# Mathematical modelling and numerical bifurcation analysis of inbreeding and interdisciplinarity dynamics in academia 

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#### Abstract

We address a mathematical model to approximate in a coarse qualitative the interaction between inbreeding-lobbying and interdisciplinarity in academia and perform a one and two-parameter numerical bifurcation analysis to analyse its dynamics. Disciplinary diversity is a necessary condition for the development of interdisciplinarity, which is being recognized today as the key to establish a vibrant academic environment with bigger potential for breakthroughs/innovation in research and technology. However, the interaction of several factors including institutional policies, and behavioural attitudes put significant barriers on advancing interdisciplinarity. A "cognitive rigidity" may rise due to reactive academic lobby behaviours favouring inbreeding. The proposed model consists of four coupled non-linear Ordinary Differential Equations simulating the interaction between certain types of academic behaviour and the rate of knowledge advancement which is related to the level of disciplinary diversity. The effect of a control policy that inhibits inbreeding-lobbying is also investigated. The numerical bifurcation analysis reveals a rich nonlinear behaviour including multistability, sustained oscillations, limit points of limit cycles, homoclinic bifurcations as well as codimension-two bifurcations and in particular Bogdanov-Takens and Bautin bifurcations.


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## 1. Introduction

The challenging complex problems that we are facing today, the ones with important social, health and environmental impacts (such as the climate change, the (re)emergence and the spread of infectious diseases and the mapping of the human brain connectome) are beyond the potential of any single scientific discipline to confront by itself. Their solution requires the synergy and integration of knowledge and efforts from diverse scientific disciplines. Thus, the role and importance of disciplinary diversity and its efficient integration is recognized today as a key to unlocking the potential to achieve breakthroughs in science and technology [1-8]. Recall the example of computational neuroscience, a prototypical example of an interdisciplinary subject that was born around the early 70 s fertilized by the pioneering work of Ian Hodgkin and Andrew Huxley (Nobel Prize laureates in Medicine in 1963). The need to understand the complexity of brain

[^0]development and functioning led to the integration of different traditional disciplines ranging from medicine and biology to physical sciences, social sciences, mathematics, and computer science.

Over the last decades, interdisciplinarity has been emerged in two ways [9]: (a) internally through the interaction and osmosis of different disciplines themselves (such as the case of the birth of computational neuroscience [10], and (b) externally-driven policies that increase and allocate public science funding. For example, the US government took the decision in 1988 to allocate a $\$ 3.8$ billion (through its completion in 2003- $\$ 5.6$ billion in 2010) funding the "Human Genome Project" with the objective of determining the DNA sequence of the entire euchromatic human genome within 15 years. According to studies, the benefit from this initiative added around $\$ 1$ trillion to US economy [11,12].

Today, many universities, government agencies and institutions acknowledge the importance of fostering multi- and interdisciplinary interactions both in research and teaching programs [3,6-8]. However, this process is neither monotonic nor easy to establish. Several factors including structural and behavioural aspects, conflict of interests (especially in funding) raise significant barriers towards this aim $[8,13,14]$. All in all, what is recognized as the main barrier is the resistance to change [15]. These barriers establish a "cognitive rigidity" that favours the conduction of both research and teaching within rigid boundaries of disciplines [14]. Policies and practices for the allocation of research grants, recruitment, tenure and career advancement are some of the structural barriers hindering interdisciplinarity [8,13,16]. This structural "rigidity" coupled with professional friendships and academic lobbying behaviours make established/reactive practices harder to change [8]. As in any social system, people are connected to others that share common practices that are more familiar to them. In academia, this "familiarity" is expressed both in the advancement of careers and faculty recruitment. [17] defined "academic inbreeding" as the recruitment practice of departments/institutions to hire their own graduates as faculty directly after doctoral graduation (see also [18-20]).

In the past, this practice was likely to be beneficial in terms of fast production of research results as it fosters research team cohesion and continuity and diminishes recruitment risks [21]. However, it has been widely accepted [22] that academic inbreeding enforces the closeness of universities by favouring internal over external academic information exchange, thus leading to intellectual and organizational inertia [23]. Furthermore, academic inbreeding is criticized as a non-meritocratic practice hindering academic productivity, research quality and innovation [19,21,24-26].

On the other hand, it has been recognized that the challenges of today demand openness and disciplinary diversity to innovation [7,8]. Thus, many countries, among them the USA, UK and Germany have established recruitment policies against academic inbreeding in order to facilitate dynamic interaction among academics with the aim of enhancing cross-disciplinary research [18,27,28].

Nowadays, it has been argued that the concept of academic inbreeding needs to be re-examined beyond its traditional "institutional" definition to include also "intellectual inbreeding" and "social"-related inbreeding: inbreeding has been associated with the re-production of learned knowledge, research activities, hiring practices, and a consolidation of social structures [29,30]. In other studies, academic inbreeding is defined as the promotion of academic practices (hiring process, promotion of careers) on the basis of social/personal relationships (most often between senior faculty and former students), rather than on the basis of academic merits [19,27], all in all what is called lobbying. Furthermore, inbreeding solidifies hierarchical relationships within departments, enhances the power of senior faculty members [19], consequently decreases disciplinarity diversity, thus leading to a vicious circle.

At this point we should note that there is no general consensus about what causes inbreeding and academic lobbying. Several reasons have been suggested including weak national academic labour markets, traditions of immobility in both employment and society, hiring mechanisms that involve personal/social ties and national language policies [19]. Interestingly, in most countries where inbreeding prevails, experts stress the importance of social ties (see for example the discussion in [19]).

Regarding scientific productivity, many studies have shown the negative effect of inbreeding in scientific productivity [21,31,32] and world's research output in terms of innovation [19,24,32,33]. However, as also noted in several studies [17,19,21,34], the consequences of inbreeding are less harmful in "leading" universities, as their graduates are well above the average in terms of academic achievements and are well integrated into the international academic community.

Here, based on the above concepts, we construct and analyse a nonlinear ODE model that attempts to approximate qualitatively some features of inbreeding dynamics and its interplay with disciplinary diversity and the advancement of innovation. Our model (see Fig. 1 for a conceptual scheme) contains three types of individuals (adopting terms which are used in the literature [21,27,31,35]: those that favour (are proponents to) academic inbreeding (lobbying), thus hindering cross-disciplinary research (we call them "Inbreds"), those that do not favour academic inbreeding and favour the advancement of disciplinary diversity (we call them "Outbreds") and the "Neutrals".

Because there is extensive literature on academic inbreeding, we will use the propensity to practice inbreeding as a proxy for being a proponent of disciplinary diversity. We recognize that these two concepts are slightly different, as inbreeding involves hiring graduates of the same university while disciplinary diversity means working with people from different research disciplines, but we suggest they are related as we also discussed above, thus adopting a more general concept/meaning of the above terms (also given in other studies (see e.g. [19,36,37]), than that usually given in the literature. For example the term "Inbreds" is most often used to characterize faculty who perform research and teaching in the university in which they had received all or any part of their training [17,34,38]. Similarly, the term "Outbreds" (or non-Inbreds) is most-often used to characterize the faculty [sic] "working in an university other than the one where the doctoral degree was awarded and worked on several universities during their academic career" [19,27]. However,


Fig. 1. Conceptual overview of the model. Schematic representation of an academic system structure and related effects on rates of breakthrough knowledge advancement. Black filled symbols refer to faculty that foster interdisciplinarity (here named Outbreds), triangles represent faculty members that favour inbreeding against interdisciplinary (named Inbreds/Lobbyists), and open circles are Neutral individuals. In (A), the system is characterized by high disciplinary diversity and small-sized inbreeding, thus maintaining sustainable conditions for positive rates of knowledge advancement and consequent strong stimulus to innovation. In (B), the system presents large sized groups of Inbreds and therefore low discipline diversity, producing a negative feedback on the rate of knowledge "jumps" and consequent reduced stimulus to breakthroughs. The transition from $B$ to $A$ is only possible by an external policy control against inbreeding.
as discussed in other studies (see for example in $[19,36,37]$ ), the term "Inbreds" is used in a more broader sense not limited to the fore-mentioned "geographical" mobility definition; it is rather used to characterize the faculty that favours inbreeding [sic] "based on personal relationships rather than the standardized evaluation of applications or the thorough analysis of individual skills" [36].

