M3} & 0 & 5\lambda_{M5} & 0 & 7\lambda_{M7} & 0\end{bmatrix}^{\mathrm{T}}\end{aligned} \tag{10}$$

E. Power Loss Expression

Under the assumption that the dissipation in the machine is primarily due to the Joule losses in the stator windings, the instantaneous power losses can be expressed as:

$$\begin{aligned}p&=R\cdot\sum_{k=1}^{9}i_k^2=R\cdot[i_{\mathrm{ph}}]^{\mathrm{T}}\cdot[i_{\mathrm{ph}}]=\\ &=R\cdot\left([\mathrm{T}]\cdot[\mathrm{D}]^{\mathrm{T}}(\theta)\cdot[i_{\mathrm{dq}}]\right)^{\mathrm{T}}\cdot\left([\mathrm{T}]\cdot[\mathrm{D}]^{\mathrm{T}}(\theta)\cdot[i_{\mathrm{dq}}]\right)=\\ &=R\cdot[i_{\mathrm{dq}}]^{\mathrm{T}}\cdot\left([\mathrm{D}](\theta)\cdot[\mathrm{T}]^{\mathrm{T}}\cdot[\mathrm{T}]\cdot[\mathrm{D}]^{\mathrm{T}}(\theta)\right)\cdot[i_{\mathrm{dq}}]=\\ &=R\cdot[i_{\mathrm{dq}}]^{\mathrm{T}}\cdot[\mathrm{G}](\theta)\cdot[i_{\mathrm{dq}}]\end{aligned} \tag{11}$$

In general, the instantaneous power depends on the instantaneous electrical rotor position θ. The average power losses can be easily found by averaging the instantaneous power p along a full rotor cycle.

If the $[i_{\mathrm{dq}}]$ set is controlled independently from θ, one immediately obtains:

$$\begin{aligned}P&=\frac{1}{2\pi}\int_0^{2\pi}p(\theta)\mathrm{d}\theta=\frac{1}{2\pi}\int_0^{2\pi}R\cdot[i_{\mathrm{dq}}]^{\mathrm{T}}\cdot[\mathrm{G}](\theta)\cdot[i_{\mathrm{dq}}]\mathrm{d}\theta=\\ &=R\cdot[i_{\mathrm{dq}}]^{\mathrm{T}}\cdot\left(\frac{1}{2\pi}\int_0^{2\pi}[\mathrm{G}](\theta)\mathrm{d}\theta\right)\cdot[i_{\mathrm{dq}}]=R\cdot[i_{\mathrm{dq}}]^{\mathrm{T}}\cdot[\mathrm{H}]\cdot[i_{\mathrm{dq}}]\end{aligned} \tag{12}$$

For a symmetrical winding configuration, due to the orthogonality of [T] (i.e. $[\mathrm{T}]^{\mathrm{T}}\cdot[\mathrm{T}]=[\mathrm{C}]\cdot[\mathrm{T}]=[I_{9\times9}]$), it follows that $[\mathrm{G}](\theta)=[\mathrm{H}]=[I_{9\times9}]$ and $p=P=R\cdot[i_{\mathrm{dq}}]^{\mathrm{T}}\cdot[i_{\mathrm{dq}}]$ (which explains the "power invariant" nomenclature for [T]). Vice versa, for an asymmetrical machine as the one studied here, the relationship between the average power and the transformed currents depends on a modified scalar product associated to the symmetric and positive definite matrix [H]. Given the transformation matrices (5)-(7), the following results:

