

JOSEPH PETZOLDT AND THE SYSTEMATIZATION OF ERNST MACH'S PHILOSOPHY

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ABSTRACT

Ernst Mach's claim that the objects are bundles of sensation was (and still is) widely interpreted in a phenomenalist sense, as if he was asserting that perceptual experiences is the only reality. The talk will reconstruct the philosophical system of Mach's pupil Joseph Petzoldt, to propose a different interpretation of Mach's work, that put it into the wider context of the polemic against the widespread neo-Kantianism of that time.

KEYWORDS

ERNST MACH. JOSEPH PETZOLDT. PHENOMENALISM. POSITIVISM. NEO-KANTIANISM. CAUSALITY

1. INTRODUCTION

Ernst Mach is one of those thinkers whose success is also his curse. He never wanted to be considered as a philosopher and he never attempted to establish a philosophical system. Nonetheless, because of the widespread attention received by his works in the philosophical community, his ideas were stretched in all directions, to fit into different philosophical labels.

One of the most discussed aspects of Mach's work is his famous assertion that "thing, body, matter, are nothing apart from the complex of the colors, sounds, and so forth" (MACH, 1886, p. 5). Already during Mach's life, his idea that objects are bundles of sensations was interpreted on the line of a radical empiricism that ends up into subjective idealism or phenomenalism, analogous to the philosophy of Berkeley. For example, the physicist Max Planck summed up Mach's ideas as follows: "This view holds that there are no other realities than one's own perceptions. [...] The essential and only elements of the world are perceptions" (PLANCK, [1908] 1992, p. 129). For this reason, Planck regarded Mach's position as a threat to the healthy realism of science. Moving from a different preoccupation – i.e. that Mach's rebuttal of the metaphysical notion of "matter" could endanger the triumph of Marxian materialism – Lenin criticized Mach on similar grounds, condemning his position as a form of Berkeleyan idealism (LENIN, [1909] 1927). But this idealistic reading of Mach was defended also by Popper, who wrote a paper entitled *A Note on Berkeley as Precursor of Mach* (POPPER, 1953). And even those who did not go as far as to say that Mach was a Berkeleyan, like Moritz Schlick, still regarded his philosophy as having an anti-realists tendency, that made it impossible to explain the existence of objects independently from human cognition, and the presence of a single object for multiple observers at once (cf. SCHLICK, 1918, p. 177).

However, it is not our task to provide a complete catalog of the phenomenalistic readings of Ernst Mach. Suffice it to say, that this interpretation

is still so common that Erik C. Banks recently wrote several works aiming at its confutation (BANKS, 2013, p. 2014). For us, the acknowledgment of the enduring controversy about Mach's (real or presumed) phenomenalism is just a first step to introduce the real protagonist of this paper: Joseph Petzoldt, who was the most prominent follower of Mach between the end of the nineteenth century and the first decades of the twentieth century. This paper aims at demonstrating that Joseph Petzoldt, with his attempt to incorporate Mach's ideas into a broader philosophical system, may help us to shed light on the issue of Mach's alleged phenomenalism.

To avoid the risk of overlapping and confusing the thought of Mach and Petzoldt, we will start with the latter, trying to sketch his philosophical system. Afterward, we will compare his thought with that of Mach, in order to establish what they agreed upon and what they disagreed about.

2. PETZOLDT IN CONTEXT

2.1 Philosophical Background

Joseph Petzoldt (1862–1929) began studying philosophy at a time when German culture was dominated by two fundamental trends. On the one hand, the second scientific revolution (WATSON, 2010) had fueled a resurgence of the mechanical-materialistic worldview. The recent advances in the study of chemical and electromagnetic processes, the formulation of the law of conservation of energy, the development of experimental physiology, and, soon after, Darwin's theory of natural selection, all contributed to the idea that the whole world – living beings not excluded – could and should be explained in terms of physical laws. During the middle of the century, this revival of materialism led to a vibrant debate, known to history as *Materialismusstreit* (see BAYERTZ; GERHARD; JAESCHKE, 2007).

On the other hand, the backlash against Hegelianism resulted in a rediscovery of Kantian philosophy, epitomized by Otto Liebmann's motto "Back to Kant!" (LIEBMANN, 1865). This neo-Kantian trend included a variety of great thinkers, both philosophers and scientists, that would be impossible to summarize (see BEISER, 2017). Nevertheless, together with the aforementioned materialist trend, this Kant renaissance also created a more generic philosophical attitude, that became the common way of thinking for educated people in the second half of the nineteenth century, especially in scientific circles. This simplified materialist/neo-Kantian position grew out of the popular success of Friedrich Albert Lange *History of Materialism* (1866) and Emil du Bois-Reymond *On the Limits of our Knowledge of Nature* (1872), who both acknowledged the importance of the materialist worldview while also accepting a Kantian framework.

The Kantian core of this sort of philosophy-of-the-educated-man consisted roughly in Kant's Copernican revolution: since all knowledge begins and ends with consciousness, we cannot grasp reality-in-itself; thus, science does not deal with the world-in-itself, but is the consequence of the functioning of our mind; therefore, science is a necessary activity, grounded on the forms of our knowledge, but science has also inherent limits, that are the limits of our own understanding; thus, beyond these limits, there can be room for something else (free will, values, religion...). This neo-Kantianism thus allowed a "philosophical justification for an attitude of 'neither-nor' or one of 'this as well as that'" (KÖHNKE, 1991, p. 96), that tried to reconcile materialism and idealism on the basis of a certain degree of skepticism. The idealist conception – according to which we cannot know anything about a mind-independent reality, and we only deal with our representations – could go hand in hand with materialism, since the latter was regarded as the fundamental aim of scientific understanding, but not necessarily as a true picture of the world. In other words, even if the very functioning of our minds demands that we explain all natural phenomena –

living beings not excluded – in terms of purely physical laws, thus creating the world-picture of materialism, we cannot really know if this world-picture is true, because we never grasp reality in itself.

Petzoldt grew up in this milieu. In a short autobiographical piece, he lists as his first readings: Kant's *Prolegomena*; *Force and Matter*, by the materialist philosopher Ludwig Büchner; Darwin's *Origins of Species*; and the already mentioned works by Lange and du Bois-Reymond. At that time he was a fervent supporter of the new scientific enterprises, but philosophically he considered himself a "neo-Kantian and agnostic" (PETZOLDT, 1904). This means that he shared the philosophical attitude we were trying to sketch: full commitment to the scientific goal of materialism (i.e. interpreting *all* natural phenomena in terms of physical processes), combined with the acknowledgment of the fact that we are confined into our consciousness, incapable of fathoming what is behind it.

However, Petzoldt soon grew dissatisfied with this philosophical position for a number of reasons. Firstly, he couldn't accept such agnostic, or even skeptic, outcome anymore. He wanted a philosophy that was not afraid to assert something about the real features of the world, and that had the audacious goal to provide a positive knowledge about reality. Secondly, he rejected the subjectivist tendency of that conception, according to which we are always stuck with our own representation, so that everything we know is actually a product of our own minds. Thirdly, he refuted its underlying dualism, that counterposes the reality in itself on the one hand, and the domain of representations and consciousness on the other hand. Petzoldt aimed at mending this split in the fabric of reality by reinstating a unitary understanding of the world. Lastly, he believed that such a unitary understanding could also solve the problem of the relations between matter and mind, which stems from that pernicious dualism.