In our model, as reported in several studies (see e.g. [21,27], Inbreds/Lobbyists show a clear tendency to work more within their sectorial/disciplinary expertise rather than on new ideas outside their discipline, i.e. on ideas that require interdisciplinary research. On the other hand, "Outbreds" refer to the individual researchers that do not favour inbreeding, and foster interdisciplinarity. So, individuals belonging to this category have an interest and tendency to work across traditional disciplinary boundaries. In fact, this attitude has been explicitly recognized as a transdisciplinary orientation characterizing researchers with higher production of interdisciplinary research articles [5]. Their interactions and relative dominance create the scientific-cultural environment affecting the capability of the system to achieve breakthroughs in knowledge and innovation (see e.g. the discussion in [24]). These two opposite behaviours objectively produce feedbacks on the diversity levels of the academic scenario, with the Inbreds/Lobbyists and Outbreds to decrease and increase, respectively in the long term, the rate of knowledge innovation. Neutral individuals represent those researchers that may not provide strong contribution to knowledge breakthroughs, but at the same time do not actively favour inbreeding. However, these individuals may change their status either joining the Inbreds/Lobbyists or the Outbreds. Such a decision is influenced (in an analogy to the concept of "cultural attractors") by two opposing factors: the "power" of Inbreds and the level of attractiveness of interdisciplinarity fostered by the Outbreds, respectively [39,40]. We also address a fourth variable (the potential for breakthroughs/innovation, $B$ ), representing the effect of the level of knowledge integration from different fields due to disciplinary diversity. By definition, $B$ is enhanced by Outbreds and inhibited by Inbreds/Lobbyists, as disruptive technologies and/or breakthrough concepts most likely rise from cross-border interactions rather than from data accumulation within an established field (see the discussion in [7,8]).

We then perform a numerical bifurcation analysis to detect the critical points that mark the onset of phase changes in the academic structure and the related capability of innovation. Within this context, the bifurcation diagrams were constructed in the one and two-dimensional parametric space with respect to parameters related to the "influence" of Inbreds/Lobbyists and the external policy intensity aiming at establishing disciplinary diversity. The one-parameter
numerical bifurcation analysis reveals a complex dynamical behaviour including multistability of equilibria, sustained oscillations, limit points of limit cycles and homoclinic bifurcations. The two-parameter numerical bifurcation analysis revealed the existence of Bogdanov-Takens and Bautin (Generalized Hopf) bifurcations.

Finally, we discuss some (loose) connections between our model and phase transitions observed in interdisciplinary research in biology reported from 1989 to 2000 due to the flow of Russian scientists in the USA, the Human Genome Project and the Internet diffusion.

## 2. Mathematical model and assumptions

We address a mean field compartmental model describing in a coarse qualitative way the dynamics of certain academic behaviours and the related scientific potential to breakthroughs. The variables and the main assumptions of our model are the following:

- Assumption \#1. Inbreds/Lobbyists (I) favour academic inbreeding based on either "disciplinary" affinity and/or personal ties with other faculty [19,36,37], thus hindering disciplinary diversity (see e.g.[18,21,28,31]). The lobbyists' main scope is the increase of their academic power (number of researchers in their group and career advancement for their group members). Possibly they can also find agreements between different groups creating "cartels" sharing similar goals, thus inducing an auto-catalytic reinforcement of their specific research sector. This assumption for the negative correlation between academic inbreeding and disciplinary diversity as discussed also in the introduction, has been shown in many studies (see e.g.[18,21,28,31]).
- Assumption \#2. Outbreds $(0)$ are individuals not favouring inbreeding practices, fostering disciplinary diversity and thus enhancing the potential of interdisciplinary research and innovation. We define these individuals as those who [sic] "disrupt the status quo in their institutions although constrained by environmental and institutional factors" [24] linked to the inbreeding practice. Their presence fosters innovation as reported in several studies (see e.g. in [24]).
- Assumption \#3. Neutrals $(N)$ are members of the academia (either new entries or already inside), which are "politically" passive in their preference regarding the advance of inbreeding or disciplinary diversity. As stated above, Neutral individuals do not actively participate to lobby groups but may (depending on the "departmental dynamics") either joining a lobby group or becoming an independent researcher. Such a change is influenced by two opposing factors: the "power" of lobbyists and the level of attractiveness of interdisciplinarity [39,40].
- Assumption \#4. The potential of achieving breakthroughs/innovation products $(B)$ is enhanced by the presence of Outbreds, $O$, and inhibited by the presence of Inbreds, $I$. Our assumption here is that, as reported in several studies, disciplinary diversity is a necessary condition for the growth and establishment of interdisciplinary research and innovation (see e.g. [30]). This assumption of the association between disciplinary diversity and interdisciplinarity is supported by an extensive literature which implements bibliometric and social network analysis techniques to measure interdisciplinarity [30,41-46]. For example, a most common measure of interdisciplinarity is based on the information entropy defined as $-d \log (d)$, where $d$ is the percentage of faculty in a given discipline. This measure reflects both the number of disciplines and the distribution of faculty between them [30].
However, as discussed in other studies (see e.g. [2]) disciplinary diversity is a necessary but not a sufficient condition to guarantee knowledge integration that can lead to truly interdisciplinary research; the latter rather depends on the successful/efficient interaction between different disciplines.
Negative (positive) values of $B$ correspond to low (high) levels of such a potential. Negative and positive values of $B$ could be defined in relation to an average level of scientific research performance, which is set to zero. According to the above, $B$ is modelled through the following equation ( $\dot{x} \equiv \frac{d x}{d t}$ ):

$$
\begin{equation*}
\dot{B}=\theta \cdot O-\eta \cdot I-\epsilon \cdot B \tag{1}
\end{equation*}
$$

where, $\theta$ is the rate of growth of $B$ per Outbred, $\eta$ is the rate of decline of $B$ per Inbred, and, $\epsilon$ is the fade out rate of $B$ in the absence of any "stimulus".

Based on the above assumptions, the model dynamics evolve according to the following rules:

- Individuals leave the system (getting into pension or leaving the department for another institute) with a rate $\mu$.
- Recruitment of Inbreds. In an analogy to the infection rate in networked epidemic models [47,48], an Inbred may influence/"infect"/convince a Neutral to join the Inbreds through direct interaction/contact. This is modelled through the transmission rate $\beta \equiv p \cdot s$, where $p$ is the per-contact(with an Inbred)-probability that a Neutral will be "convinced" to join Inbreds, and $s$ is the average number of contacts with Inbreds per unit time. Hence, $\beta$ is the average rate of contacts a Neutral makes with Inbreds that are sufficient to make him/her to join Inbreds. This rate reflects also new (inbreeding) recruitment (e.g. students of the Inbreds that get hired in the same department). Thus the mean field conversion/hiring rate of Neutrals to Inbreds reads:

$$
\begin{equation*}
r_{N I}=\beta \cdot N \cdot I \tag{2}
\end{equation*}
$$

- Recruitment of Outbreds. A positive, $B$, reflects a relatively high level of disciplinary diversity/interdisciplinarity. Thus, the more positive $B$ is, the easier it is for a Neutral to join Outbreds (or equivalently an Outbred to be recruited by the department). More specifically, Neutrals join Outbreds with a rate:

$$
\begin{equation*}
r_{N O}=\lambda \cdot f(B) \cdot N \tag{3}
\end{equation*}
$$

where $f(B)$ is the logistic function:

$$
\begin{equation*}
f(B)=\frac{1}{1+e^{-b \cdot B+c}} \tag{4}
\end{equation*}
$$

The parameter $b$ controls the steepness of $f(B)$ and the ratio $\frac{c}{b}$ controls the midpoint of $f(B)$, i.e, the value of $B$ where $f(B)$ is $1 / 2$, i.e. the half of its maximum value. For large positive (negative) values of $B, f(B)$ approaches one (zero).
Here, for our simulations we have set $b=15, c=2.25$; hence, the midpoint is at $B=\frac{c}{b}=0.15$ and the slope of $f(B)$ at the midpoint is $\frac{d f(B)}{d B}=\frac{15}{4}$. Note that there is a nonzero (small) probability for a $N$ to behave as/become an $O$ even for (small) negative values of $B$.