$$[\mathrm{G}](\theta)=\begin{bmatrix}[I_{2\times2}] & [0_{2\times2}] & [0_{2\times2}] & [0_{2\times2}] & [0_{2\times1}]\\ [0_{2\times2}] & \begin{bmatrix}G_{d3,d3}(\theta) & G_{d3,q3}(\theta)\\ G_{q3,d3}(\theta) & G_{q3,q3}(\theta)\end{bmatrix} & [0_{2\times2}] & [0_{2\times2}] & \begin{bmatrix}G_{d3,\emptyset}(\theta)\\ G_{q3,\emptyset}(\theta)\end{bmatrix}\\ [0_{2\times2}] & [0_{2\times2}] & [I_{2\times2}] & [0_{2\times2}] & [0_{2\times1}]\\ [0_{2\times2}] & [0_{2\times2}] & [0_{2\times2}] & [I_{2\times2}] & [0_{2\times1}]\\ [0_{1\times2}] & \begin{bmatrix}G_{\emptyset,d3}(\theta) & G_{\emptyset,q3}(\theta)\end{bmatrix} & [0_{1\times2}] & [0_{2\times2}] & 9\end{bmatrix} \tag{13}$$

where the functions of θ are:

$$\begin{aligned}&G_{d3,d3}(\theta)=5-4\cos(6\theta+2\varphi_3+\pi/3)\\ &G_{q3,q3}(\theta)=5+4\cos(6\theta+2\varphi_3+\pi/3)\\ &G_{d3,q3}(\theta)=G_{q3,d3}(\theta)=4\sin(6\theta+2\varphi_3+\pi/3)\\ &G_{d3,\emptyset}(\theta)=G_{\emptyset,d3}(\theta)=-3\sqrt{6}\sin(3\theta+\varphi_3)-3\sqrt{2}\cos(3\theta+\varphi_3)\\ &G_{q3,\emptyset}(\theta)=G_{\emptyset,q3}(\theta)=3\sqrt{2}\sin(3\theta+\varphi_3)-3\sqrt{6}\cos(3\theta+\varphi_3)\end{aligned} \tag{14}$$

It can be noted that all the non-diagonal elements of the matrix are either zero or simple trigonometric functions of the electrical angle θ. The integration therefore leads to a simple diagonal matrix:

$$[\mathrm{H}]=\frac{1}{2\pi}\int_0^{2\pi}[\mathrm{G}](\theta)\mathrm{d}\theta=\begin{bmatrix}1&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0\\0&0&5&0&0&0&0&0&0\\0&0&0&5&0&0&0&0&0\\0&0&0&0&1&0&0&0&0\\0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&9\end{bmatrix} \tag{15}$$

from which it can be concluded that, in contrast to the symmetrical configuration, the $d3$-$q3$ subspace is weighted 5 times more than the other planes in the power losses evaluation (the zero-sequence component $\emptyset$ can be neglected since it is constrained to zero by the single neutral point of the windings).

F. Transformed Electrical Equations

By applying the VSD and rotational transformations (4)-(6) to the system of equations (2) with the matrices defined as per (5)-(7), the following electrical equations for the d-q variables are obtained:

$$\begin{cases}[u_{\mathrm{dq}}]=R\cdot[i_{\mathrm{dq}}]+[L_{\mathrm{dq1}}](\theta)\cdot\dfrac{\mathrm{d}}{\mathrm{d}t}[i_{\mathrm{dq}}]+\dots\\ \qquad\dots+\omega\cdot[L_{\mathrm{dq2}}](\theta)\cdot[i_{\mathrm{dq}}]+[e_{\mathrm{dq}}]-v_{ON}\cdot[g](\theta)\\ [0\ \ 0\ \ \cdots\ \ 0\ \ 1]\cdot[i_{\mathrm{dq}}]=i_{\emptyset}=0\end{cases} \tag{16}$$

with the new matrices and vectors defined as:

$$\begin{aligned}&[L_{\mathrm{dq1}}](\theta)=[\mathrm{D}](\theta)\cdot[\mathrm{C}]\cdot[L]\cdot[\mathrm{T}]\cdot[\mathrm{D}]^{\mathrm{T}}(\theta)\\ &[L_{\mathrm{dq2}}](\theta)=[\mathrm{D}](\theta)\cdot[\mathrm{C}]\cdot[L]\cdot[\mathrm{T}]\cdot\left(\partial[\mathrm{D}]^{\mathrm{T}}(\theta)/\partial\theta\right)\\ &[g](\theta)=[\mathrm{D}](\theta)\cdot[\mathrm{C}]\cdot[1]\end{aligned} \tag{17}$$