These were Petzoldt's goals throughout his intellectual career. These were the problems he was trying to address with his philosophy. All his forces were directed against that sort of common-sense Kantianism, that had become so

popular and deeply-rooted that even the scientists – who were supposed to be the vanguard of mankind’s efforts to understand the world – had internalized their own self-defeat, and gave up the task of actually knowing reality. On the contrary, Petzoldt believed that science and philosophy are capable to know the world. Therefore, not only he tried to develop a philosophical system aimed at establishing a positive, realist, unitary understanding of the world on the basis of the advances of science, but he also devoted his energies to the promotion of the dialogue between philosophers and scientists. In 1912 in Berlin, he established the *Gesellschaft für positivistische Philosophie* (Society for positivist philosophy), open “to all researchers interested in philosophy, whatever their field of study, and to all philosophers that hope to reach tenable convictions through the careful study of empirical facts” (PETZOLDT, 1912a). The Society included important figures of the time, such as Albert Einstein and Sigmund Freud, the mathematicians Georg Helm, David Hilbert and Felix Klein, the positivist historian Karl Lamprecht, the evolutionary biologist Wilhelm Roux, the botanist Henry Potonié, the sociologist Ferdinand Tönnies, the experimental physiologists Max Verworn and Georg Elias Müller, the philosophers Wilhelm Schuppe, Theodor Ziehen, and Wilhelm Jerusalem, and – needless to say – Ernst Mach. Even though the Society was disbanded during the First World War, in 1927 Petzoldt re-established it with the new name *Gesellschaft für empirische Philosophie* (Society for empirical philosophy). This new Society was frequented by members of the so-called Berlin Group – i.e. the circle of logical positivists revolving around Hans Reichenbach – who took up its direction after Petzoldt’s death in 1929, and transformed it into their institutional organization (Hentschel 1990).

Petzoldt’s cultural activism is already proof that – even if he did not have a successful academic career, and had to become a high school teacher, who only taught philosophy part-time as *Privatdozent* at Berlin Technische Universität – he was nonetheless a respected member of the German philosophical milieu.

Further evidence is his role in the philosophical debate about Einstein's theory of relativity. Petzoldt wrote on the topic as early as 1912, thus becoming the second philosopher after the neo-Kantian Paul Natorp to deal with this subject (NATORP, 1910; PETZOLDT, 1912b). One of his first publications on the theory of relativity (Petzoldt 1914) was also praised by Einstein himself, who recommended its reading for those interested in the "*erkenntnistheoretische* and *naturphilosophische* assumptions and implications" of the new physical theory (EINSTEIN, 1914; see also HOWARD, 1992). But Petzoldt's interpretation of the theory of relativity – according to which there is an inherent bond between Mach's thought and Einstein's theory – was also discussed by the most prominent exponents of the debate about relativity. Moritz Schlick examined Petzoldt's ideas in his book *General Theory of Knowledge*, as well as in his papers on relativity (SCHLICK, 1918; 1915; 1921). Ernst Cassirer argued against Petzoldt in his book about *Einstein's Theory of Relativity* (CASSIRER, 1921). And Hans Reichenbach addressed Petzoldt's position in his paper *The Present State of the Discussion on Relativity* (REICHENBACH, 1922), which also led to an exchange of letters between the two (HENTSCHEL, 1990).

2.2 The relationship between Mach and Petzoldt

However, it is not within the scope of this paper to reconstruct Petzoldt's role in the early debate on relativity. We touched on the topic to demonstrate that Petzoldt was acknowledged by his peers in the philosophical community as a valued representative of Machian philosophy. But, even more importantly, he was not just a self-appointed advocate of Mach, because Mach himself let him become his public defender. Even though they were both aware of their disagreements over some philosophical issues, they regarded themselves as on the same side. As Mach once said to Hugo Dingler, "I met Petzoldt only later in life. We never agreed completely on everything", but "one must stick together" (WOLTERS, 1987, p. 410).

The first epistolary contact between Mach and Petzoldt dated back to 1891, when Mach wrote to Petzoldt after reading his *Maxima, Minima und Ökonomie* (1890), that contained an internal critique of Mach's principle of thought-economy, while embracing and praising the anti-metaphysical tendency of Mach's thought. Those first letters already set the tone that characterized all their correspondence throughout the years: a strong kinship and, at the same time, an open discussion of their differences, with the aim of achieving greater insight into philosophical problems. As Mach wrote in a letter to Petzoldt towards the end of his life: "I thank you for the clarification that I have drawn from your modified version of my own conception. I am deeply grateful for this. This is what discussion and debate can do, when conducted properly" (BLACKMORE; HENTSCHEL, 1985, p. 141).

In 1898 Mach suffered a stroke that left him hemiplegic and burdened with serious health issues. Therefore, Mach started to believe that he had not long to live, and decided to retire in 1901. For the same reason, he assigned Petzoldt the task to edit all future editions of the *Science of Mechanics*: Petzoldt had to leave the text unchanged, but could add an afterword written by him, and even gained a share of the revenues. To secure his legacy, Mach also tried to get Petzoldt appointed as his successor for the chair of philosophy of the inductive sciences at the Wien university. However, when the plan failed, Mach still managed to help Petzoldt obtaining the *Habilitation* from Berlin Technisches Universität (WOLTERS, 1987, p. 174-75).

In the following years, as Mach – against all odds – continued to live despite his serious medical problems, his personal and epistolary relationship with Petzoldt grew closer. Petzoldt visited Mach several times in Wien, and their correspondence intensified (WOLTERS, 1987, p. 161; p.173-76). Confined into his house, unable to directly participate in the scientific and philosophical community, Mach derived much comfort from his friendship with Petzoldt. The devotion of the latter probably pleased him, while the ongoing discussions about

their differences avoided the risk of falling into a sheer adulation that would have been in contrast with Mach's humble character. We might say that Mach let Petzoldt become his representative on the philosophical stage not because they agreed on everything, but precisely for the opposite reason: because Petzoldt's intellectual autonomy prevented him from being a mere sycophant.¹

3. PETZOLDT'S PHILOSOPHICAL SYSTEM

So far, we introduced Petzoldt and established: 1) that he started his intellectual journey by distancing himself from the agnostic/neo-Kantian beliefs that he had held during his university studies; and 2) that at the peak of his career he was a respected representative of Machian philosophy, acknowledged as such by his peers in the philosophical community, as well as by Mach himself. Now we have to reconstruct the content of his philosophy. For this purpose, we will start with his first works.

3.1 The tendency towards stability

When Petzoldt abandoned the agnostic neo-Kantism that was in vogue at the time, he believed to have discovered the basis for a new philosophy in the works of three authors: Gustav Theodor Fechner, Richard Avenarius, and Ernst Mach.²

In Fechner's *Einige Ideen zur Schöpfungs- und Entwicklungsgeschichte der Organismen* (Some Ideas on the History of Creation and Evolution of the

¹ Not all letters between Mach and Petzoldt have been published. A selection can be found in Blackmore and Hentschel (1985). However, all digitalized manuscripts of the correspondence can be found online at <https://digital.deutsches-museum.de>, from signature NL-174-2425 to NL-174-2500.

² Avenarius and Mach were both influenced by Fechner. This is very likely the root of the similarities between their philosophies, that led to the custom of reuniting them under the same philosophical label (e.g. "empiriocriticism"). See RUSSO KRAUS, 2019, p. 28ss.

Organisms, 1873), Petzoldt found the “principle of tendency towards stability,” which states that, due to the effects of internal forces, in the long run all systems will tend toward more stable states. According to Fechner, this principle applies both to the organic and the inorganic world, and ultimately to the universe itself, insofar as it can be regarded as an all-encompassing system. Organic systems, like living beings, are only relatively stable, since they only display an approximate stability; which means that their elements change their mutual positions, but periodically return to a state in which they had already been. On the contrary, the elements of inorganic systems always maintain their mutual positions. Therefore, unlike all traditional theories of evolution, that derive organisms from inorganic matter, Fechner claimed that the universe evolves by moving from an original “cosmorganic” state, full of motion and instability, towards the formation of more stable systems: organic systems first, and inorganic systems later, until one day it will reach a state of complete, inorganic stability (FECHNER, 1873, p. 43). Fechner’s conception (evidently influenced by the first formulations of the law of entropy) was an attempt to rethink teleology to make it compatible with causality. Natural processes could then be explained thanks to two principles: the conservation of energy, which focuses on the “quantitative relations”, and the tendency towards stability, which is its “qualitative completion” (cf. FECHNER, 1873, p. 34–35; see also HEIDELBERGER, 2004, p. 249–55).

For Petzoldt, Avenarius’ *Philosophie als Denken der Welt gemäß dem Princip des kleinsten Kraftmasses* (Philosophy as Thinking of the World in Accordance with the Principle of the Least Amount of Energy, 1876) and Mach’s *Science of Mechanics* (1883) established a general theory for the functioning of mind and knowledge. Even though Avenarius’ principle of the least amount of energy and Mach’s principle of thought-economy are not exactly the same, Petzoldt focused on their similarities. According to him, they both affirmed that the mind operates by constantly trying to optimize the limited available forces: i.e. developing

general concepts that can be used to sum up a great deal of information, and applying those concepts that carry out the greatest performance with the least use of energy.