- Decline of Inbreeding. Inbreds are "neutralized", i.e. inbreeding declines with a rate:

$$
\begin{equation*}
r_{I N}=\gamma \cdot f(B) \cdot I \tag{5}
\end{equation*}
$$

We have used the same values of the parameters $b, c$. Thus, as above, there is a nonzero small rate at which $I$ are neutralized even for (small) negative values of $B$.
Note that $B$ exerts an inverse effect on the rate of the decline $r_{I N}$ (given by Eq. (5)) and the rate of growth of outbreds $r_{\mathrm{NO}}$ (given by Eq. (4)) i.e. both rules share the same $f(B)$ : as the rate of growth of Outbreds increases (nonlinearly) by $f(B)$ the decline rate of inbreeding decreases analogously.

- Decline of Outbreds. When $B$ is negative, reflecting a relatively low level of interdisciplinarity/potential for breakthroughs, the easier is for an Outbred to be neutralized (or leave the department).
More specifically, Outbreds leave the department/neutralize with a rate:

$$
\begin{equation*}
r_{O N}=\zeta \cdot g(B) \cdot O \tag{6}
\end{equation*}
$$

where $g(B)$ is the logistic function:

$$
\begin{equation*}
g(B)=1-f^{\prime}(B) \tag{7}
\end{equation*}
$$

The function $f^{\prime}(B)$ is a logistic function of the same form of Eq. (4) with generally different values for the parameters $b, c$. Here, for $f^{\prime}(B)$, we have set $b=15$ (the same as in Eq. (4)), and $c=-2.25$ (i.e. the opposite of the value of $c$ in Eq. (4)); hence, the midpoint is at $B=\frac{c}{b}=-0.15$ and the slope of $g(B)$ at the midpoint is $\frac{d g(B)}{d B}=-\frac{15}{4}$. For large positive (negative) values of $B, g(B)$ approaches zero (one).
The above "neutralization" of Outbreds accounts to the natural assumption that when disciplinary diversity remains low, Outbreds realize at some point that their efforts in changing the status quo have no long-term prospects and just give up efforts/ or leave the department.
Note, that there is a nonzero (small) rate at which $O$ are neutralized for small positive values of $B$.

- Control policy against inbreeding. At low levels of innovation, as reflected by negative values of $B$, an external to the department action (e.g. institutional/governmental "control policy") is imposed aiming to depress academic inbreeding. This control policy can be implemented for example by (re)allocating new faculty positions/ and or funding to other departments/institutions where inbreeding does not prevail.
Accordingly, inbreeding is declined by an external feedback policy with a rate:

$$
\begin{equation*}
r_{I N}^{e x t}=\kappa \cdot g^{\prime}(B) \cdot I \tag{8}
\end{equation*}
$$

where, $g^{\prime}(B)$ is of the same form of Eq. (4), but with generally different values of the parameters of the logistic function $f^{\prime}(B)$. For the control policy, in $g^{\prime}(B)$, we have set $b=15$, and $c=-7.5$; thus the midpoint is at $B=\frac{c}{b}=-0.5$ and the slope of $g^{\prime}(B)$ at the midpoint is $\frac{d g^{\prime}(B)}{d B}=-\frac{15}{4}$. For large positive (negative) values of $B$, $g(B)$ approaches zero (one).
The inverse of the rate $r_{I N}$ can be interpreted as the mean latent period, i.e. the period between the application of the policy and depression of the inbreeding practices. See that with this selection of the parameter values, the external control action is activated considerably for relatively large negative values of $B$, as the midpoint is at $B=-0.5$.

Our choice to model the rate of transformations with respect to $B$ by the logistic function is based not only on well established biological/population dynamics theoretical concepts and experimental studies. Recent social dynamics studies including experimental data have shown that the logistic function models also the impact of social interconnected relationships (e.g.cooperation, friendship, communication, influence, consensus formation and decision making) [49,50].

In summary, based on the above assumptions, the mean field model reads:

$$
\begin{array}{r}
\dot{N}=\mu \cdot(1-N)+\kappa \cdot g^{\prime}(B) \cdot I+\zeta \cdot g(B) \cdot O \\
-\lambda \cdot f(B) \cdot N-\beta \cdot N \cdot I+\gamma \cdot f(B) \cdot I \tag{9}
\end{array}
$$

Table 1
Model parameters and variables.

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $N$ | density of Neutrals | dimensionless |
| I | density of Inbreds (Lobbyists) | dimensionless |
| 0 | density of Outbreds | dimensionless |
| B | potential for breakthroughs/innovation | dimensionless |
| $\mu$ | probability at which individuals get into pension, and/or leaving their department | years ${ }^{-1}$ |
| $\beta$ | average rate of contacts a Neutral makes with Inbreds that are sufficient to make him/her to be recruited by Inbreds | contacts $\cdot$ years ${ }^{-1}$ |
| $\zeta \cdot g(B)$ | rate at which an Outbred becomes Neutral or leaves the department; $g(B)$ is the logistic function scaling the rate w.r.t. $B$ | years ${ }^{-1}$ |
| $\lambda \cdot f(B)$ | rate at which a Neutral is recruited by Outbreds; $f(B)$ is the logistic function scaling the rate w.r.t. $B$ | years $^{-1}$ |
| $\gamma \cdot f(B)$ | rate at which an Inbred becomes Neutral; $f(B)$ is the logistic function scaling the rate w.r.t. B | years $^{-1}$ |
| $\kappa \cdot g(B)$ | rate at which external policy neutralizes inbreeding. Its inverse can be regarded as the mean latent period, i.e. the period between the implementation of the policy and neutralization; $g(B)$ is the logistic function scaling the rate w.r.t. $B$ | years $^{-1}$ |
| $\theta$ | rate of growth of $B$ per Outbred | years $^{-1}$ |
| $\eta$ | rate of decline of $B$ per Inbred | years ${ }^{-1}$ |
| $\epsilon$ | rate of decline of $B$ per neutral | years $^{-1}$ |

$$
\begin{align*}
& \dot{I}=-\mu \cdot I+\beta \cdot N \cdot I-\kappa \cdot g^{\prime}(B) \cdot I-\gamma \cdot f(B) \cdot I  \tag{10}\\
& \dot{O}=-\mu \cdot O+\lambda \cdot f(B) \cdot N-\zeta \cdot g(B) \cdot O  \tag{11}\\
& \dot{B}=\theta \cdot O-\eta \cdot I-\epsilon \cdot B \tag{12}
\end{align*}
$$

Note, that if the initial conditions are chosen so that $N+I+O=1$, the above system preserves density as $\dot{N}+\dot{I}+\dot{O}=0$. Hence, Eq. (11) can be omitted from a numerical analysis point of view.

Computations were performed using MATCONT [51]. The numerical continuation of solutions past critical points was performed using the Moore-Penrose continuation [51] and the absolute and relative error for the Newton-Raphson iterations were set equal to $1 . E-06$. The tolerance of test functions used to detect criticalities was set equal to $1 . E-05$. The computations of limit cycles were performed using 20 mesh points and 4 collocation points.

A summary of the model parameters and variables is shown in Table 1.

## 3. Numerical bifurcation analysis

The dynamics of the model Eq. (9)-(12) were analysed using the tools of numerical bifurcation analysis. In particular, we used as bifurcation parameters the intensity of the power of academic inbreeding to recruit neutrals (represented by the parameter $\beta$ ) and the intensity of the external policy against academic inbreeding (represented by the parameter $\kappa$ ). The values of the other parameters were set to $\mu=0.05, \lambda=0.1, \gamma=0.05, \zeta=0.07, \theta=1, \eta=0.05, \epsilon=0.02$.

At this point we should note, that the above values of the parameters are based on intuition rather than quantitative measures, as such (except from the rate of retirement which is expressed by the variable $\mu$ ) have not (to the best of our knowledge) been reported yet in the literature. However, there is a rationality on the selection of the values of the parameters. For example $\mu=0.05$ years $^{-1}$, reflects an average of 20 years of service of a faculty in the same department. Also the selection of $\lambda=0.1$ years $^{-1}$ reflects a recruitment rate of one Outbred every 10 years (i.e. approximately at the half of the life time of an (Outbred) faculty in a department), when $B$ is at its maximum value. The values of the other parameters have been selected on an analogous basis. In the next section we perform also a sensitivity analysis to study the robustness of the obtained bifurcations to parameter changes. Our work could initiate further advancements and research questions in the field for the quantification of such parameters with the aid of Scientometry and Sociometry (we talk about this issue also in the Discussion).