By explicitly writing the resulting system of differential equations it can be verified that, as in the symmetrical case, the

$d1$-$q1$, $d5$-$q5$ and $d7$-$q7$ subspace equations take the usual decoupled form:

$$\begin{cases} u_{dh} = R \cdot i_{dh} + L_h \cdot \dfrac{\mathrm{d}\, i_{dh}}{\mathrm{d}t} - h \cdot \omega \cdot L_h \cdot i_{qh} \\ u_{qh} = R \cdot i_{qh} + L_h \cdot \dfrac{\mathrm{d}\, i_{qh}}{\mathrm{d}t} + h \cdot \omega \cdot L_h \cdot i_{dh} + e_{qh} \end{cases} \tag{18}$$

with $h = 1,5,7$ and $e_{qh} = \sqrt{9/2}\, h\, \omega\, \lambda_{Mh}$. The motional back-EMF depends on the h-th spatial harmonic equivalent inductance L_h.

To the contrary, the $d3$-$q3$ and 0 subspaces are coupled with each other through the effect of v_{ON}. Indeed, one gets:

$$\begin{cases} u_{d3}=Ri_{d3}+L_3\dfrac{\mathrm{d}\, i_{d3}}{\mathrm{d}t} - 3\omega L_3 i_{q3} - 2\sqrt{2}\cos\left(3\theta+\varphi_3-\pi/3\right) v_{ON} \\ u_{q3}=Ri_{q3}+L_3\dfrac{\mathrm{d}\, i_{q3}}{\mathrm{d}t} + 3\omega L_3 i_{d3} + e_{q3} - 2\sqrt{2}\cos\left(3\theta+\varphi_3+\pi/6\right) v_{ON} \\ v_{ON}=\dfrac{L_{m3}}{9}\left[\left(2\sqrt{2}\sin(3\theta+\varphi_3+\pi/6)\right)\left(\dfrac{\mathrm{d}\, i_{d3}}{\mathrm{d}t} - 3\omega i_{q3}\right)+\ldots \right. \\ \left. \ldots+\left(2\sqrt{2}\sin(3\theta+\varphi_3+\pi/6)\right)\left(\dfrac{\mathrm{d}\, i_{q3}}{\mathrm{d}t}+3\omega i_{d3}\right)\right]+\dfrac{e_0-u_0}{3} \end{cases} \tag{19}$$

with $e_{q3} = \sqrt{9/2} \cdot 3\,\omega\, \lambda_{M3}$, $e_0 = -6\,\omega\,\lambda_{M3} \sin(3\theta + \varphi_3 - \pi/3)$ and $L_{m3} = L_3 - L_l$ (where L_l is the winding leakage inductance).

Not only the $d3$-$q3$ voltages u_{d3} and u_{q3}, but also the zero sequence voltages u_0 and e_0 can influence i_{d3} and i_{q3}. This unusual behaviour is due to the fact that $\cos(3\cdot[\alpha])\cdot[1] = 3 \neq 0$ and $\sin(3\cdot[\alpha])\cdot[1] = 3\sqrt{3} \neq 0$, meaning that, due to the transformation matrix [C], not only u_0, but also u_{x3} and u_{y3} can be affected by a common mode voltage injection. Moreover, the coupling depends on θ and is, therefore, time-varying.

III. Optimal Third Harmonic Current Injection

The standard Field Oriented Control (FOC) for magnetically isotropic PMSMs only control the i_{q1} current, while keeping all the other components of the $[i_{dq}]$ set to zero. This strategy guarantees a direct proportionality relation between the desired torque T_{em}^* and the reference current $i_{q1}^* = T_{em}^*/\kappa_{q1}$.

However, given the presence of higher order spatial harmonics in the PMs' induced flux density in the air-gap, some other components of the $[i_{dq}]$ can be exploited as useful degrees of freedom for the torque control. In particular, the simplest enhancement can be obtained through the control of the i_{q3} current component which, in steady state conditions, due to the 3θ rotation in the $[D](\theta)$ matrix, corresponds to the third harmonic current injection into the phase variables.