Petzoldt's first step towards the creation of his philosophical system was reuniting the principles developed by Fechner, Avenarius, and Mach into one, which he named "principle of tendency towards stability" or "principle of the minimum disruptions" (PETZOLDT, 1887, p. 189ss.). In so doing, Petzoldt could bring together under one unifying law not only the organic and the inorganic world (like Fechner had already done) but also the mental processes. According to Petzoldt's principle, a system rests on a certain equilibrium, on a certain stability, as long as all its elements maintain their mutual relations. When a variation occurs to a system, this variation constitutes a threat to its equilibrium, a disruption of its stability. Consequently, the system will keep on changing, trying to reduce the disruption to a minimum, until it reaches a new state of stability. We could express this process with the following formula: $S + \Delta S = S'$. However, it is important to note that the pursuit of a new stable state should not be interpreted teleologically, as a deliberate act; rather, it stems from the self-evident fact that unchanging systems do not change, and changing systems do change, until they change into systems that do not change, so that eventually all systems reach non-changing, stable states.³

Following Fechner, Petzoldt presents as the emblem of this process a planetary system, in which all celestial bodies maintain their mutual relations by having the same orbit (PETZOLDT, 1890, p. 217ss.). When a foreign celestial body arrives into the system, it produces some variations that disrupt the usual trajectories of the planets. However, eventually, the system will "adapt" to the

³ Petzoldt's stability principle may be called a teleomatic principle, to use Ernst Mayr's definition: "all objects of the physical world are endowed with the capacity to change their state, and these change strictly obey natural laws. They are end-directed only in a passive, automatic way, regulated by external forces or conditions, that is by natural laws. I designated such processes as teleomatic to indicate that they are automatically achieved" (MAYR, 1992, p. 125).

disturbing element, by encompassing it into a new equilibrium: the foreign celestial body will have its own regular trajectory, whereas the other planets will have regained their regular orbits (either the same they had before or new ones). But, according to Petzoldt, we can apply the same model to organisms too, for example in the context of metabolic processes, where the physiological system maintains its equilibrium by constantly elaborating the nutritive substances and stimuli coming from the external environment. In this case, Petzoldt's source was Avenarius (AVENARIUS, 1888-1890), who had elaborated a theory of organic activity based on the constant rebalance of disequilibria between nutrition and work. Finally, Petzoldt believes that the same principle of the minimum disruption applies also to mental activity, since we elaborate new information by reducing it to familiar concepts, or by developing new concepts that include the new information. Here Petzoldt's primary sources were Mach and Avenarius, but also the Herbartian tradition, that explained mental activity in terms of "apperception" processes, where the new, unusual representations are assimilated through older, usual representations (see STEINTHAL, 1871).

We cannot go into much detail on the principle of the minimum disruptions. However, it is important to note that it already reveals Petzoldt's philosophical goal, insofar as it is an overarching law that describes a fundamental feature of the whole reality, thus reuniting the material world, the living beings, and the mental sphere. Moreover, Petzoldt regarded his principle as founded upon scientific advances, such as the studies on the stability of the solar system (BHATTACHARYA; LICHTMAN, 2016, p. 23-30), the discoveries in the field of physiology (AGUTTER; WHEATLEY, 2008), and the recent development of scientific psychology (see GREENWOOD, 2015, p. 236ss.).

3.2 The law of univocalness

The principle of stability constitutes the first pillar of Petzoldt's philosophical system, that will remain in place throughout the development of

his philosophy. The second pillar is the so-called law of *Eindeutigkeit*, which can be translated as unambiguity, unequivocalness, uniqueness, or – following in the footsteps of Howard (1992; 1996) – as univocalness.

Petzoldt first introduces the notion of univocalness in a paper entitled *Das Gesetz der Eindeutigkeit* (The law of univocalness, 1895). This concept is designed to play a role analogous to that of stability. According to Petzoldt, stability is the scientifically rigorous redefinition of the metaphysical and anthropomorphic concept of finality. Traditionally, philosophers conceived finality along the lines of human purposiveness, as the free choice of ends and means to ends. In so doing, finality conflicted with causality by positing a second type of determination for the phenomena and the possibility of multiple outcomes to choose from. But, in rigorous scientific terms, there is no room for choice or multiple outcomes, since in nature everything is what it is, and could have not been different from what it is. The concept of stability avoids these mistakes by focusing on the *actual outcome* of natural processes, rather than on allegedly freely chosen goals. According to Petzoldt, the principle of stability only states the *fact* that natural phenomena keep changing until they reach a stable state. The notion of univocalness plays a similar role, insofar as it is supposed to redefine causality to remove all its metaphysical and anthropomorphic elements. Naively, humans tend to think of the relationship between the cause and the effect following the model of human agency: the cause *acts on* the effect and forces it to do something, as if the cause is the master and the effect the slave.

Of course, Petzoldt's criticism of causality was far from new. Hume and Comte already criticized the notion of causality for its metaphysical implications and naïve anthropomorphism. However, Petzoldt made no pretense of novelty, since he (once again) explicitly drew on the work of Fechner, Avenarius, and Mach, who all proposed to substitute the metaphysical notion of causality with the more rigorous concept of functional relation (FECHNER, 1860, p. 1:8; AVENARIUS, 1876, p. 42 ss.; [1891] 1905, p. 18–19; MACH, 1872, p. 33ss.; 1883, 455ss.; see also HEIDELBERGER, 2010). According to these thinkers, the

empirical core of causality is the bare fact that in natural phenomena it is possible to find such relationships between variables, that when one variable changes the other variable changes jointly.

The concept of functional relation was intrinsically connected with the concept of mathematical functions, conveying the fact that natural phenomena can be described in terms of physical laws expressed as equations. Evidently, the idea of substituting causality with functional relations was influenced by the recent formulation of the law of conservation of energy, which – stripped of all improper metaphysical contaminations regarding the indestructibility of force and matter – boils down to the claim that there must exist mathematical equations that express the fixed conversion rates between the various forms of energy.

Even though Petzoldt agreed with the proposal of getting rid of causality in favor of functional relations, he also believed that the latter, *per se*, do not suffice to express the connection of phenomena. Indeed Fechner, Avenarius, and Mach (following Hume) rejected the necessity that was traditionally implied in the notion of causality. Consequently, they allowed a certain degree of indeterminism in the natural phenomena, that Petzoldt regarded as inadmissible. According to them, functional relations do not express what *must* happen, in deterministic sense; they do not express the necessity of the course of nature. Rather, they simply express that there are certain relations among what *actually* happens. For example, assuming that there is a certain set of conditions, there is no necessary determined outcome that must result from those conditions. Instead, there are various possible outcomes, and the functional relation only states that – whatever possible outcome will actually occur – there will be certain relations between certain variables (or, more specifically, between certain quantitative parameters) of the conditions and of the resulting phenomena.

This indeterminacy is not implicit in the thought of Fechner, Avenarius, and Mach. On the contrary, they openly defended this conception. For example, Fechner stated that:

[...] the principle of conservation of force tells us *nothing* about the course or the manner of the transformation between the living force and the potential force⁴; *nothing* about the state in which a system must be at a given time; rather, this depends upon the particular conditions and circumstances of every specific system, which no general principle can determine, but *can only be drawn from the experience*. The principle of conservation of force only tells us that – regardless of how the transformation between the living force and the potential force occurs [...], it can only occur in such a way that their constant sum is maintained; thereby *there still remains freedom for that to occur in infinitely different ways* (FECHNER, 1860, 1, p. 35 emphasis mine).

As we can see, Fechner rejects the idea that the principle of conservation of force dictates the course of nature. The functional relations only regard the quantitative framework within which natural phenomena happen, but no general law can pre-determine what must and will happen. On the contrary, since natural events can always take different paths, only the experience can tell us what happens in a specific case.

