1 Introduction

After World War II, the demographic transition exhibited features of convergence from developing countries with countries that pioneered this important phenomenon since the 19th century. The decline of fertility is characterized across the world by heterogeneity and non-linearity of the time paths. Indeed, the timing of the onset of the transition varies substantially across countries, and in several cases fertility declines along non-monotonic time paths with trend inversions. This study analyzed convergence of fertility in 190 countries for the period of 1950–2017 and extends the analysis to United Nations (UN) projections until the year 2100.

We make three contributions to the literature.[1] First, we studied convergence and clustering of fertility transitions applying the econometric methods of Phillips and Sul (2007, 2009), which allow non-linearity and heterogeneity of the time series. Second, we used this appraoch to cluster countries according to fertility in 1950 with a reverse convergence regression. Third, we investigated convergence in the UN projections of fertility until 2100. The next section describes data and methodology. Section 3 presents the results. Section 4 concludes our findings.

2 Modeling Fertility Convergence

Data on the Total Fertility Rate (TFR) refer to the annual time series of 190 countries from 1950 to 2017.[2] The source of the data is the World Bank's (2020) World Development Indicators for the years 1960–2017. The years 1950–1959 are covered by the estimates from the United

[1] Dorius (2008) investigates β-convergence and σ-convergence of the TFR of a panel of 195 countries from 1955 to 2005 and finds some evidence for fertility convergence in the world after the 1980s.
[2] Table S1 in Supplementary Material provides a list of the countries.

Nations World Population Prospects, 2019 Revision. The last source provides five–year projections of TFR between 2020–2025 and 2095–2100. These probabilistic projections are produced under different assumptions that underlie medium, low, and high fertility variants of the estimates. For the last two variants, 80% and 95% prediction intervals are made available.[3]

We studied convergence using the *logt* test proposed by Phillips and Sul (2007) who assume the variable x_{it} follows a factor model for individual i at time t:

$$x_{it} = \theta_{it}\pi_t, \tag{1}$$

where unobserved common factors π_t and time-varying idiosyncratic components θ_{it} can be distinguished. In this model, the distance between the common factor and the systematic part of x_{it} is specific to the individual i and changes over time.

Relative convergence among the series x_{it} defines as the long-run equilibrium of their ratios:

$$\lim_{t\to\infty} \frac{x_{it}}{x_{jt}} = \lim_{t\to\infty} \frac{\theta_{it}}{\theta_{jt}} = 1 \text{ for all } i \text{ and } j. \tag{2}$$

Hence, relative convergence is equivalent to: $\lim_{t\to\infty} \theta_{it} = \theta$. Phillips and Sul (2007) assume the following model for the coefficients θ_{it}:

$$\theta_{it} = \theta_i + \frac{\rho_i}{L(t)t^{\alpha}}\varepsilon_{it},$$

where $\rho_i > 0$, $L(t)$ is a slowly varying function as $\log(t)$, ε_{it} is $iid(0,1)$, weakly dependent and stationary over time. The null hypothesis of convergence is defined by the conditions: $\theta_{it} = \theta$ and $\alpha \geq 0$. The alternative is defined by: $\theta_{it} \neq \theta$ and $\alpha < 0$. The *logt* test of convergence is defined in terms of the relative transition coefficients, $h_{it} = \frac{x_{it}}{\hat{x}_t}$, where $\hat{x}_t$ denotes the cross-sectional average.

[3] Details on the UN fertility projections can be found in United Nations (2019).

Convergence now implies $h_{it} \to 1$, and the cross-sectional variance V_t,

$$V_t = \frac{1}{N}\sum_{i=1}^{N} (h_{it} - 1)^2, \tag{3}$$

converges to zero. The test is based on V_t. Phillips and Sul (2007) show the test can be performed with the estimation of the regression equation

$$\log\left(\frac{V_1}{V_t}\right) - 2\log[\log(t)] = a + b\log(t) + \mathrm{u}_{t,} \tag{4}$$

where $b = 2\hat{\alpha}$ and $\hat{\alpha}$ is an estimate of α in the null hypothesis. The *logt* test of convergence is simply the test of the null $b \geq 0$. In this case, $\log(V_1/V_t)$ diverges to ∞, meaning that V_t tends to zero as t goes to infinity. In the case of divergence, $b < 0$, and V_t converge to a positive value. We apply the *logt* test using the t-statistic calculated with a heteroskedasticity- and autocorrelation-consistent estimator of the standard error. This statistic is asymptotically distributed as a standard normal. Phillips and Sul (2009) show how this approach can test for absolute convergence when the component π_t follows a random walk with drift or a trend stationary process. In this case, $\alpha \geq 1$ and the *logt* test can be easily applied.