An optimal strategy can be achieved when this component is controlled in order to minimize the average power losses in the machine for a given reference electromagnetic torque. Once all the other components of $[i_{dq}]$ are set to zero, the expressions (9) -(12) can be rewritten as:

$$T_{em} = \kappa_{q1}\, i_{q1} + \kappa_{q3}\, i_{q3}\ ; \quad P = R \cdot (i_{q1}^2 + 5 i_{q3}^2) \tag{20}$$

By imposing $T_{em}=T_{em}^*$ and solving for i_{q1} the average power is:

$$P = R \cdot \left[i_{q1}^2 + 5 \cdot (T_{em}^* - \kappa_{q1}\, i_{q1})^2 \big/ \kappa_{q3}^2 \right] \tag{21}$$

The expression (21) is a quadratic function of i_{q1}, whose minimum is obtained for:

$$\left.\frac{\partial P}{\partial i_{q1}}\right|_{i_{q1,\min P}} = 0 \quad \Leftrightarrow \quad i_{q1,\min P} = \frac{5\kappa_{q1}}{5\kappa_{q1}^2 + \kappa_{q3}^2} \cdot T_{em}^* \tag{22}$$

The reference currents i_{q1}^* and i_{q3}^* are then chosen to be the optimal ones with reference to the average power minimization. By substituting the values of κ_{q1} and κ_{q3}, defined in (10), the following relationships are obtained:

$$i_{q1}^* = \frac{5\lambda_{M1}}{5\lambda_{M1}^2 + 9\lambda_{M3}^2} \cdot \sqrt{\frac{2}{9}} \cdot \frac{T_{em}^*}{P_p}\ ; \quad i_{q3}^* = \frac{3\lambda_{M3}}{5\lambda_{M1}^2 + 9\lambda_{M3}^2} \cdot \sqrt{\frac{2}{9}} \cdot \frac{T_{em}^*}{P_p} \tag{23}$$

The optimal ratio is then easily found to be:

$$k_{\mathrm{opt}} = \frac{i_{q3}^*}{i_{q1}^*} = \frac{3\lambda_{M3}}{5\lambda_{M1}} \tag{24}$$

and the corresponding power losses, if compared with the case when only the fundamental component is exploited, are reduced by a factor:

$$\eta_{\mathrm{opt}} = \frac{P_{\mathrm{THI,opt}}}{P_{\mathrm{FUND}}} = \frac{\lambda_{M1}^2 \cdot (25\lambda_{M1}^2 + 45\lambda_{M3}^2)}{(5\lambda_{M1}^2 + 9\lambda_{M3}^2)^2} \tag{25}$$

It should be noted that the proposed approach, by only focusing on the Joule losses, does not maximize the overall efficiency, which also depends on the iron losses. If needed, these losses could be included in (21), but their relationship with the stator currents would require a more refined machine model than (2) and the resulting optimal injection ratio would depend on the rotor speed (which affects both the supplying frequency and the magnitude of the back-EMFs).

IV. Strategy Validation

The proposed strategy has been numerically tested with respect to the parameters of the real nine-phase machine described in [19], which can be properly rearranged to yield an asymmetrical configuration. The main parameters of the considered machine are summarized in Table I.

TABLE I NINE -PHASE MACHINE PARAMETERS

$P_p = 1$	$R = 31.3\ \Omega$	$L_l = 84$ mH
$\lambda_{M1} = 385$ mWb	$\varphi_1 = 0°$	$L_1 = 147$ mH
$\lambda_{M3} = 119$ mWb	$\varphi_3 \cong 180°$	$L_3 = 92$ mH
$\lambda_{M5} = 38$ mWb	$\varphi_5 \cong 0°$	$L_5 = 88$ mH
$\lambda_{M7} = 7$ mWb	$\varphi_7 \cong 165°$	$L_7 = 87$ mH

A. Theoretical Optimal Results

The machine flux linkages of interest are $\lambda_{M1} \approx 385$ mWb and $\lambda_{M3} \approx 119$ mWb, with $\varphi_3 \approx 180°$. Fig. 2 shows the average power losses obtained by varying the current ratio $k = i_{q3}/i_{q1}$ in the interval [0; 1] with a fixed reference torque, normalized by the value obtained without the third harmonic current injection. In accordance with (24), the minimum value is obtained for $k_{\mathrm{opt}} \approx 0.19$ and, as per (25), the corresponding power ratio is $\eta_{\mathrm{opt}} \approx 0.85$ (i.e. the third harmonic injection allows a theoretical reduction of the losses by 15%).