In his first book, Avenarius embraced Hume's reduction of the necessity of nature to a psychological attitude that we have towards future phenomena. According to him:

We do not experience the *necessity* of motion [...]. What we do experience is always just that one follows the other. [...] Of course, necessity can also mean that every time that A occurs, then B will follow; therefore, strictly speaking, a certain degree of probability (i.e. certainty) with which the occurrence of the consequence is and should be expected" (AVENARIUS, 1876, p. 45–46).

This means that for Avenarius we can be psychologically certain that an event will follow from a given set of conditions, but there is no real necessity for

⁴ Fechner identified the law of conservation of force with the law of conservation of mechanical energy (that states the constancy of the sum between kinetic and potential energy), since he believed that "in the exact study of nature all physical processes [...], whatever their names (chemical, imponderable, not excluding the organic processes), can be reduced to processes of motion, either of big masses or of tiny particles" (FECHNER, 1860, 1, p. 26).

that event to happen, since we know nothing about the actual necessity of natural phenomena.

Finally, in his *History and Root of the Principle of Conservation of Energy* Mach wrote:

The law of causality is identical with the supposition that between the natural phenomena $\alpha, \beta, \pi, \delta, \dots \omega$ certain equations subsists. The law of causality says nothing about the number or form of these equations [...] but it is clear that if the number of the equations were greater than or equal to the number of the $\alpha, \beta, \pi, \delta, \dots \omega$, all the $\alpha, \beta, \pi, \delta, \dots \omega$ would be thereby *overdetermined or at least completely determined*. The fact of the varying of nature therefore proves that the number of the equations is less than that of the $\alpha, \beta, \pi, \delta, \dots \omega$. But with this a certain *indefiniteness* in nature remains behind. [...] It may not be unimportant for the investigator of nature to consider and recognize *the indetermination which the law of causality leaves over* (MACH, 1872, p. 35–37 emphasis mine).

So, we may say that Fechner, Avenarius, and Mach all held that the notion of functional relation – that had to take the place of the metaphysical, anthropomorphic view of causality – must do away with the concept of natural necessity, either because they believed in an indeterministic universe (Fechner), or because they believed that natural necessity cannot be experienced (Avenarius), or because they believed that we can only find a limited degree of order and regularity in the ever-changing flow of nature (Mach).

As anticipated, Petzoldt did not agree with his masters in this regard (PETZOLDT, 1895, p. 170ss.). Therefore, he developed the notion of univocalness as a way to address the notion of natural necessity, without falling back on the metaphysical concept of the cause as a force that compels the effect. According to Petzoldt, natural phenomena must be completely determined. This means that only *one* outcome must follow from a given set of conditions. Consequently, the real case, the outcome that actually occurs, is something *unique*. And it is precisely this feature that is highlighted by the law of univocalness. This law affirms that, since the real case is unique among all the infinite possible ones:

For every process means of determination [*Bestimmungsmittel*] may be found that determine it uniquely [*eindeutig*], in such a manner that for every variation in this process that one can conceive as being determined by the same means, one can find at least one other variation that would then be determined in the same way as, and thereby be equivalent to, the first, and thus would have, as it were, the same right to be realized as the first (PETZOLDT, 1900–1904, 1:p. 39; for the translation see HOWARD, 1992, p. 170; see also PETZOLDT, 1895, p. 168).

This means that we must investigate natural phenomena so as to find some means of determination (like measurements, quantitative parameters, and so on) that indicate the real case as something unique. In other words, our means of determinations work properly when they provide a single model for the real case and multiple equivalent models – for which no selection criterion can be established – for the merely possible cases. Put another way, our means of determination should provide a univocal (*eindeutig*) description of the actual reality, and an ambiguous, equivocal, or, more precisely, plurivocal (*vieldeutig*) description of the possible cases.

The best illustration of this principle is the law of the parallelogram of forces. We have a billiard ball traveling at a given speed, when it gets hit by another ball. We want to determine what will happen to the ball. Which trajectory and speed will it have after being struck by the other ball? Potentially, we may imagine infinite possible trajectories, infinite possible outcomes. But nature is univocal, *eindeutig*, because the real case is unique. The conditions determine only *one* outcome. Therefore, we can find means of determinations that point univocally to the real case. In the example of the parallelogram of forces, this means of determinations are the vectors. Thanks to the vectors, we can describe the real trajectory and velocity of the ball through the diagonal of the parallelogram. Since the diagonal is unique, it describes univocally the real case, which is unique too (cf. PETZOLDT, 1900–1904, 1: p. 34ss.).

In light of the above, it is apparent that Petzoldt's law of univocalness has two sides. On the one hand, it is a metaphysical principle, that states that phenomena are univocally determined, meaning that in nature only *one* outcome

must occur, given the set of existing conditions. On the other hand, it is an epistemological principle, that requires science to describe phenomena in such a way that the real case is univocally determined, meaning that a theory or model must point univocally to the actual reality, describing it as something unique.

In summary, while embracing the criticism of causality and the notion of functional relations proposed by Fechner, Avenarius, and Mach, Petzoldt claims that we must retrieve the necessity of nature. We cannot throw the baby of the complete determination of phenomena with the bathwater of the metaphysical and anthropomorphic account of causality. It is not enough to say that there are functional relations among the phenomena; these relations must be *univocal* functions, that result in only *one* description of the state of affairs, just like the state of affairs itself is *singular*. In Petzoldt's worldview there is no room for indeterminism, and univocal determination is the concept designed to wipe out any trace of indeterminism left by Fechner, Avenarius, and Mach.

3.3 *Psycho-physical parallelism*

After dealing with the first two pillars of Petzoldt's philosophical system – i.e. the principle of tendency towards stability and the law of univocalness – we can now turn to the third pillar: the principle of psycho-physical parallelism. Here too Petzoldt draws on the work of Fechner, Avenarius, and Mach. Fechner had tried to get rid of the dualism of mind and matter by positing the physical and the psychical as two different standpoints on the same reality (FECHNER, 1860, p. 1ss.). Avenarius and Mach left behind Fechner's metaphysical panpsychism, while maintaining the idea that the physical and the psychical are simply two different ways of considering a reality that *per se* is neither physical nor psychical (cf. AVENARIUS, 1894–1895, 1: p. 412ss.; MACH, 1886, p. 13ss.). However, for all three thinkers, the rebuttal of the ontological distinction between mind and matter as two separate realms of reality did not imply the rebuttal of psycho-physical parallelism; on the contrary, psychophysical

parallelism had to be reaffirmed in a non-metaphysical sense. Instead of regarding the relations between the two substances, this new psychophysical parallelism consisted in the assumption that all mental phenomena must have a physiological correlate in the brain activity. Since matter and mind are not different domains of reality, there is no problem in assuming that all variations in the mental contents must be accompanied by parallel variations in the cerebral substrate. Moreover, when we apply the aforementioned concept of functional relations to the connection between mind and brain, we avoid the risk of interpreting this relation in materialistic sense, as if the mind were a product or an effect of the brain activity. Rather, psychophysical parallelism only states the connections between two sets of variations, without aiming at reducing one to the other (cf. FECHNER, 1860, 1: p. 8ss. AVENARIUS, 1894–1895, 2: p. 13ss. MACH, 1900, p. 46ss. see also HEIDELBERGER, 2004, p. 165ss.).

Petzoldt was committed to this notion of psychophysical parallelism, but also connected it to his own concept of univocalness. According to him, there is no univocal determination between mental contents. Put another way, mental contents cannot be used as means of determination for the occurrence of other mental contents (PETZOLDT, 1900–1904, 1: p. 57ss. 1902, p. 313ss.). The only way to regard the psychical processes as univocally determined is by functionally connecting them to the brain activity. We cannot explain why *one* thought – or feeling, sensation, idea, etc. – occurs at a given time, instead of another of the infinite possible ones, unless we analyze the physiological substrate that is the direct condition of that thought (or feeling, sensation, idea, etc.). In so doing, Petzoldt completely rejects any notion of mental causality by stating that it is the activity of the brain that univocally determines the mental phenomena.