## One-dimensional bifurcation diagrams in the absence of control policy

In the absence of a control policy, i.e. for $\kappa=0$, and for $\beta=0.14$, the system exhibits multiplicity of states. Depending on the initial conditions $(N(0), I(0), O(0), B(0))$, two different stable equilibrium points can be reached (see Fig. 2A-D).


Fig. 2. Model dynamics without control policy $(\kappa=0, \beta=0.14)$. Time evolution of the state variables. (A) $B$ and (C) $N, I, O$ with initial conditions $N(0)=0.334, I(0)=0.333, O(0)=0.333, B(0)=-4,(\mathbf{B}) B$ and (D) $N, I, O$ with initial conditions $N(0)=0.334, I(0)=0.333$, $O(0)=0.333, B(0)=-1$. One-dimensional bifurcation diagrams with respect to $\beta$. E. $N, \mathbf{F} . I, \mathbf{G} . O, \mathbf{H} B$. There are two turning points at $\beta \approx 0.058$ $((N, I, O, B)=(0.862,0.136,0.001,-0.284))$, and at $\beta \approx 3.588((N, I, O, B)=(0.028,0.917,0.054,0.428))$; a transcritical bifurcation appears also at $\beta \approx 0.3((N, I, O, B)=(0.333,0,0.667,33.333))$. Solid lines correspond to stable equilibrium and dotted lines to unstable equilibrium.

Starting for example with equal densities for all three types of individuals and for $B(0)=-4$, the system converges to $(N, I, O, B) \approx(0.357,0.643,0,-1.185)$. This state is characterized by a low level of innovation capability and domination of the inbreeding practice (Fig. 2A,C). Setting as initial condition $B(0)=-1$, the system converges to an inbreeding-free state $(N, I, O, B) \approx(0.333,0,0.667,33.333)$ characterized by a very high level of breakthrough potential and dominance of the Outbreds (Fig. 2B,D).

To systematically investigate the system's behaviour with respect to $\beta$, in the absence of a control policy, we performed a one-parameter numerical bifurcation analysis (see Fig. 2E-H). Our analysis revealed two turning points at $\beta \approx 0.058$ $((N, I, O, B) \approx(0.862,0.136,0.001,-0.284))\left(T P_{1}\right)$ and at $\beta \approx 3.588((N, I, O, B) \approx(0.028,0.917,0.054,0.428)) T P_{2}$, as well as a transcritical bifurcation at $\beta \approx 0.3((N, I, O, B) \approx(0.333,0,0.667,33.333)) T R$. The two turning points mark the appearance and disappearance of solutions and the transcritical bifurcation marks the exchange of the stability between the solution branches. Thus, for $\beta<0.058$, the only stable solution is the inbreeding-free state characterized by the dominance of Outbreds and therefore by very high levels of potential for innovation. At this branch of solutions, a transcritical bifurcation appears at which the inbreeding-free state loses its stability in favour of another branch of stable equilibria that ends up to the turning point $T P_{2}$. This branch is now characterized by the co-existence of all


Fig. 3. One-dimensional bifurcation diagrams with respect to $\kappa$ for $\beta=0.14$ ( $\mathbf{A}-\mathbf{D}$ ) and for $\beta=0.18$ (E-H). Solid lines correspond to stable equilibria and dotted lines to unstable equilibria; filled (open) circles correspond to maximum and minimum values of stable (unstable) oscillations. In $\mathbf{D}$, $\mathbf{H}$ the inset shows the bifurcation diagram of the period of oscillations with respect to $\kappa$. For $\beta=0.14$, there is a subcritical Andronov-Hopf bifurcation $(A H)$ at $\kappa \approx 6.147((N, I, O, B)=(0.824,0.171,0.005,-0.197))$. A stable inbreeding-free solution $(N, I, O, B)=(0.333,0,0.66,33.333)$ (not shown) also exists for all values of $\kappa$. For $\beta=0.18$ there is a supercritical Andronov-Hopf bifurcation $(A H)$ at $\kappa \approx 17.55((N, I, O, B) \approx$ $(0.707,0.281,0.011,-0.137))$. The branch of limit cycles has a turning point $(L P C)$ at $\kappa \approx 18.508$. A stable inbreeding-free solution $(N, I, O, B)=$ ( $0.333,0,0.667,33.333$ ) (not shown in Figure) also exists for all values of $\kappa$.
three "species" and thus by relatively high levels of disciplinary diversity. For example, on this branch at $\beta \approx 0.5$, $(N, I, O, B) \approx(0.2,0.4,0.4,19)$, while at $\beta \approx 1.0,(N, I, O, B) \approx(0.1,0.7,0.2,8.313)$. Beyond $T P_{1}$, the solutions become unstable up to the other turning point $\left(T P_{2}\right)$. At $T P_{1}$ a branch of stable equilibria is born. This branch is characterized by the absence of Outbreds $(0=0)$ (see Fig. 2G) and thus low levels of potential of breakthroughs (see Fig. 2H). For increasing values of $\beta$, this branch leads asymptotically to the absolute dominance of inbreeding. Indeed, on this branch at $\beta \approx 0.125,(N, I, O, B) \approx(0.4,0.6,0,-1.5)$, while for $\beta \approx 0.5,(N, I, O, B) \approx(0.9,0.1,0,-2.25)$.

## One-dimensional bifurcation diagrams in the presence of control policy

To analyse the system's behaviour in the presence of the control policy against inbreeding, i.e. for $\kappa>0$, we constructed the bifurcation diagrams with respect to $\kappa$. Fig. 3 depicts the one-dimensional bifurcation diagrams for $\beta=0.14$ (Figs. 3A-D) and $\beta=0.18$ (Figs. 3E-H).


Fig. 4. Homoclinic orbit $(N(t), B(t))$ at $\kappa \approx 5.475$ for $\beta=0.14$.

For $\beta=0.14$, a stable inbreeding-free solution $(N, I, O, B)=(0.333,0,0.667,33.333)$ exists for all values of the bifurcation parameter $\kappa$ (not shown in Fig. 3A-D).

For $\kappa<6.147$, the co-existence of all three types of academic behaviour is possible. This co-existence is characterized by high densities of Neutrals (around $80 \%$ of the total staff, see Fig. 3A), moderate densities of Inbreds I (around 19\% of the total staff, see Fig. 3B), low densities of Outbreds $O$ (around $1 \%$ of the total staff, see Fig. 3C) and a relatively low/moderate potential for breakthroughs $B$ (around -0.3 , see Fig. 3D). At $\kappa \approx 6.147((N, I, O, B)=(0.824,0.171,0.005,-0.197)$ ), the equilibrium state looses its stability by a subcritical Andronov-Hopf bifurcation $(A H)$ which marks the onset of a branch of unstable limit cycles. This branch of unstable limit cycles disappears at a homoclinic bifurcation (HB) at $\kappa \approx 5.475$ where the unstable limit cycle hits the saddle equilibrium. Fig. 4 depicts a homoclinic orbit for $\beta=0.14$ appearing at $\kappa \approx 5.475$.

For $\kappa>6.147$ the only stable solution is the inbreeding-free one characterized by a very high level of disciplinary diversity. For even higher values of $\kappa$, a turning point $(T P)$ appears at $\kappa \approx 12.1782((N, I, O, B)=(0.616,0.367,0.017,-0.11))$, which however does not change the stability of solutions; on either side of the turning point, the equilibria are saddles.

For $\beta=0.18$, we get the bifurcation diagrams with respect to $\kappa$ depicted in Fig. 3E-H. Now, the subcritical AndronovHopf bifurcation $(A H)$ turns into supercritical at $\kappa \approx 17.55((N, I, O, B)=(0.707,0.281,0.011,-0.137))$, giving birth to a branch of stable limit cycles. The stability of limit cycles is then lost through a limit point of limit cycles (LPC) at $\kappa \approx 18.508$. The branch of unstable limit cycles disappears through a homoclinic bifurcation $(H B)$ at $\kappa \approx 18.48$ where the limit cycle hits the saddle equilibrium. For $\kappa>18.508$ the only stable solution is the inbreeding-free state characterized by a very high level of potential for breakthroughs. For even higher values of $\kappa$, a turning point (TP) appears at $\kappa \approx 21.349$ $((N, I, O, B)=(0.554,0.426,0.02,-0.094))$ that does not change the stability of equilibria as on either side of the branch, these are saddles.