The corresponding phase current waveforms for varying θ can be obtained as $[i_{ph}] = [T]\cdot[D]^T(\theta)\cdot[0\ i_{q1}^*\ 0\ i_{q3}^*\ 0\ 0\ 0\ 0\ 0]^T$. They are graphically depicted in the first column of Fig. 3, normalized by $I_{FUND} = \sqrt{2/9} \cdot T_{em}^*/\kappa_{q1}$, which is the peak phase

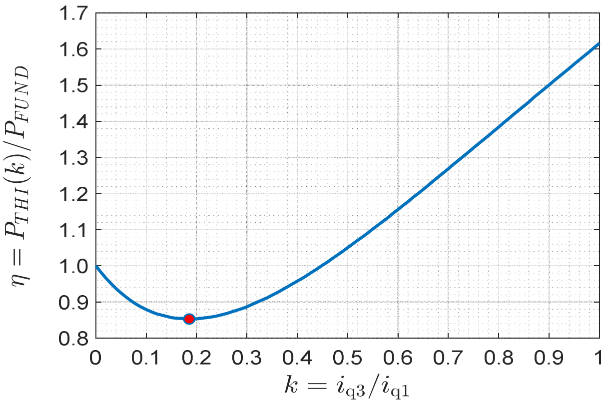

Fig. 2. Normalized power losses for a varying injection ratio.

current needed to supply the same average torque without any third harmonic injection. Their harmonic spectra, obtained through a Fourier series expansion, are represented in the second and third column of the same picture.

As expected, only the first and third harmonics have a non-zero magnitude. The current waveforms in each $\{a,b,c\}$ set are symmetrical and equally shifted by 120°. Nevertheless, it can be immediately observed that the three three-phase sets behave differently from each other.

Indeed, while the first harmonic components for each winding only differ for their angle, the third harmonic components also show different magnitudes. This behaviour is needed to guarantee that the sum of all the currents is zero and it can be easily verified from the angle dispositions in the spectra that the magnitude of the third harmonic components in the set $\{a_2,b_2,c_2\}$ needs to be $\sqrt{3}$ times higher than in the other two sets where, instead, it has the same value.

The different harmonic magnitudes lead to a different RMS value in the different three-phase windings and, as a result, to an unbalanced distribution of the power losses. In particular, for the examined machine, it can be verified that that the losses in the second set are about 37.4% of the total, while for the other two sets they are almost 31.3% of the total. Naturally, since the third harmonic components have the same phase in each $\{a,b,c\}$ set, the proposed enhancement cannot be achieved in case of three isolated neutral points. Indeed, for such configuration the current components i_{x3} and i_{y3} cannot be arbitrarily controlled given the constraints on the common mode currents imposed on each three-phase windings set.

B. Simulation Results

The theoretical results have been numerically tested in the Matlab/Simulink environment. A standard proportional-integral (PI) controller based FOC is performed to drive the currents in each *d-q* subspace of the machine, with the exception of the *d*3-*q*3 subspace which, as previously stated, is coupled with the **0** subspace. An improved FOC, involving a proper compensation action on the motional induced back-EMFs appearing in (19), is able to overcome this drawback.

The machine is subject to a constant 2 Nm load torque and a feedback control loop guarantees a constant speed of 500 rpm. The test has been performed by linearly varying the current ratio $k = i_{q3}/i_{q1}$ in the interval [0; 1] during a time window of 20 s. The results are depicted in Fig. 4; the average power losses P have been evaluated by averaging the instantaneous losses over a moving time window of 12 ms, while $T_{em,1} = \kappa_{q1} i_{q1}$ and $T_{em,3} = \kappa_{q3} i_{q3}$ identify the contributions of the fundamental and of the third harmonic components on the overall torque.