However, there is also another connection between psychophysical parallelism and the law of univocalness, since the latter can be founded on psychophysical parallelism. According to Petzoldt, since all mental activity rests upon brain activity, we must find the basis for the law of univocalness in the cerebral substrate. But the brain is a physiological organ that is the product of a

long adaptation to the environment; it is an organic system that results from a long evolution in accordance with the principle of stability. Therefore, following the evolution of the brain, we find the ultimate root of the law of univocalness in the world itself (PETZOLDT, 1895, p. 167ss.). Put differently, since nature is univocally determined, humans (as organisms that live in such a natural environment) evolve according to this feature of nature. Consequently, univocalness becomes embedded into us, into our own physiological structure, into our own brains. Insofar as univocalness was a condition of our evolution, we are shaped according to it, and need it for our own survival.

As we can see, Petzoldt considers univocalness both as a postulate and as a fact. It is a postulate insofar as we must regard nature as univocally determined, insofar as we must search for univocal determination in the world. In this sense, univocalness is a sort of condition of possibility of our knowledge or, more radically, of our very existence. On the other hand, univocalness is also a fact, since it is primarily an actual feature of the world, confirmed by our own existence. To use the words of Petzoldt himself:

The strength of our principle does not come from a sum of single experiences, but from the fact that we demand its validity from nature. Before being a law, it is a principle with which we turn to reality, a postulate. It is comparatively a-priori valid, independently of every single experience. But it would be bad for a philosophy of pure experience to teach about a-priori truth and to fall back in the most sterile metaphysics. Its a-priority can be nothing but a logical a-priority, never a psychological or metaphysical. [...] The strength of our principle has its roots in very general experiences [...]. These are the facts of our own existence and of the existence of the world; the fact that we are thinking and acting beings; the fact that there is an evolution. None of this would be possible without the complete determination of natural events, which is the most general necessary conditions for all that (PETZOLDT, 1900-1904, 1: p. 40).

As we can see, in this regard Petzoldt defends a completely anti-Kantian position. All Kantian philosophy regards the necessity of nature as a postulate that we impose on our empirical experience. Conversely, Petzoldt rejects the idea that we are the law-giver of nature. Nature has its own law, the law of

univocalness, and we make it ours by living, evolving, and adapting to nature. Kantians are correct in claiming that we must regard nature in its necessity, but if we start from here we cannot explain why we are compelled to do so and why nature meets our postulate, namely what the origin of our a priori forms is and how comes that nature complies with them (cf. PETZOLDT, 1911, p. 126ss.). On the contrary, if we make a Copernican counter-revolution, and accept that reality itself is univocally determined, everything comes full circle: univocal determination is a feature of nature; since we live and evolve in nature, univocal determination gets inscribed into ourselves and our brains; therefore our cerebral activity, as well as the mental processes that depend on it, requires univocal determination; so we demand the world to act according to univocal determination and the world meets this demand because it was univocally determined all along.

3.4 Relativistic positivism

In the previous paragraphs, we saw how – building on the works of Fechner, Avenarius, and Mach – Petzoldt developed the principles of stability, univocalness, and psychophysical parallelism. For him, these principles were interrelated insofar as psychophysical parallelism tells us that knowledge depends upon the brain, and the brain evolves according to the principle of stability in a univocally determined world. Therefore, the more we know nature in its univocal determination the more our brains develop a stable relationship with the natural environment. As already noted, this conception represents a complete departure from Petzoldt's early Kantian agnosticism. Not only he now believes that stability and univocalness provide positive knowledge of the world, rather than being mere regulative principles for the organization of the experiences, devoid of any true grip on reality itself; but with the help of the principle of psychophysical parallelism, he also believes to have overcome the dualism between thought and reality, since mental activity (thanks to its physiological substrate) no longer has its own transcendental rules, but abides

by the same laws of all reality: i.e. the principles of stability and univocal determination.

Those three principles represent the cornerstones of Petzoldt's thought; upon these cornerstones he builds a philosophical system, which he himself names "relativistic positivism" (PETZOLDT, 1911). To understand what he means by that, we should first clarify his interpretation of the terms "positivism" and "relativism". Firstly, Petzoldt regards as the main feature of positivism its opposition to metaphysics. In particular, this implies the rejection of the search for a substance as the never-changing substrate of reality. Secondly, but closely related, Petzoldt defines relativism as the philosophical position that is based upon the refusal of the dichotomy between appearance and reality (cf. Petzoldt 1906b). Thus, relativistic positivism is described primarily in negative terms, in opposition to the mainstream philosophy, characterized by this kind of wrong reasoning: the thinker searches for an unchanging substance to serve as the foundation of all things; he finds it in some particular aspect or part of the totality of things, which is therefore generalized to become a property of everything; since not everything is identifiable with that aspect or part of reality, the thinker claims that things *appear* different, even though *in reality*, underneath, they are nothing but that substance. This mistake is then doubled by the invention of not one but two substances: a material and a mental one (or whatever they are called: matter and mind; nature and soul; extension and thought; the physical and the psychical; world and representation). The duplication of the substance reinforces the juxtaposition of reality and appearance, because it leads to the idea that the mental domain is a mere image of the material reality, separated from it and unable to fully grasp it (PETZOLDT, 1906a, p. 6ss.).

According to Petzoldt, this faulty line of thought runs throughout the history of philosophy, from Thales, who claimed that everything is water, up to materialism and idealism, who claim that everything is matter or thought, respectively (PETZOLDT, 1906a, p. 70ss., p. 113ss.). Even Kant, who purported

to fight against metaphysics, actually based his philosophy upon the dichotomy between the things-in-themselves and the I-in-itself, and therefore ended up deeming our experiences as appearances (PETZOLDT, 1906a, p. 122ss.). But also all post-Kantian philosophers, who pretend to get rid of Kant's thing-in-itself by stating that our experience is the only reality there is, make the same mistake. Insofar as they regard the experience as something subjective, as representations in our consciousness, they still think within the same framework, even if unwittingly. They still begin with the assumption that there must be two different "somethings", two substances, and that our experience belongs to one of these two domains. Indeed, stating that our experience (and therefore reality) is subjective has meaning only when one distinguishes between subject and object, consciousness and matter, the I and the not-I. If *everything* we know were really subjective, this notion would have no meaning, because the term would cease to indicate something specific. Therefore, by stating that everything there is is subjective, post-Kantian philosophers are surreptitiously maintaining the idea of an objective reality beyond our experience (cf. PETZOLDT, 1901, p. 140ss.; 1900-1904, 2: p. 304ss.).

For Petzoldt, relativistic positivism marks a break with this tradition. Instead of assuming that there is the reality on the one hand, and its appearance in our representations on the other hand, it affirms that our experience is not just an image of reality, *it is the reality*. Simply, it is not the reality *in itself*, it is the reality *in relation to us*. However, what matters is that the things-in-relation-to-us are not just some kind of second-hand, phenomenal reality. Since relativistic positivism rejects the notion of substance, it rejects the notion of "in itself" altogether: everything is related to each other, and it is what it is only in relation to everything else (PETZOLDT, 1906a, p. 59ss., p. 141ss.). In his own words:

We can only think of things just as we find them and not as no-one finds them. We can only ever think of them from the perspective in which we find them, and not from a perspective from which we cannot think of them, or in general from no perspective at all. There is no absolute perspective and there is no absence of perspectives, there are only relative perspectives (PETZOLDT, 1906a, p. 142-43).

Considering that our experience is the reality in relation to us, our experience is not a separate domain from reality. There is no sphere of consciousness juxtaposed to the sphere of things. Rather, our conscious experience is the relationship between ourselves (i.e. our bodies) and the other things of the world. My experience of an object is not an image or the phenomenal appearance of that object; it is the relationship between my body and that object. But it is meaningless to ask what is that object in itself, beyond my experience, since nothing is ever “in itself”. There is no substance serving as an unrelated and immutable substrate for all relations and all changes.

Moreover, for Petzoldt, the relationships between our bodies and the other objects of the world are univocally determined. Therefore, my experience of an object is not only real, but also necessary (cf. PETZOLDT, 1906a, p. 66). This further undermines the juxtaposition of reality and appearance. Things do not *appear* to us different from what they *really* are. The experience is not a veil of Maya that covers and hides reality; it is a real process that is governed by the same laws of all reality. We do not fail to grasp reality because the functioning of our mind stands in the way of our understanding of things in themselves, as Kantian philosophy believes. On the contrary, we do grasp reality because the relationship between our body (perceptive organs and brain) and the objects of the world is univocally determined. In brief, the experience is a lawful relation between real objects, not a process in a mental substance that skews the objective reality.