By further increasing the value of $\beta$ to $\beta=0.2$ we get the bifurcation diagram w.r.t. $\kappa$ illustrated in Fig. 5.
The Andronov-Hopf bifurcation disappears and we end up with a saddle-node bifurcation at $\kappa \approx 26.344,(N, I, O, B) \approx$ ( $0.53,0.45,0.02,-0.088$ ).

Taken all together, the results of the above one-dimensional bifurcation analysis suggest the existence, in the twodimensional parameter space ( $\kappa, \beta$ ), of at least one Bautin (generalized Hopf) bifurcation marking the boundary between sub and supercritical Andronov-Hopf bifurcations, and potentially, of a Takens-Bogdanov bifurcation which marks the coincide of turning points with Andronov-Hopf bifurcations.

## Two-parameter numerical bifurcation analysis

To explore the overall system's behaviour in the two-parameter dimensional space ( $\kappa, \beta$ ), we constructed the twodimensional bifurcation diagram depicted in Fig. 6. We also illustrate phase portraits of a sustained oscillation within the region of existence of stable limit cycles for $\kappa=18, \beta=0.18$ and initial conditions $N(0)=0.729, I(0)=0.259$, $O(0)=0.012, B(0)=-0.15$. The period of oscillations is of the order of several decades.

As it is shown, there are two Bogdanov-Takens (BT) bifurcations, where the Andronov-Hopf bifurcations coincide with the turning points at $(\kappa, \beta) \approx(0.045,0.059),(N, I, O, B) \approx(0.859,0.139,0.002,-0.253)$, and at $(\kappa, \beta) \approx(23.006,0.187)$, $(N, L, I, B K) \approx(0.546,0.434,0.02,-0.092)$.


Fig. 5. One-dimensional bifurcation diagrams with respect to $\kappa$ for $\beta=0.2$. A. Neutrals, $N, \mathbf{B}$. Inbreds, $I, \mathbf{C}$. Outbreds, $O$, D. $B$. There is a turning point $(T P)$ at $\kappa \approx 26.344((N, I, O, B) \approx(0.53,0.45,0.02,-0.088))$. A stable inbreeding-free solution $(N, I, O, B)=(0.333,0,0.667,33.333)$ (not shown in the figure) also exists for all values of $\kappa$. Solid lines correspond to stable equilibria and dotted lines to unstable equilibria.

There are also two Bautin bifurcations (generalized Hopf bifurcations (GH)), which set the boundaries between sub and supercritical Andronov-Hopf bifurcations at $(\kappa, \beta) \approx(10.748,0.1639),(N, I, O, d) \approx(0.788,0.205,0.007,-0.173)$, and at $(\kappa, \beta) \approx(21.842,0.185),(N, I, O, B K) \approx(0.607,0.376,0.017,-0.108)$. On the branch of the limit points of limit cycles that connects the two GH points (shown with (green) solid line), there is a cusp point at ( $\kappa, \beta) \approx(7.455,0.1503)$. The region of stable sustained oscillations is bounded between the branch of Andronov-Hopf bifurcations connecting the two $B T$ points (shown with dotted (blue) line), and the branch of limit points of branches of limit cycles (shown with (green) solid line).

At this point we should note, that due to the test function that MATCONT uses to perform continuation of AndronovHopf points, MATCONT reports also a branch of neutral saddles, that emerges from the BT bifurcation at $(\kappa, \beta) \approx$ ( $21.842,0.185$ ). Neutral saddles satisfy also the test function of MATCONT for Andronov-Hopf points (as at these points the sum of two (non zero) real eigenvalues becomes zero), yet these points are not bifurcations. Hence, we do not show this branch in the two dimensional bifurcation diagram.

As observed from Fig. 3 but also from the two-parameter bifurcation diagram shown in Fig. 6, the type and location of Andronov-Hopf bifurcation points depend on the interplay between the power of lobbyists (reflected by $\beta$ ) and the intensity of the control policy (reflected by $\kappa$ ) aiming at inhibiting academic inbreeding-lobbying. In particular, in Fig. 3 there are reported two different bifurcation diagrams for increasing values of the parameter $\beta, \beta=0.14$ (A-D) and for $\beta=0.18(\mathrm{E}-\mathrm{H})$, which is related to the power of the lobbyists. For $\beta=0.14$ the limit cycles arise with a subcritical Andronov-Hopf bifurcation which means that the oscillations are unstable for these parameter values. Whereas for $\beta=0.18$, the Andronov-Hopf bifurcation becomes super-critical (through a Bautin bifurcation) and the arising limit cycles are stable. The appearance of these stable oscillations reveals a back and forth competitive process between openness and closeness to new ideas. By increasing values of both the power of lobbyists and the intensity of control policy, the oscillations disappear through a Bogdanov-Takens bifurcation where turning points coincide with Andronov-Hopf bifurcations including the existence of a homoclinic orbit where the period of oscillations becomes infinite. Beyond the Bogdanov-Takens bifurcation do not anymore exist oscillations. Lobbying, when it exists is stable and disappears only beyond a turning point for very high values of the control policy (see Fig. 5).

## Sensitivity of the system dynamics with respect to the parameter values

Because of the lack of experimental data for many of the parameters in our model (e.g. for the rate of the fade out of $B$, or for the rates of conversion/recruitment by Inbreds/Outbreds etc.), a question that arises is if the above described


Fig. 6. Two-dimensional bifurcation diagram with respect to $\kappa$ and $\beta$. There are two Bogdanov-Takens (BT) bifurcations and two Bautin (generalized Andronov-Hopf (GH)) bifurcations. The dotted (blue) line is the branch of Andronov-Hopf bifurcations. The branch of turning points of the limit cycles that connects the two GH bifurcations is shown with the solid (green) line; this branch exhibits a cusp point at ( $\kappa, \beta$ ) $\approx(7.455,0.1503)$. The branch of the continuation of turning points passing through the two BT points is shown with the (black) solid line. Phase portraits of sustained oscillations for $\kappa=18, \beta=0.18$ are also shown. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
dynamics are present also for other values of the parameters. For that purpose, we have changed an order of scale the values of the parameters of $\eta, \epsilon$ and we have set similar values to the other rates. In particular, to test the robustness of the model with respect to changes in the parameter values, we performed a numerical bifurcation analysis using the following set of parameter values: $\mu=0.04, \lambda=0.1, \gamma=0.05, \zeta=0.1, \theta=1.5, \eta=0.3, \epsilon=0.4$, while keeping $\beta$ and $\kappa$ as bifurcation parameters. The results of this analysis are depicted in Fig. 7. As it is shown, for the case without a control policy (Fig. 7A), the bifurcation diagram with the new set of parameter values is qualitatively the same with the one shown in (Fig. 2F). Furthermore, the bifurcation diagram with respect to $\kappa$ is again qualitatively similar to the one shown in Fig. 3B. Thus, as demonstrated by the numerical bifurcation analysis, (significant) changes in the values of the model parameters do not alter qualitatively the behaviour of the systems dynamics.

## 4. Analogy to real-world observed dynamics

In the presence of an external control action against inbreeding and moderate rates of influence of Inbreds/Lobbyists, our model predicts bifurcation points beyond which inbreeding disappears and there are again two stable states both characterized by zero levels of inbreeding. One of them is characterized by the dominance of Neutrals and low presence of Outbreds and moderate levels (potential) of research breakthroughs. The other one corresponds to very high levels/potential for research breakthroughs due to the dominance of Outbreds that favour disciplinary diversity.

It should be noted, that the intensity of external action that is necessary to moderate inbreeding is relatively large with respect to the values of the other rates. This reflects the fact that (as also reported in [8]) in order to tackle conservative attitudes, [sic] "a high level of awareness and effort is required". For higher values of the power of influence of Inbreds/Lobbyists, the system behaviour is enriched with stable sustained oscillations in a relatively small window of the parameter space. These oscillations can be interpreted as steps back and forth dynamics between openness and closeness to new ideas as known to occur in societies with opposite tendency induced by concurrency [52].