Phillips and Sul (2007) also propose a procedure to identify clusters of units that converge in the long run. The algorithm starts by ordering all units in the panel according to the value of the last available observation, x_{iT}. Then, a core subgroup of units can be detected and the *logt* test can assess whether another unit belongs to the same group. This procedure can be iterated until every unit in the panel is classified. In the following section, we apply this clustering method to the fertility rate in a panel of 190 countries.

3. Results

Table 1 presents the results of the *logt* test for the whole sample of 190 countries between 1950–2017, and in periods that include the years of UN projections. The null of convergence in fertility rates around the world is strongly rejected at a 5% level. The same result emerges from the *logt* regression between a more recent period, 1980–2017, which focuses further on countries where the demographic transition has already begun. Given the remarkable generality of the *logt* test, these results strongly suggest the demographic transition will be an open-ended question in the future of many developing countries.

The projections of future fertility produced by the UN shed light on this question. Table 1 shows fertility across the world will converge before the year 2100, according to the UN. More specifically, statistical evidence of global convergence exists after the year 2050. This phenomenon would occur under the different projection variants the UN researchers assumed.

It is possible the alternative to global convergence is convergence within groups. We have pursued the investigation of such an alternative configuration of fertility trends by applying Phillips and Sul's (2007) clustering algorithm to the TFR data for the period of 1950–2017[4]. We found five groups reduced to four after the merge of the third and fourth groups, as suggested by the *logt* test ($\hat{b} = -0.095$ and $t_b = -1.052$).[5]

Table 2 presents the *logt* test statistics for the four groups, which we denote Club Highest, Club High, Club Intermediate, and Club Low, hinting at average fertility in each club. Club Highest includes Congo, Niger, and Somalia, three countries where fertility remains very high. The same can be said of Chad and Mali, which compose Club High. The *logt* test for the merge of the two clubs

[4] We ordered all countries in the panel according to the TFR of the last observation from the highest to the lowest.
[5] See Supplementary Material Table S1 for the list of countries in each convergence club.

rejects the null with $\hat{b} = -0.349$, and $t_b = -4.154$. Club Intermediate is composed of 33 countries that, between 1950–2017, began a demographic transition that is far from concluding, given the average TFR is greater than four births per woman (4.638) in 2017. The last convergence club derives from running the convergence test on the time series of the remaining countries. In this case, the *logt* test does not reject the null of convergence ($t_b = -0.306$), although its value indicates slow convergence. Club Low consists of 152 countries with low TFR at the end of the 1950–2017 period (2.183); Club Low includes all countries where the onset of the fertility transition occurred before 1950 and several countries where the same process started after 1950.

This distinction better characterizes the demographic transition in developing countries and the reproductive behavior (past and perspective) of low fertility countries. According to Reher (2004), a group of 23 countries can be classified as *Forerunners* because the decline of fertility began before 1935. This group includes countries from Europe (16), North America (5), and South America (2). However, Reher's classification refers to 145 countries and does not consider other countries in our dataset. Referring to our panel data, the main question is how to distinguish countries according to their TFR in 1950. In other words, can we cluster countries according to the starting value of TFR? To answer this question, we apply a clustering algorithm to the time series of TFR reverting the time arrow: from the last year to the first year of the time period. Accordingly, referring to x_{it}, we define a new variable: $k_{it} = x_{i,T+1-t}, \quad t = 1 \dots T$, and apply the methods of Section 2 to k_{it} to investigate club convergence of TFR in 1950.