To fully comprehend the distance between Petzoldt’s relativistic positivism and what he regards as the incorrect traditional philosophy, we should look at their different interpretations of two issues: perceptual illusions, and the differences in the experiences of various persons.

Regarding the first, let us take an example from Mach’s work on perception: the so-called Mach bands. As he discovered, the functioning of the retina – which

amplifies the discontinuity in gradients of light intensity – creates the illusory perception of lighter or darker bands at borders of regions with changes in luminance (MACH, 1865). Leaving aside the question of the complex physiological mechanism behind this illusion, what matters is how this phenomenon is interpreted philosophically. In a traditional framework, one may say that *in reality* there are no bands, but these *appear* to us, since our eyes distort the reality while reading it. Therefore, our perceptual apparatus *mistakenly* perceives something that is not really there. On the one hand there is the object-in-itself, which displays a gradient of brightness; on the other hand there is the image in our consciousness, our experience of the object, which contains the bands; and since the object and our representation of it do not coincide, our representation is not a faithful account of the object.

Conversely, according to Petzoldt, it makes no sense to regard the perception of the bands as a mistake, an illusion, an appearance. In natural phenomena (including perception) there are no mistakes, because everything happens according to necessary laws, everything is univocally determined. As Mach himself once wrote:

The expression “sense-illusion” proves that we are not yet fully conscious, or at least have not yet deemed it necessary to incorporate the fact into our ordinary language, that *the senses represent things neither wrongly nor correctly*. All that can be truly said of the sense-organs is, that, *under different circumstances they produce different sensations and perceptions* (MACH, 1868, p. 38; see also 1900, p. 7n).

This means that it is a *real* and *necessary* fact that our eyes function in such a way that they increase the contrast around the edges of a luminance gradient. There is no real object on the one hand, and our mental representation of it on the other hand; there is only a real world and we are a part of it, and the relationship between our perceptual apparatus and the luminance gradient is also a part of it. The brightness gradient *in relation to our eyes really has* the lighter or darker bands at the edges, and there is no brightness gradient in itself, regardless of any relationship with other objects of the world. Of course, we may imagine that a different perceptual apparatus, with eyes that work differently than ours, would

not perceive any bands. But that would only mean that the object *in relation to that perceptual apparatus* would not have any bands. Which leads us to the second issue, regarding the experiences of different persons.

Let us imagine two individuals, and one of them is color-blind. They both observe an apple, so the first one states that the apple is red, while the second one affirms that the apple is grey. According to the traditional interpretation, their experiences of the object are not only separated from the real object, but also from one another. Since both individuals only perceive their own experience, they have no grasp of the experience of the other individual. Both observers are confined within their own representations, unable to ever fathom what is happening in the separate domain that is the consciousness of the other individual. On the one hand there is the object in itself, the domain of reality, and on the other hand there is the object in their representations, in their mental spheres. Hence, we have three different regions, all separated by unbridgeable gaps.

Petzoldt believes that the fact that one object may be experienced differently by two observers is not an argument in favor of the thesis that things *appear* different for different people, and that beyond those appearances there is the way the things *really* are (PETZOLDT, 1906a, p. 143). The apple *is* red for the first observer, and it *is* grey for the color-blind observer. But these statements do not contradict themselves, precisely because they state something about the thing *in relation to the observers*. They would be contradictory only if they both tried to claim something about the apple *in itself*. But, as we already know, for Petzoldt it makes no sense to speak of things in themselves. Furthermore, since the way the two observers perceive the object is univocally determined by the physiological functioning of their bodies, each observer can also acknowledge the reality of the experience of the other individual. This means that the first observer knows that a color-blind person must necessarily perceive the apple differently. Therefore, for him it is a fact both that the apple is red in relation to their own body, but it

is also a fact that the apple is grey in relation to the body of the other individual. Consequently, there is no unbridgeable gap between the experiences of the two observers. The differences in the experiences of distinct individuals are not an epistemological mystery, but simply a fact of nature, that result from the differences in their bodies. If the experience is a lawful relationship between a body and an object, different bodies will experience the object differently. As Petzoldt writes: “we conceive the diversity of the worldviews as lawful [*Gesetzmässig*] insofar as we think them as dependent upon the diversity of the individuals” (PETZOLDT, 1906a, p. 144).

In summary, Petzoldt’s relativistic positivism rejects the traditional philosophical conception according to which there is the material world on the one hand, and the mental world of representations on the other hand, and that we are stuck with what appears in this second world, unable to grasp the reality that is beyond that, as well as what is in the consciousness of other people. His relativistic positivism affirms that there are no different substances; instead, there is only one world, where everything is related to everything else, and we are a part of this world. Our experience is simply an example of the many relationships that exist in the world; it is the relationship between our bodies and the objects around us. And, like other natural relationships, it is also univocally determined.

Consequently, Petzoldt affirms that his relativistic positivism differs from the traditional interpretations of relativism. First, its claim that we can only know reality in relation to ourselves does not end up in a form of subjectivism, because it doesn’t claim that we only know what appears to us, that we only know the subjective appearances of reality. Second, Petzoldt’s relativism is not a form of skepticism, since it does not claim that we do not know the reality; on the contrary, it affirms that we *do know* the reality, i.e. we know it in relation to ourselves (and we ourselves are a part of reality). Third, Petzoldt’s relativism does not imply solipsism, because it rejects the assumption that – since we know reality only in relation to ourselves – we have no bridge towards the experiences of other people. For Petzoldt, the univocal determination is this bridge, because

it allows us to determine how reality must be in relation to other individuals, in relation to other perceptual apparatus.

For all these reasons, Petzoldt's relativism is a form of positivism, because it claims that our experience of the world is not just a shadow, a representation, a sheer phenomenon, but a real knowledge of the world. Furthermore, since everything in the world is related to everything else, in order to know the world one must know the relations. According to Petzoldt, true positive knowledge is relative knowledge, since there is no knowledge other than the knowledge of relations.⁵

For Petzoldt, the strive for the things in themselves is just bad metaphysics. We should get rid of this impossible and meaningless quest for the unchanging substance that is supposed to be the substrate of all relations. Yet, it is natural and healthy to want the world to be stable, because stability is as much a biological need as univocalness. But the stability we need is not the stability of the metaphysical substance, of an *absolute* (literally: untied, not connected) reality that is supposed to be beyond relations. Rather, it is the stability of the univocalness principle, of a general law that governs all changes and all relations. To attain a true and stable knowledge of the world we must look at it in its univocal determination, we must look at it *sub specie univocitatis*, so to speak. For Petzoldt, this is the goal of science, and the physiology of the perceptual apparatus, the investigation of the brain, will help us regard our own experience too in such a way, by discovering the lawful activity of the nervous system upon which the experience depends.

⁵ In contemporary terms, Petzoldt's relativistic positivism would fall into the category of perspectivism. To use Deleuze and Guattari's definition: "Perspectivism, or scientific relativism, is never relative to a subject: it constitutes not a relativity of truth but, on the contrary, a truth of the relative" (DELEUZE; GUATTARI, 1996, p. 130).

4. PETZOLDT'S INTERPRETATION OF MACH'S THOUGHT

4.1 Mach's overcoming of post-Kantian philosophy

After having described Petzoldt's philosophical system, we can now go back to Mach and see how this relativistic positivism is supposed to provide an interpretation – if not an advancement – of Mach's thought.

As quoted at the beginning of this paper, Mach affirmed that things are nothing apart from the complex of sensations. Petzoldt believes that we should not read this statement as meaning that the *things in themselves* are bundles of sensations or as if the objects are *made of* sensations. On the contrary, Mach is stating the same as relativistic positivism, which is: 1) that things *in relation to us* are bundles of sensations; and 2) that there are no things in themselves.

According to Petzoldt, Mach does not want to detach the sensations from ourselves, making them the constituents of the world. Nor does he want to project the world into the subject, making it a sheer perceptual image. When Mach speaks of sensations, we should keep in mind that he means them as functional relations between the objects of the world and our bodies (which, too, are objects of the world). The sensation is therefore a natural phenomenon that *connects* certain objects of the world, not a subjective event in the mysterious domain of consciousness, nor a stuff the world is made of. As Mach himself wrote:

The elements A B C [an object], therefore, are not only connected among one another, but also with K L M [our own body]. *To this extent, and to this extent only, do we call A B C... sensations [...].* In this way, accordingly, we do not find the gap between bodies and sensations above described, between what is without and what is within, between the material world and the spiritual world (MACH, 1886, p. 12 emphasis mine).