Fig. 7. One-dimensional bifurcation diagrams of $I$ with respect to $\beta$ and $\kappa$ for $\mu=0.04, \lambda=0.1, \gamma=0.05, \zeta=0.1, \theta=1.5, \eta=0.3, \epsilon=0.4$. A Bifurcation diagram with respect to $\beta$ for $\kappa=0$. As in Fig. 2 in the absence of control policy, there are two turning points at $\beta=0.0544$ $((N, I, O, B) \approx(0.746,0.243,0.01,-0.145))$ and at $\beta=0.913((N, I, O, B) \approx(0.096,0.672,0.231,0.362))$. B Bifurcation diagram with respect to $\kappa$ for $\beta=0.1$. As in Fig. 3 a turning point occurs at $\kappa=17.391((N, I, O, B) \approx(0.631,0.318,0.051,-0.052))$ and a subcritical Andronov-Hopf bifurcation (first Lyapunov exponent:3.059) occurs at $\kappa=16.6((N, I, O, B) \approx(0.674,0.287,0.039,-0.0684))$. Solid lines correspond to stable equilibria and dotted lines to unstable equilibria.


Fig. 8. Phase transitions in disciplinary publications and related breakthrough knowledge. Dots correspond to number of interdisciplinary papers connecting mathematics and molecular biology resulting from the flow of Russian scientists in the USA in 1990 (RJ), the raise in funding by the Human Genome Project in 1992 (HG) and internet diffusion in 2000 (ID) as reported in [53]. Solid line corresponds to model simulations with calibrated changes of the parametric values. This has been obtained as follows (see also Fig. 6a): initial conditions at 1984: $(N, I, O, B)=(0.45,0.4,0.15,0.1)$, run from 1984 until 1990 using $\mu=0.04, \lambda=0.05, \beta=0.5, \gamma=0.05, \zeta=0.1, \theta=1.5, \eta=0.3, \epsilon=0.4$; then at 1990 we introduced $30 \%$ more Outbreds, reducing equivalently the number of Neutrals and have set $\lambda=0.4, \beta=0.4$; then at 1992 we have set $\lambda=0.2, \beta=0.3$; then at 2000 we have set $\lambda=0.3, \beta=0.2$.

In a (loose) analogy to the "Departmental/Institutional" scale, one could think of generalizing the above concepts at a country level. For example, real-country-level phase-transitions in interdisciplinary research have been recently reported in [53]. In particular, it has been reported a sudden and astonishing jump/phase transitions of both the number of publications and breakthroughs in mathematical and molecular biological sciences resulted by the increased flow of Russian scientists in the USA, after the dissolution of the Soviet Union starting in 1989. Moreover, other similar phase transitions, though to a lower extent to the above reported "Russian jump" (RJ) were recorded in the period 1992-1996 with the $\$ 3.8$ billion funding of the Human Genome (HG) fostering interdisciplinary research and, later on, in association with the Internet diffusion (ID) in 2000 [53]. Our model was used to approximate qualitatively these observations by a simple exploratory simulation exercise (see Fig. 8).

It is important here to note, that the phase-transitions can be in principle approximated in parts using a simpler model, for example a first order linear differential equation. Our intention here is not to show that with some changes in the model parameters, we can approximate the reported dynamics. We aim more at showing the "physical connection/ interpretation" of certain parameters and state variables of our model and the observed dynamics. In this spirit, the RJ
effect on interdisciplinary research can be simulated by a sudden change of disciplinary diversity due to the entrance in the academic system of many new scientists with very different backgrounds. At the same time, the increase of resources due to HG project and the Internet-diffusion jump can be reflected by changes to $\lambda$ and $\beta$ so that a faster exchange/recruitment between Neutral and Outbreds is facilitated. We began our simulations by setting as initial values $(N, I, O, B)=(0.45,0.4,0.15,0.1)$; the choice of a $15 \%$ of Outbreds to USA was chosen based on coarse estimates on the percent of full time foreign-born faculty members at that period (see [54]; the values of the parameters were set similar to the ones used on creating the bifurcation diagram shown in Fig. 7.

## 5. Discussion

As well known in biology and ecology, diversity is a key factor for selection processes, system evolution [55] and ecosystem productivity [56]. The negative effects of the excess of inbreeding are well known, producing worsening of genetic diversity and consequently lower competitive performance in the long term [57]. In analogy to this, our model addresses a reciprocal negative feedback between the potential of knowledge breakthroughs and the growth of academic inbreeding due to the related effects on disciplinary diversity.

An in-depth analysis of the universities in the USA [1] demonstrated that the potential for scientific breakthroughs is significantly higher in relatively small and flexible research structures, whereas institutions characterized by high level of organization isomorphism, i.e. reduced disciplinary diversity in their research structures, show clear decline of their scientific innovation performances, due to the intrinsic tendency to work in established problems areas.

In Europe, several studies have revealed high levels of academic inbreeding, where in some countries it reaches as much as $95 \%$ [26,32]. On the other hand, the importance of intellectual and scientific diversity has been recognized in American universities, which in general do not hire their own Ph.D. students. In USA, around 93\% of candidates to academic positions were reported as externals [35,58].

In fact, it has been reported that inbreeding inhibits the entrance of new and fresh ideas as research partners within the same disciplines with strong ties over long periods may "naturally" be entrapped to a clique [4,59]. On the other hand, interdisciplinarity (as measured in several studies by quantifying disciplinary diversity (see e.g. [30,41-46]) has been associated with research breakthroughs [1]. However, we have to keep in mind that its quantitative and objective assessment still remains an open subject of research [2]. Therefore, purposely, we did not associate interdisciplinarity with the size of scientific production, but with disciplinary diversity as reflected by the density of the Outbreds. Thus, we do not claim that Inbreds and Neutrals necessary publish less or worse quality papers than Outbreds. Here, instead, we focused on a certain attitude of Inbreds/Lobbyists as an opposite/reactive force to the disciplinary diversity, and therefore interdisciplinarity, the necessary (yet not sufficient) condition for achieving breakthroughs in complex problems with important social impact.

Building up on previous studies and conceptions, our modelling approach is in analogy to social contagion processes borrowing ideas from epidemiological models [60,61]. For example, [61] have addressed a mathematical model for the spread of rumours where the population is divided into three "social" states: ignorant, spreaders, and stiflers. Interestingly, our modelling approach share parallels to another more recent work by [62], where the authors use concepts related to the spread of infections to model the diffusion of scientific ideas between research communities of the USA, Japan, and the USSR in the period immediately after World War II. In their model (in analogy to ours) they use parameters such as recruitment rate, contact rate per capita and the probability of adoption per contact and transitions rates between adopters of the idea, sceptics (i.e. opponents of the idea) and susceptibles (that may or not affected by the new idea).

Our mathematical model incorporated two categories of parameters, reflecting both internal and external factors that advance/inhibit disciplinary diversity/interdisciplinarity [4,63,64]. The internal factors reflect the direct and indirect interactions of individuals in an academic environment, while the external factor mirrors the control policy against inbreeding, thus favouring the enhancement of disciplinary diversity.

At this point, we should underline that the purpose of our work is not to show that interdisciplinary research implies superiority over disciplinary research, nor that disciplinary research drives necessarily to the creation of inbreeding. Furthermore, we should also say that our intention was not to address a detailed/accurate model that can approximate in a quantitative manner the real-world academic behaviours and practices. An obvious difficulty in any mathematical model that would aspire to do so is the quantification for example of "contribution to breakthroughs", "breakthroughs" and the various rates of "transitions". Hence, a limitation of our model is that the values of the model parameters (still, to the best of our knowledge) lack an experimental quantification. However, we think that such parameters could be quantified through the measurement and analysis of appropriate databases with the aid of scientometric and sociometric explorations in order to measure and "reveal the hidden structures that give a group its form: the alliances, the subgroups, the hidden beliefs, the forbidden agendas, the ideological agreements etc" [65-67]. For example one could think of measuring the social contact rates (and thus social relations and consequently influence) on the basis of communications through social platforms [68].