We catagorize a reliable partition of 190 countries from 1976 to 1950 into two groups according to the initial fertility rate when the time period is close to 1950.[6] The first group includes 146 countries with high average TFR in 1950: 6.148; *Forerunners* and the other 21 countries

[6] See Table S2 in the Supplementary Material.

compose the second group characterized by low average TFR in 1950: 2.862. The outcome of the Phillips and Sul (2007) clustering methodology is quite satisfying because the average 1950 TFR in low initial fertility countries is so close to that of the *Forerunners*: 2.802. Combining the two club classifications shows the average woman had many children after WWII in a large part of the world, but in the following decades fecundity declined in many countries. This is the case of clubs Highest, High, and Intermediate where TFR recently began to follow a declining path. Today, those countries do not show convergence with the rest of the world, but they will before 2100, according to the projections of the UN.

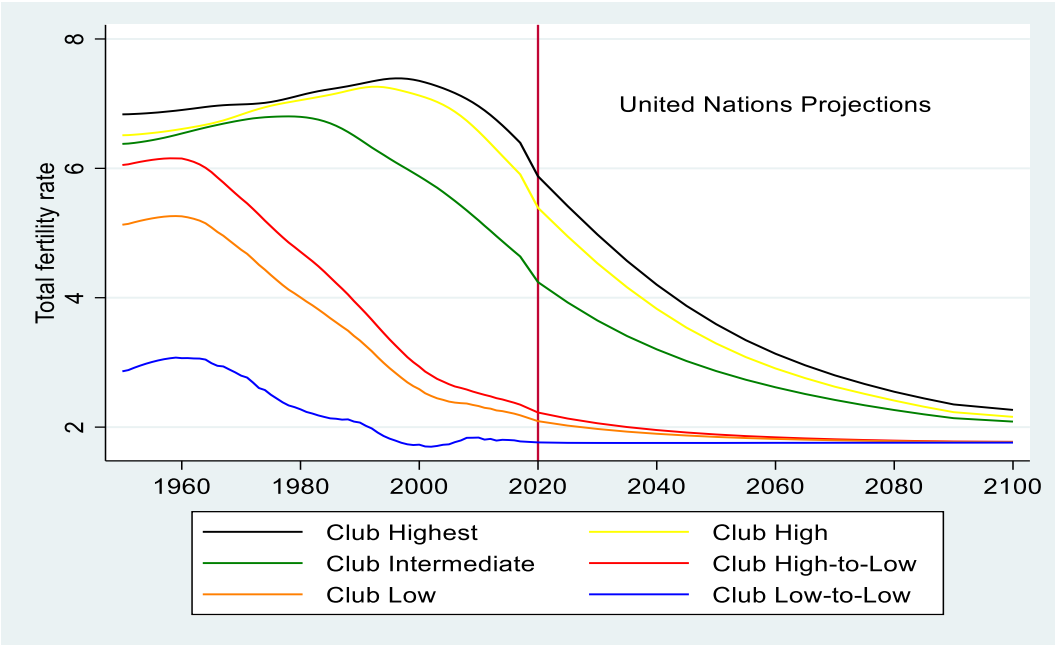

Fig. 1. Average fertility in convergence clubs (1950–2100).

Figure 1 depicts the average TFR for each club over the 1950–2100 period, and shows some interesting patterns. The most impressive phenomenon is the strong process of fertility reduction experienced by many countries in Club Low after WWII. In Club Low we distinguish a group of 108 countries that we also classified as high initial fertility, and in 2017 converge into the club of low initial TFR. We denote this group as Club High-to-Low. Some sub-Saharan countries (e.g., Eritrea,

Ghana, Kenya, South Africa, Zimbabwe) enter Club High-to-Low, which also includes many other countries from Latin America, Asia, Africa, and Oceania. Figure 1 shows the average onset of the fertility decline in this group occurs during the sixties, before other developing countries, and then proceeds along a steep path. The remainder of Club Low, which we denote as Club Low-to-Low, includes those 44 countries where fertility is low in 1950 and in 2017 and after the 1980's it further declines below the significant value of two.