Consequently, Machian philosophy does not coincide with immaterialism, or subjective idealism, or phenomenalism, or – more generally – with the

philosophical position according to which the world is our representation. The similarity between these two philosophical positions comes from the fact that they both aim at getting rid of the thing in itself; they both identify reality and our experience; and they both stress that we cannot know reality independent of ourselves. But, despite these similarities, there remains one fundamental difference. From the fact that we cannot know reality independent of ourselves, idealist philosophers infer that reality depends on ourselves (cf. PETZOLDT, 1900–1904, 2: p. 317). According to Petzoldt, Machian philosophy overcomes precisely this crucial mistake. Not only Mach does not claim that reality depends on ourselves, but – even more radically – he claims that *there are no selves*. Mach explicitly rejects the notion of a philosophical subject (the I, the self, the ego, or whatever it might be called), and therefore he succeeds where all post-Kantian philosophy failed (cf. PETZOLDT, 1900–1904, 2: p. 327ss.). As he famously wrote, *das Ich ist unrettbar*, the ego is unsavable:

The primary fact is not the I, the ego, but the elements (sensations). [...] The ego is not a definite, unalterable, sharply-bounded unity. [...] This content, and not the ego, is the principal thing. This content, however, is not confined to the individual. [...] The ego is unsavable (MACH, 1886, p. 17–18).

Since traditional philosophy started with the assumption of two separated realms – reality and the mental sphere – when post-Kantian philosophers tried to get over the idea of things in themselves, they could not fully achieve their purpose because they were stuck with the other half of this dichotomy: with the I in itself, the subject, the residue of the notion of a mental substance. Therefore, they ended up with this faulty reasoning: since there is no thing-in-itself, then everything must fall on the shoulders of the subject. The core of all philosophy after Kant, the fundamental result of his Copernican revolution, is that the subject became the cornerstone of all reality. Mach put an end to this line of thought by getting rid not only of the things in themselves, but also of the I in itself. In so doing, he was the first to succeed in rejecting dichotomy of subject and object as the starting point of philosophy.

According to Petzoldt, we must stress this point to understand what is so revolutionary about the philosophy of Ernst Mach. If we interpret his claim that the object is a bundle of sensations in a subjectivist sense, we have not taken one step forward in comparison to all the philosophy of the nineteenth century, and Mach becomes just another thinker that tried to get rid of the thing in itself, like Kant himself, and basically all philosophers after him. Conversely, the point of Machian philosophy is to get rid of the subject/object dichotomy altogether, so that when we reject the notion of thing-in-itself we do not slip back into subjectivism. But to get rid of the subject/object dichotomy we must get rid of the self. And once we get rid of the self, sensations are not anymore something subjective, they are not mental contents that belong to a subject, or a self, a consciousness. Rather, they are real events of the world that result from (or we may even say: that consist in) the relationship between an object and a body.

Therefore, when reading Mach, we should always keep in mind that the elements of the world are sensations *only in relationship with the body*. Sensations are neither a part of the real world (of the thing *in itself*) nor a part of the mental world (of the I *in itself*), because Mach has overcome this kind of reasoning. Sensations are a connection between certain elements of the world (the objects and our bodies), which are neither mental nor material, neither subjective nor objective.

In summary, for Petzoldt Mach marks a shift in the history of philosophy, because he moved away from the line of thought that tried to solve the dichotomy between the real world and the mental world by widening the role of the second up to the point that there was nothing left beyond the subject. Hence, Mach started a new philosophy, that instead of beginning with the two different substances, focuses on a world where everything is in relation to everything else and nothing is ever in itself. This means that, unlike what was claimed by the phenomenalist and subjectivist interpretations of his ideas, Mach did not want the concrete world to evaporate in a bundle of sensations; on the contrary, it is

the I that evaporates, since there is no longer a place for the metaphysical subject that is supposed to be the one who *has* the sensations.

4.2 Can we rely on Petzoldt's interpretation of Mach?

As anticipated in the paragraph where we reconstructed their relationship, even though Petzoldt regarded himself as a follower of Mach, they did not try to hide their different views on several issues. For this reason someone may question the legitimacy of our attempt to use Petzoldt to shed light on the ideas of Mach. Thus, we should address the problem of whether it is possible to consider Petzoldt's interpretation of Mach not only as an aspect of Petzoldt's *own* philosophy, but also as an aid for a better understanding of Mach's thought. For this purpose, we first need to clarify what they agreed upon and what they disagreed about. Then, we will try to provide some evidence in support of the claim that Mach himself shared Petzoldt's interpretation of his ideas.

When comparing the work of Petzoldt and Mach, one major difference stands out. Indeed, apart from their disagreements on specific topics, it is indisputable that they had two very different philosophical attitudes. As we saw, Petzoldt's goal was to establish a philosophical system capable of providing a positive true knowledge of the world, based on the results of science. Conversely, Mach insisted throughout his life that he was not a philosopher, and that there is no such thing as a Machian philosophy (MACH, 1905, p. VII). Unlike Petzoldt, he did not aim at defining a conclusive and coherent set of philosophical conceptions. Furthermore, he was wary of every thinker (philosopher or scientist) who had the pretension to say something definitive. Therefore, Mach's main contribution to the history of philosophy is not a philosophical system, but his historical-critical approach, that prompts us to question all fixed beliefs by retracing their origins.

This different attitude is also the root of another fundamental disagreement between Mach and Petzoldt, regarding the law of univocalness. When we introduced this concept, we already stressed that Petzoldt explicitly designed it to overcome the indeterministic aspects of Mach's position. Therefore, it is no surprise that Mach did not agree with Petzoldt's assertion that natural phenomena are necessarily and entirely determined. In the *Science of Mechanics*, Mach said that "all forms of the law of causality spring from subjective impulses, which nature is by no way compelled to satisfy" (MACH, 1889, p. 486). Even though we should not interpret these "subjective impulses" in Kantian sense, Mach's statement still means that the necessity that connects all natural phenomena is not a fact of nature; rather, it is a generalization that we make after observing the connections between natural events. Therefore, when Petzoldt countered this statement by affirming that "our own existence is proof that such a 'necessity' really exists" (PETZOLDT, 1895, p. 191n), Mach replied as follows in the next edition of his book:

I believe I am not at variance with Petzoldt in formulating the issue here at stake as follows: It first devolves on experience to inform us what particular dependence of phenomena on one another actually exists, what the thing to be determined is, and experience alone can instruct us on this point. If we are convinced that we have been sufficiently instructed in this regard, then when adequate data are at hand we regard it as unnecessary to keep on waiting for further experiences; the phenomenon is determined for us, and since this alone is determination, it is univocally determined (MACH, 1901, p. 282-83).

Despite Mach's downplaying of the divergence with Petzoldt, it is apparent that here he defends a much weaker conception of natural determination. According to Mach, the univocal determination of phenomena is simply a form of incomplete induction based on a series of empirical observations. Conversely, as we saw, for Petzoldt the principle of univocalness has a much stronger foundation and validity, since it does not come from a sheer sum of single experiences, but it is based upon our own existence and the existence of an ordered cosmos.

No wonder, the debate continued in the new edition of the *Analysis of Sensations*, where Mach wrote:

We can conclude, with Petzoldt, from our own existence and from our bodily and spiritual stability, to the stability and to the univocalness, as regards determination and direction, of the processes of nature. For not only are we ourselves a fragment of nature, but it is the presence of these very properties in our environment that determines our existence and our thought. *But it will not do to build too confidently on this foundation*, for organisms are peculiar fragments of nature, of very limited and moderate stability, which in point of fact are liable to destruction, and for the preservation of which *a proportionately moderate amount of stability in the environment is sufficient*. The most convenient course will therefore be to *recognize the limits which are everywhere manifestly set to our knowledge*, and to regard the effort towards unique determination as an ideal, which, so far as may be, we actualize in our thought (MACH, 1900, p. 237).