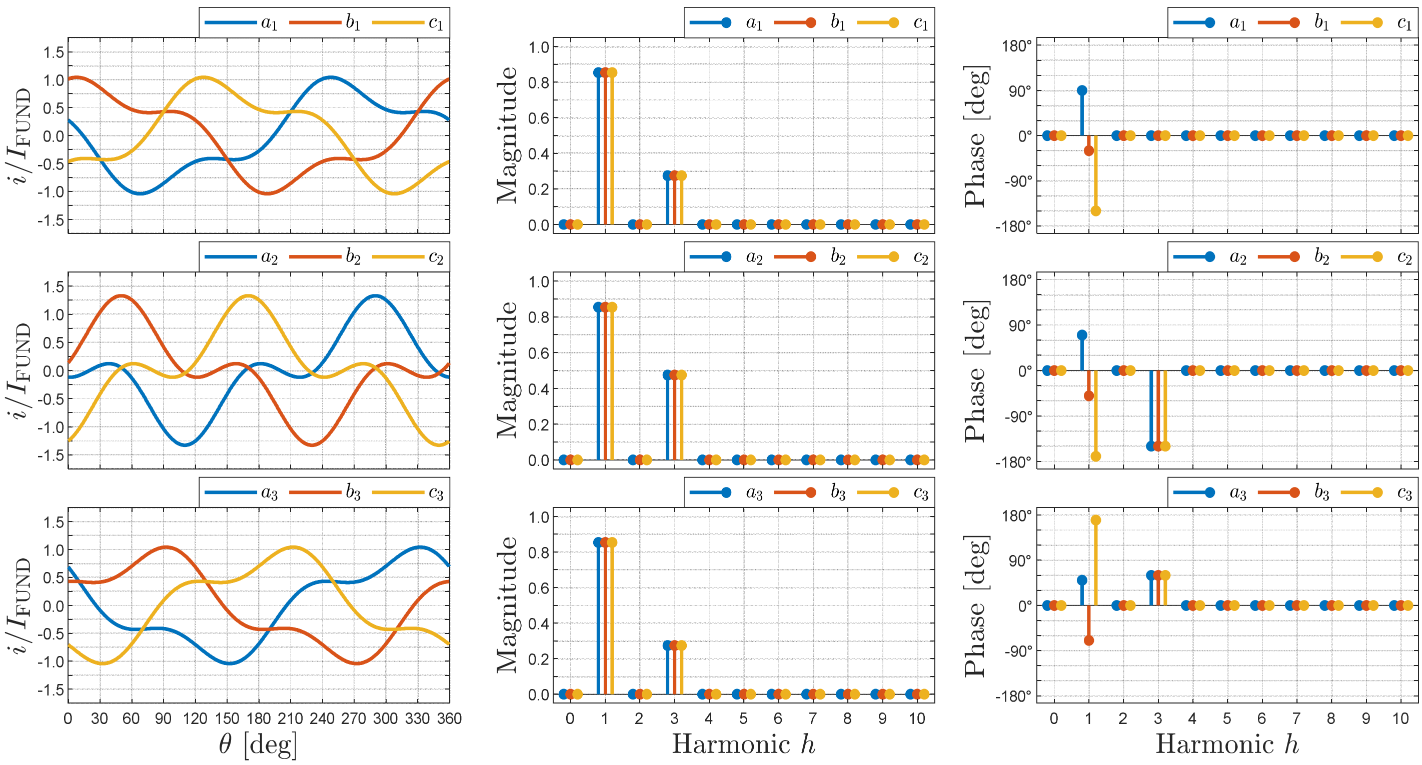

Fig. 3. Optimal phase current waveforms and harmonic spectra for the asymmetrical nine-phase machine case study.