Here, we rather propose and analyse a relatively simple mathematical model aiming at capturing in a coarse qualitative manner some (of course not all) academic attitudes and practices that have been reported that could serve as a base of further discussion, modelling efforts and quantification of the relevant parameters in the future. By performing a numerical bifurcation analysis we reveal a rich nonlinear behaviour including multistability, stable and unstable limit cycles, limit points of limit cycles, homoclinic bifurcations and codimension-two bifurcations (Bogdanov-Takens and Bautin).

To this end, we conclude by saying that in our opinion, interdisciplinarity should not be regarded as an end in itself, but as a carrier of qualitative jumps/phase transitions of knowledge and technological advancements. Advance of knowledge obviously occur in well established academic disciplines, however as it is highlighted in many studies, in order to avoid decline in such positive performances, the research institutions need to be open to external development and recruitment $[1,69]$.

## References

[1] J.R. Hollingsworth, The Dynamics of American Science: An Institutional and Organizational Perspective on Major Discoveries, vol. 1, 2006.
[2] I. Rafols, M. Meyer, Diversity and network coherence as indicators of interdisciplinarity: case studies in bionanoscience, Scientometrics 82 (2) (2009) 263-287, http://dx.doi.org/10.1007/s11192-009-0041-y.
[3] S.W. Aboelela, E. Larson, S. Bakken, O. Carrasquillo, A. Formicola, S.A. Glied, J. Haas, K.M. Gebbie, Defining interdisciplinary research: Conclusions from a critical review of the literature, Health Serv. Res. 42 (1p1) (2007) 329-346, http://dx.doi.org/10.1111/j.1475-6773.2006.00621.x.
[4] W. Oh, J.N. Choi, K. Kim, Coauthorship dynamics and knowledge capital: The patterns of cross-disciplinary collaboration in information systems research, J. Manage. Inf. Syst. 22 (3) (2005) 266-292, http://dx.doi.org/10.2753/mis0742-1222220309.
[5] S. Misra, D. Stokols, L. Cheng, The transdisciplinary orientation scale: Factor structure and relation to the integrative quality and scope of scientific publications, J. Transl. Med. Epidemiol. 3 (2) (2015) 1042.
[6] J. Silvius, Strength in diversity: a cross-disciplinary approach to graduate training in chemical biology, Nat. Chem. Biol. 2 (9) (2006) 445-448, http://dx.doi.org/10.1038/nchembio0906-445.
[7] NAS, Facilitating Interdisciplinary Research, National Academies Press, 2004, http://dx.doi.org/10.17226/11153.
[8] TheBritishAcademy, Crossing Paths: Interdisciplinary Institutions, Careers, Education and Applications, Technical Report, The Britich Academy, 2016, https://www.thebritishacademy.ac.uk/sites/default/files/Crossing\ Paths\ -\ Full\ Report_2.pdf.
[9] P. Weingart, B. Padberg (Eds.), University Experiments in Interdisciplinarity, Transcript Verlag, 2014, http://dx.doi.org/10.14361/transcript. 9783839426166.
[10] E.D. Schutter, Why are computational neuroscience and systems biology so separate? PLoS Comput. Biol. 4 (5) (2008) e1000078, http: //dx.doi.org/10.1371/journal.pcbi. 1000078.
[11] Battelle, Economic Impact of the Human Genome Project, Battelle Memorial Institute, 2013, https://www.battelle.org/docs/default-source/misc/ battelle-2011-misc-economic-impact-human-genome-project.pdf.
[12] M. Wadman, Economic return from human genome project grows, Nature (2013) http://dx.doi.org/10.1038/nature.2013.13187.
[13] M. Nissani, Ten cheers for interdisciplinarity: The case for interdisciplinary knowledge and research, Soc. Sci. J. 34 (2) (1997) 201-216, http://dx.doi.org/10.1016/s0362-3319(97)90051-3.
[14] T. Pellmar, L. Eisenberg, Bridging Disciplines in the Brain, Behavioral, and Clinical Sciences, National Academies Press, 2000, http://dx.doi.org/ 10.17226/9942.
[15] H. Ledford, How to solve the worlds biggest problems, Nature 525 (7569) (2015) 308-311, http://dx.doi.org/10.1038/525308a.
[16] P. Dilley, Creating interdisciplinarity: Interdisciplinary research and teaching among college and university faculty (review), J. Gen. Educ. 51 (2) (2002) 149-152, http://dx.doi.org/10.1353/jge.2002.0009.
[17] P.E. Jacob, B. Berelson, Graduate education in the United States, AAUP Bull. 47 (1) (1961) 70, http://dx.doi.org/10.2307/40222632.
[18] H. Horta, M. Sato, A. Yonezawa, Academic inbreeding: exploring its characteristics and rationale in Japanese universities using a qualitative perspective, Asia Pac. Educ. Rev. 12 (1) (2010) 35-44, http://dx.doi.org/10.1007/s12564-010-9126-9.
[19] P.G. Altbach, M. Yudkevich, L.E. Rumbley, Academic inbreeding: Local challenge, global problem, in: Academic Inbreeding and Mobility in Higher Education, Palgrave Macmillan, 2015, pp. 1-16, http://dx.doi.org/10.1057/9781137461254_1.
[20] O. Gorelova, M. Yudkevich, Academic inbreeding: State of the literature, in: Academic Inbreeding and Mobility in Higher Education, Palgrave Macmillan UK, 2015, pp. 17-44, http://dx.doi.org/10.1057/9781137461254_2.
[21] H. Horta, Deepening our understanding of academic inbreeding effects on research information exchange and scientific output: new insights for academic based research, Higher Educ. 65 (4) (2012) 487-510, http://dx.doi.org/10.1007/s10734-012-9559-7.
[22] D. Pelz, F. Andrews, Scientists in organizations, J. Mol. Struct. 5 (4) (1970) 323-324, http://dx.doi.org/10.1016/0022-2860(70)80035-7.
[23] J.C. Hearn, D.W. Leslie, E.K. Fretwell, Wise moves in hard times: Creating and managing resilient colleges and universities, J. Higher Educ. 69 (4) (1998) 467, http://dx.doi.org/10.2307/2649275.
[24] S. Hasanefendic, J.M. Birkholz, H. Horta, P. van der Sijde, Individuals in action: bringing about innovation in higher education, Eur. J. Higher Educ. 7 (2) (2017) 101-119, http://dx.doi.org/10.1080/21568235.2017.1296367.
[25] I. Bleiklie, R. Høstaker, Modernizing research training-education and science policy between profession, discipline and academic institution, Higher Educ. Policy 17 (2) (2004) 221-236, http://dx.doi.org/10.1057/palgrave.hep. 8300052.
[26] A. Navarro, A. Rivero, High rate of inbreeding in Spanish universities, Nature 410 (6824) (2001) http://dx.doi.org/10.1038/35065259.
[27] N. Morichika, S. Shibayama, Impact of inbreeding on scientific productivity: A case study of a Japanese university department, Res. Eval. 24 (2) (2015) 146-157, http://dx.doi.org/10.1093/reseval/rvv002.
[28] E. Cohen, S.J. Lloyd, Disciplinary evolution and the rise of the transdiscipline, Inform. Sci.: Int. J. Emerg. Transdiscipl. 17 (2014) 189-215, http://dx.doi.org/10.28945/2045.
[29] A. Yamanoi, The academic marketplace in Japan: Inbreeding, grades and organization at research universities, Higher Educ. Forum 2 (2005) 93-114.
[30] A. Wiggins, S. Sawyer, Intellectual diversity and the faculty composition of iSchools, J. Am. Soc. Inf. Sci. Technol. 63 (1) (2011) 8-21, http://dx.doi.org/10.1002/asi.21619.