Petzoldt acknowledged that Mach accepted this “weakened” version of his position, but reiterated his conviction that:

This cross-section of the world that is revealed by my bare eyes, by the telescope and the microscope, and this cross-section of the world in which I live, and know, and act, cannot be thought without the univocal connection of their elementary features; and therefore this connection exists in every space and time that my thought may cross (PETZOLDT, 1902, p. 325–26).

We quoted these passages from Mach’s and Petzoldt’s work because we think that their discussion on the topic of univocalness not only does not erase the similarities between these two thinkers on other topics, but, in a certain sense, it even confirms them. If they were not reluctant to publicly debate with each other on the pages of their books,⁶ the lack of discussion on other topics may be assumed as proof that they were in agreement about them. Hence, it is striking that in Mach’s work there is not one word where he distances himself from Petzoldt’s “positivistic-relativistic” interpretation of his thought. On the contrary, we have evidence that seems to indicate that Mach let Petzoldt become

⁶ As well as in their private correspondence, but there is not enough space in this paper to delve into it. There is an account in Italian in Russo Krauss (2020, p. 188ss.), which I hope to publish soon in English.

his public defendant on this topic. For example, there are various letters where Petzoldt talks about the philosopher Wilhelm Schuppe, who was a representative of the so-called philosophy of immanence and had personal contacts with Petzoldt and Mach. In one of these letters, Petzoldt writes:

I assured Schuppe that you are not an idealist, nor a subjectivist, nor a psychomonist, or similar, but a true positivist. He was not sure about that, since in the *Analysis of Sensations* you describe the elements of the world as "sensations," by which is often meant something merely subjective (PETZOLDT, 1905).

Even though we do not have Mach's answer to this letter, Petzoldt sounds quite confident that his reply to Schuppe will be welcomed by his mentor. Either way, this letter is indicative of the role played by Petzoldt on the philosophical scene. While Mach was forced home by his health problems, Petzoldt took up the task of fighting against the widespread subjectivist readings of Mach's ideas.

We believe that further arguments in support of the possibility to use Petzoldt as key to interpret Mach may come from a comparative study of Mach's various editions of his books. Indeed, it is our opinion that the changes made by Mach in the new editions of his works show the influence of Petzoldt. This would mean that Mach not only influenced Petzoldt, but was also influenced by him in return (not necessarily in the sense that Petzoldt determined a shift in Mach's opinion, in the sense that Mach adopted Petzoldt's argument in the subsequent expositions of his own ideas.⁷ A possible example of this influence is in the third edition of the *Analysis of Sensations*, where Mach added this section to his famous *Anti-metaphysical Introductory Remarks*:

A common and popular way of thinking and speaking is to contrast *appearance* with *reality*. A pencil held in front of us in the air is seen by us as straight; dip it into the water, and we see it crooked. In the latter case we say that the pencil *appears* crooked, but is in *reality* straight. But what justifies us

⁷ Of course, such a thorough analysis of Mach's works would require a lot of time and pages, and cannot be fulfilled in this short paper. All the more so, considering that there still does not exist a critical edition of Mach's works, that would simplify this kind of investigation. However, I hope to carry out this research in the future.

in declaring *one* fact rather than another to be the reality, and degrading the other to the level of appearance? In both cases we have to do with facts which present us with different combinations of the *elements*, combinations which in the two cases are differently conditioned. Precisely because of its environment the pencil dipped in water is *optically* crooked; but it is *tactually* and metrically straight. An image in a concave or flat mirror is *only* visible, whereas under *other* and ordinary circumstances a tangible body as well corresponds to the visible image. A bright surface is brighter beside a dark surface than beside one brighter than itself. To be sure, our expectation is deceived when, not paying sufficient attention to the conditions, and substituting for one another different cases of the combination, we fall into the natural error of expecting what we are accustomed to, although the case may be an unusual one. The facts are not to blame for that. In these cases, to speak of *appearance* may have a practical meaning, but cannot have a scientific meaning. Similarly, the question which is often asked, whether the world is real or whether we merely dream it, is devoid of all scientific meaning. Even the wildest dream is a fact as much as any other. [...] Where there is no contrast, the distinction between dream and waking, between appearance and reality, is quite otiose and worthless. The popular notion of an antithesis between appearance and reality has exercised a very powerful influence on scientific and philosophical thought. We see this, for example, in Plato's pregnant and poetical fiction of the Cave, in which, with our backs turned towards the fire, we observe merely the shadows of what passes. But this conception was not thought out to its final consequences, with the result that it has had an unfortunate influence on our ideas about the universe. The universe, of which nevertheless we are a part, became completely separated from us, and was removed an infinite distance away (MACH, 902, p. 8-9).

As we can see, this passage displays various typical Petzoldt's themes, such as: 1) the rejection of the dichotomy between reality and appearance; 2) the idea that if *all* experience is considered merely subjective (an appearance, a dream) the very meaning of this word becomes empty, since it ceases to indicate something in contrast to something else; 3) a criticism of the philosophical tradition – that has his most prominent representative in Plato – according to which our experience does not give us a true grasp of the world.

In the Foreword of the edition that features this new piece, Mach himself wrote that he decided to change some passages of the *Anti-metaphysical Introductory Remarks* because they “were often understood in a onesided idealistic sense, an interpretation which I in no wise intended” (MACH, 1902, p. VIII). If

we consider this statement on the background of the original aim of the book, we may propose a hypothesis about what must have happened.

Mach became interested in epistemological questions due to his growing aversion towards the metaphysical mechanistic-materialism that was the more or less explicit framework of scientific investigation, especially in the domain of physics. The success of mechanics had led to the idea that all physical processes could be reduced to mechanical processes, thus devaluating the phenomena that did not fit in that model, such as the sensations. As Mach wrote in *The Science of Mechanics*: “The view that makes mechanics the basis of the remaining branches of physics, and explains all physical phenomena by mechanical ideas, is in our judgment a prejudice” (MACH, 1883, p. 467).

This mistake was felt particularly clear by Mach because he worked in the field of psychophysics, and thus had to deal all the time with sensations. Therefore, he started looking for “a point of view that need not be changed the moment our glance is carried over into the domain of another science; for, ultimately, all must form one whole” (MACH, 1900, p. 21n).

Therefore, in the pursuit of his fundamental goal of exposing the error of “ascribing a kind of higher reality to the spatial and to the temporal than to colors, sounds, and odors” (MACH, 1900, p. 6), Mach emphasized the role of sensations in the formation of knowledge. Since he was not interested in the purely philosophical debate of that era, he did not anticipate that his arguments could be prone to subjectivist readings. Later, when his position was indeed equivocated as an attempt to get rid of the concrete world in favor of sheer mental representations, Mach needed to straighten this issue out. Petzoldt was the right person for this job, because he came from a philosophical education and had become a follower of Mach’s thought precisely because he regarded it as an antidote to the subjectivist tendency of post-Kantian philosophy. For these reasons, Mach relied on Petzoldt to explain that his position not only did not imply any form of subjective idealism, but was actually opposed to this

philosophical trend. Consequently, he never challenged Petzoldt's work on this topic; he probably let Petzoldt argue in his defense in the philosophical circles; and perhaps he even adopted Petzoldt's signature arguments in the new editions of his own books.

5. CONCLUSIONS

We tried to show that the work of Joseph Petzoldt not only provides an interesting attempt to further develop the thought of Ernst Mach into a philosophical system, but can also be used to shed light on the ideas of Mach himself. In particular, we believe that Petzoldt's own relativistic positivism may help us to counterbalance the still too widespread phenomenalist and subjectivistic interpretations of Mach's thought. Despite the disagreements between Petzoldt and Mach on various issues (first and foremost on the topic of univocalness), Petzoldt correctly stressed two cornerstones of Mach's position, that distinguished him from the post-Kantian tradition: his rejection of the notion of philosophical subject (the I, the ego, the self); and his realistic-relativistic reading of the sensations as functional relationships between objects and human bodies.

Even though our study of Petzoldt's philosophy is just the first step in this direction, we hope to further support this research hypothesis in the future, with a closer study of the correspondence between Petzoldt and Mach, and with a philological comparative investigation of the various editions of Mach's books.

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