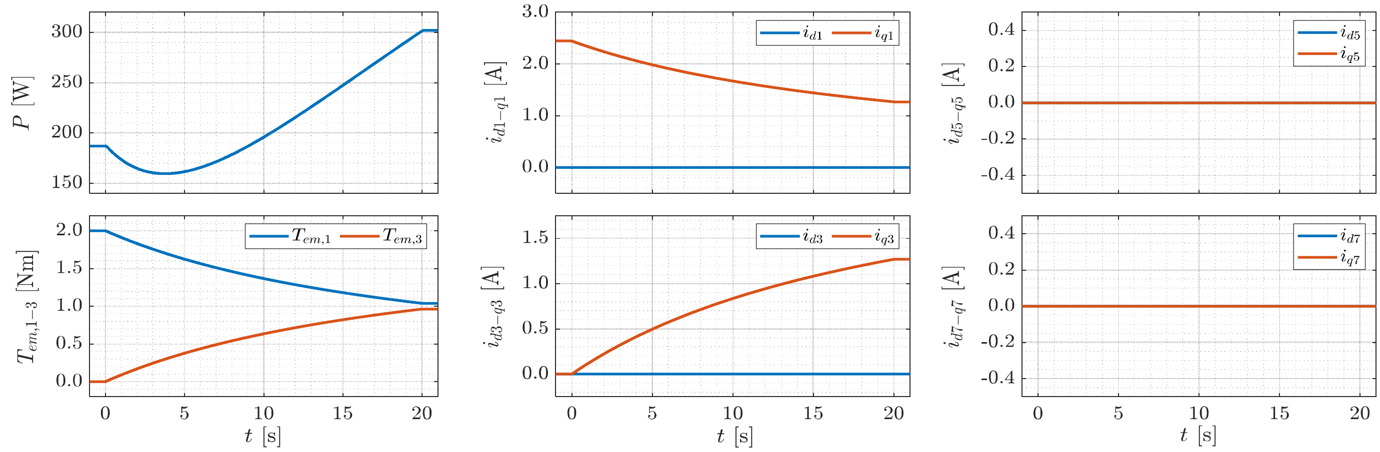

Fig. 4. Power losses and *dq* current waveform simulation results for a linearly varying current ratio *k* at constant speed and torque.

At first, only i_{q1} (i.e. only the fundamental component) is responsible for the entire torque and the corresponding power losses are $P \approx 188$ W. Then, the gradual increase of i_{q3} (i.e. the increase of the third harmonic magnitude) allows a reduction of the fundamental component. The average stator power losses follow the same convex behaviour depicted in Fig. 2 and reach the minimum value $P \approx 160$ W for $k \approx 0.19$, thus perfectly matching the theoretical results. Since the machine model neglects the effects of the spatial harmonics with order $h \geq 9$, both the torque contributions $T_{em,1}$ and $T_{em,3}$ don't show any ripple and the overall electromagnetic torque $T_{em} = T_{em,1} + T_{em,3}$ is constant and always equal to the load torque (2 Nm).

V. Conclusion

The paper presented a strategy to exploit the third harmonic current injection to optimally reduce the average power losses for an asymmetrical nine-phase PMSM with non-sinusoidal back-EMFs and a single neutral point.

An analytical solution has been found first through a detailed study of the effect of the VSD and rotational transformation matrices on both the torque and the power loss expressions. In contrast to the symmetrical configuration, the quadrature components of the synchronous current set are differently weighted in the different subspaces, resulting in a different optimal ratio between the fundamental and the third harmonic components that are to be injected. Moreover, to guarantee a zero sum of all the currents, not only the phase shift, but also the magnitude of the third harmonic components is not the same for each winding. The standard FOC is applied next to each subset of the transformed variables, with the exception of the $d3$-$q3$ subspace, which requires a slight modification due to its coupling with the zero sequence one. The theoretical results have been successfully tested through numerical simulations, thus validating the proposed approach.

Future developments will be related both to the experimental validation of the proposed approach and to its generalization for the application to other machines and/or winding configurations and for the optimal exploitation of other harmonic components.

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