[31] H. Horta, F.M. Veloso, R. Grediaga, Navel gazing: Academic inbreeding and scientific productivity, Manage. Sci. 56 (3) (2010) 414-429, http://dx.doi.org/10.1287/mnsc.1090.1109.
[32] M. Soler, How inbreeding affects productivity in Europe, Nature 411 (6834) (2001) 132, http://dx.doi.org/10.1038/35075637.
[33] O. Inanc, O. Tuncer, The effect of academic inbreeding on scientific effectiveness, Scientometrics 88 (3) (2011) 885-898, http://dx.doi.org/10. 1007/s11192-011-0415-9.
[34] R. Smyth, V. Mishra, Academic inbreeding and research productivity and impact in Australian law schools, Scientometrics 98 (1) (2013) 583-618, http://dx.doi.org/10.1007/s11192-013-1052-2.
[35] S. Pan, A Study of Faculty Inbreeding at Eleven Land-Grant Universities (Ph.D. thesis), Iowa State University, 1993, http://dx.doi.org/10.31274/rtd-180813-9613.
[36] O. Godechot, A. Louvet, Academic inbreeding: An evaluation, 2008, http://www.booksandideas.net/Academic-Inbreeding-An-Evaluation.html.
[37] F. Rocca, In Spain inbreeding threatens academe, Voprosy Obrazovaniya/ Educ. Stud. Moscow (1) (2009) 162-169, http://dx.doi.org/10.17323/ 1814-9545-2009-1-162-169.
[38] L.L. Hargens, G.M. Farr, An examination of recent hypotheses about institutional inbreeding, Am. J. Sociol. 78 (6) (1973) 1381-1402, http://dx.doi.org/10.1086/225470.
[39] N. Claidière, D. Sperber, The role of attraction in cultural evolution, J. Cognit. Culture 7 (1-2) (2007) 89-111, http://dx.doi.org/10.1163/ $156853707 \times 171829$.
[40] A. Buskell, What are cultural attractors? Biol. Philos. 32 (3) (2017) 377-394, http://dx.doi.org/10.1007/s10539-017-9570-6.
[41] X. Wang, Z. Wang, Y. Huang, Y. Chen, Y. Zhang, H. Ren, R. Li, J. Pang, Measuring interdisciplinarity of a research system: detecting distinction between publication categories and citation categories, Scientometrics 111 (3) (2017) 2023-2039, http://dx.doi.org/10.1007/s11192-017-2348-4.
[42] A. Stirling, A general framework for analysing diversity in science, technology and society, J. R. Soc. Interface 4 (15) (2007) 707-719, http://dx.doi.org/10.1098/rsif.2007.0213.
[43] A.L. Porter, D.E. Chubin, An indicator of cross-disciplinary research, Scientometrics 8 (3-4) (1985) 161-176, http://dx.doi.org/10.1007/bf02016934.
[44] L. Cassi, W. Mescheba, É. de Turckheim, How to evaluate the degree of interdisciplinarity of an institution? Scientometrics 101 (3) (2014) 1871-1895, http://dx.doi.org/10.1007/s11192-014-1280-0.
[45] L. Leydesdorff, Betweenness centrality as an indicator of the interdisciplinarity of scientific journals, J. Am. Soc. Inf. Sci. Technol. 58 (9) (2007) 1303-1319, http://dx.doi.org/10.1002/asi.20614.
[46] A.-M. Mugabushaka, A. Kyriakou, T. Papazoglou, Bibliometric indicators of interdisciplinarity: the potential of the Leinster-Cobbold diversity indices to study disciplinary diversity, Scientometrics 107 (2) (2016) 593-607, http://dx.doi.org/10.1007/s11192-016-1865-x.
[47] F. Brauer, C. Castillo-Chavez, Mathematical Models in Population Biology and Epidemiology, Springer New York, 2012, http://dx.doi.org/10.1007/ 978-1-4614-1686-9.
[48] M.J. Keeling, K.T. Eames, Networks and epidemic models, J. R. Soc. Interface 2 (4) (2005) 295-307, http://dx.doi.org/10.1098/rsif.2005.0051.
[49] H. Shirado, F. Fu, J.H. Fowler, N.A. Christakis, Quality versus quantity of social ties in experimental cooperative networks, Nature Commun. 4 (1) (2013) http://dx.doi.org/10.1038/ncomms3814.
[50] A. Ahmed, E.P. Xing, Recovering time-varying networks of dependencies in social and biological studies, Proc. Natl. Acad. Sci. 106 (29) (2009) 11878-11883, http://dx.doi.org/10.1073/pnas.0901910106.
[51] A. Dhooge, W. Govaerts, Y.A. Kuznetsov, MATCONT, ACM SIGSAM Bull. 38 (1) (2004) 21, http://dx.doi.org/10.1145/980175.980184.
[52] O.E. Klapp, Opening and closing in open systems, Behav. Sci. 20 (4) (1975) 251-257, http://dx.doi.org/10.1002/bs.3830200406.
[53] J. Phillips, Phase transitions in the web of science, Physica A 428 (2015) 173-177, http://dx.doi.org/10.1016/j.physa.2015.02.013.
[54] D. Kim, S. Twombly, L. Wolf-Wendel, International faculty in American universities: Experiences of academic life, productivity, and career mobility, New Dir. Inst. Res. 2012 (155) (2012) 27-46, http://dx.doi.org/10.1002/ir.20020.
[55] D. Charlesworth, B. Charlesworth, Inbreeding depression and its evolutionary consequences, Annu. Rev. Ecol. Syst. 18 (1) (1987) 237-268, http://dx.doi.org/10.1146/annurev.es.18.110187.001321.
[56] S. Mazzoleni, G. Bonanomi, F. Giannino, G. Incerti, S.C. Dekker, M. Rietkerk, Modelling the effects of litter decomposition on tree diversity patterns, Ecol. Model. 221 (23) (2010) 2784-2792, http://dx.doi.org/10.1016/j.ecolmodel.2010.08.007.
[57] P.W. Hedrick, S.T. Kalinowski, Inbreeding depression in conservation biology, Annu. Rev. Ecol. Syst. 31 (1) (2000) 139-162, http://dx.doi.org/ 10.1146/annurev.ecolsys.31.1.139.
[58] A. Blackwell, L. Wilson, A. Street, C. Boulton, J. Knell, Techincal Report, UCAM-CL-TR-760, University of Cambridge, 2009.
[59] D. Gambetta, G. Origgi, The LL game:the curious preference for low quality and its norms politics, Politics Philos. Econ. 12 (1) (2012) 3-23, http://dx.doi.org/10.1177/1470594x11433740.
[60] Y. Xiao, D. Chen, S. Wei, Q. Li, H. Wang, M. Xu, Rumor propagation dynamic model based on evolutionary game and anti-rumor, Nonlinear Dynam. 95 (1) (2018) 523-539, http://dx.doi.org/10.1007/s11071-018-4579-1.
[61] D.J. Daley, D. Kendall, Stochastic rumours, IMA J. Appl. Math. 1 (1) (1965) 42-55, http://dx.doi.org/10.1093/imamat/1.1.42.
[62] L.M.A. Bettencourt, A. Cintron-Arias, D.I. Kaiser, C. Castillo-Chavez, The power of a good idea: Quantitative modeling of the spread of ideas from epidemiological models, Physica A 364 (2006) 513-536, http://dx.doi.org/10.2172/990668.
[63] W.R. Kerr, W.F. Lincoln, P. Mishra, The dynamics of firm lobbying, SSRN Electron. J. (2011) http://dx.doi.org/10.2139/ssrn.1978910.
[64] C. Cao, N. Li, X. Li, L. Liu, Reforming Chinas S \& T system, Science 341 (6145) (2013) 460-462, http://dx.doi.org/10.1126/science.1234206.
[65] J. Harding, Sociometry, experimental method and the science of society: An approach to a new political orientation, Psychol. Bull. 49 (3) (1952) 283-284, http://dx.doi.org/10.1037/h0049709.
[66] B. Arons, Forbidden agendas: Strategic action in groups, Int. J. Group Psychotherapy 42 (2) (1992) 312-313, http://dx.doi.org/10.1080/00207284. 1992.11490698.
[67] M. Rogers, H.H. Jennings, Sociometry in group relations: A work guide for teachers, Sociometry 12 (4) (1949) 354, http://dx.doi.org/10.2307/ 2785603.
[68] A.Y. Mikami, D.E. Szwedo, A. Khalis, M. Jia, J.J. Na, Online social interactions predict academic and emotional adjustment in the transition to university, J. Res. Adolescence 29 (1) (2018) 210-224, http://dx.doi.org/10.1111/jora. 12377.
[69] J.A. Jacobs, S. Frickel, Interdisciplinarity: A critical assessment, Annu. Rev. Sociol. 35 (1) (2009) 43-65, http://dx.doi.org/10.1146/annurev-soc-070308-115954.


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