Figure 1 also depicts the UN projections of average fertility for each club after 2017. The graph highlights the different paths to global convergence that would follow across the world in the next 80 years if the predictions are correct. The strongest fertility decline occurs in clubs Highest and High, which approach Club Intermediate.[7] The same graph suggests absolute convergence of predicted TFR among countries in Club Low. The investigation of absolute convergence of TFR would provide interesting clues to the descriptive analysis of global fertility convergence. We already know (from Tables 1 and 2) the data do not support the hypothesis of absolute convergence in the four clubs we identified for the period of 1950–2017. However, we find different results applying the *logt* test on data of Club Low-to Low, Club High-to-Low, and the *Forerunners*.[8] When the sample is restricted to the period of 1950–2017 we find evidence of relative but not absolute convergence in the three clusters. However, when including UN projections in the sample, absolute convergence of fertility cannot be rejected in the case of Club Low-to Low and the *Forerunners*.

4 Conclusions

[7] The *logt* test does not support the null of convergence among the three clubs in 2100.
[8] See Table S3 in the Supplementary Material.

This paper analyzes the fertility transition in many developing countries that has occurred since World War II from a new empirical perspective. The results highlight the persistence of different models of human reproduction across the world. Although, fertility rates are low in many countries today, the process of convergence has not come to an end. The future could see the global fertility convergence (the UN projections) but the conditions for this long-term outcome are not clear today.

Table 1

Logt test of convergence in fertility. All countries in different periods.

	1950-2017	1980-2017	1950-2030	1950-2040	1950-2050	1950-2060	1950-2070	1950-2080	1950-2100
TFR	-0.978*	-0.533*							
	(-32.705)	(-9.716)							
TFR_MED			-0.892*	-.729*	-0.507*	-0.286	0.030	0.394	1.157
			(-7.156)	(-3.583)	(-1.793)	(-0.814)	(0.073)	(0.863)	(2.205)
TFR_U95			-0.869*	-0.699*	-0.497*	-0.324	-0.089	0.161	0.612
			(-5.868)	(-3.602)	(-2.260)	(-1.482)	(-0.426)	(0.842)	(4.272)
TFR_U80			-0.876*	-0.707*	-0.501*	-0.317	-0.063	0.212	0.728
			(-6.070)	(-3.537)	(-2.063)	(-1.223)	(-0.238)	(0.843)	(3.354)
TFR_L80			-0.912*	-0.767*	-0.555*	-0.324	0.020	0.436	1.351
			(-11.190)	(-4.183)	(-2.056)	(-0.894)	(0.042)	(0.757)	(1.983)
TFR_L95			-0.922*	-0.784*	-0.588*	-0.389	-0.100	0.243	0.945
			(-17.627)	(-4.528)	(-2.384)	(-1.244)	(-0.258)	(0.545)	(2.227)

Notes: TFR_MED is the medium-fertility variant projection; TFR_U95, and TFR_U80 refer to the 95% and 80% prediction intervals of the upper-fertility variant projection, while TFR_L95, and TFR_L80 refer the the same intervals of the low-fertility variant. *t*-statistics are in parentheses. * denotes significance at the 5% level.

Table 2

Club convergence

	logt	*t*-stat.	N° Countries	Mean TFR 2017
Club Highest	0.104	1.137	3	6.395
Club High	0.77	0.527	2	5.907
Club Intermediate	-0.095	-1.052	33	4.638
Club Low	-0.028	-0.306	152	2.183

References

Dorius, Shawn F., "Global Demographic Convergence? A Reconsideration of Changing Intercountry Inequality in Fertility," *Population and Development Review* 34:3 (2008), 519-537.

Phillips, Peter C. B., and Donggyu Sul, "Transition Modeling and Econometric Convergence Tests," *Econometrica* 75:6 (2007), 1771-1855.

Phillips, Peter C. B., and Donggyu Sul, "Economic Transition and Growth," *Journal of Applied Econometrics* 24:7 (2009), 1153-1185.

Reher, David S., "The demographic transition revisited as a global process," *Population, Space and Place* 10.1 (2004), 19-41.

United Nations, Department of Economic and Social Affairs, Population Division, "*World Population Prospects 2019: Methodology of the United Nations population estimates and projections.*" New York, United Nations, 2019.