Inconsistencies and Lurking Pitfalls in the Magnitude–Frequency Distribution of High-Resolution Earthquake Catalogs

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Key Points:

- High-resolution earthquake catalogs do not preserve the exponential magnitude distribution that characterizes ordinary catalogs. (127 characters)
- Common methods to estimate the completeness level do not guarantee an exponential magnitude distribution and lead to biased b-values. (133 characters)
- High-resolution catalogs should be used with caution for estimating any property of the magnitude distribution. (110 characters)

Keywords:
statistical seismology, magnitude–frequency distribution, earthquake catalog, earthquake statistics, hypothesis testing

Abstract:

Earthquake catalogs describe the distribution of earthquakes in space, time, and magnitude, which is essential information for earthquake forecasting and seismic hazard/risk assessment. Available high-resolution (HR) catalogs raise the expectation that their abundance of small earthquakes will help to better characterize the fundamental scaling laws of statistical seismology. Here we investigate whether the ubiquitous exponential-like scaling relation for magnitudes (Gutenberg–Richter, GR, or its tapered version, TGR), can be straightforwardly extrapolated to the magnitude–frequency distribution (MFD) of HR catalogs. For several HR catalogs such as of the 2019 Ridgecrest sequence, the 2009 L'Aquila sequence, the 1992 Landers sequence, and entire southern California, we determine if the MFD agrees with an exponential-like distribution using a statistical goodness-of-fit test. We find that HR catalogs usually do not preserve the exponential-like MFD toward low magnitudes and depart from it. Surprisingly, HR catalogs that are based on advanced detection methods depart from an exponential-like MFD at a similar magnitude level as network-based HR catalogs. These departures are mostly due to an improper mixing of different magnitude types, spatio-temporal inhomogeneous completeness, or biased data recording/processing. Remarkably, common-practice methods to find the completeness magnitude do not recognize these departures and lead to severe bias in the b-value estimation. We conclude that extrapolating the exponential-like GR relation to lower magnitudes cannot be taken for granted, and that HR catalogs pose subtle new challenges and lurking pitfalls that may hamper their proper use. The simplest solution to preserve the exponential-like distribution toward low magnitudes may be to estimate a moment magnitude for each earthquake.

Plain Language Summary:

Earthquake catalogs contain information about the location and size of earthquakes. With advanced methods, this information becomes increasingly available at higher resolution: Detection methods populate catalogs with smaller earthquakes, whereas relocation methods resolve earthquake origins more precisely. The higher resolution promises new benefits for earthquake forecasting and seismic hazard/risk assessment. At the example of the 2019 Ridgecrest sequence, the 2009 L'Aquila sequence, the 1992 Landers sequence, and entire southern California, we investigate whether high-resolution catalogs offer a better foundation for using earthquake size information. We find that the size information of the small complementary earthquakes in these catalogs does usually not comply with the most fundamental assumption to estimate the occurrence probability of large earthquakes, the Gutenberg–Richter relation. In fact, when using this relation rigorously, high-resolution catalogs do not seem to offer a crucial benefit over ordinary catalogs. This impediment is mostly due to mixed earthquake size information, spatio-temporally varying detection capabilities, or distorted data processing. Common methods to apply the Gutenberg–Richter

relation do not detect these discrepancies and produce misleading results. Our findings are relevant for both producers of high-resolution catalogs and users of earthquake size information contained in such catalogs.

1 Introduction

Enriching seismic catalogs with smaller earthquakes (i. e., below $M2.0$) offers many benefits due to the higher spatio-temporal resolution of seismicity [*Ebel* 2008; *Brodsky* 2019]. For example, such microearthquakes help to identify the location and extent of active faults [e. g. *Fischer and Horálek* 2003; *Waldhauser et al.* 2004; *Piccinini et al.* 2009; *Chiaraluce et al.* 2007; *Improta et al.* 2019], allow faster and/or new inferences about seismotectonic processes [e. g., *Hatzfeld et al.* 2000; *Bohnhoff et al.* 2006; *Bulut et al.* 2009; *Valoroso et al.* 2013; *Marzorati et al.* 2014; *Meng and Peng* 2016; *Shelly et al.* 2016; *Hainzl et al.* 2016; *Ross et al.* 2020], and provide potentially better conditions for seismicity and hazard analyses with their application to forecasting models [e. g. *Wiemer and Schorlemmer* 2007; *Werner et al.* 2011; *Mignan* 2014; *Tormann et al.* 2014; *Gulia and Wiemer* 2019].

With the increasing availability of high-resolution (HR) earthquake catalogs, these expectations might easily be taken for granted. Their refined locations suggest an improved resolution, whereas their abundance of smaller events suggest an improved completeness. Scientists may be tempted to blindly assume that the popular Gutenberg–Richter (GR) scaling relation, or its tapered version (TGR), observed in ordinary earthquake catalogs (i. e., older network-based catalogs that span a limited range of magnitudes) holds also in the low-magnitude range of HR catalogs.

Many seismicity studies use the magnitude–frequency distribution (MFD) to estimate seismicity rates and the b-value (the slope of the GR relation), from which the occurrence probability of larger events and eventually the seismic hazard and risk can be inferred. The earthquake magnitude is usually expected to follow an exponential-like distribution according to the unbounded, tapered, and truncated GR relation when the maximum (corner) magnitude is about $\geq M_c + 3$ [*Marzocchi et al.* 2020], where M_c is the lower magnitude cutoff, or magnitude of completeness. (In the following we refer to 'exponential-like' simply as 'exponential'.) An exponential distribution above M_c is a necessary and sufficient condition to calculate the b-value [*Marzocchi et al.* 2020]—otherwise, the physical meaning of the b-value becomes questionable. Various methods exist to determine M_c [e. g., *Wiemer and Wyss* 2000; *Wössner and Wiemer* 2005; *Amorèse* 2007; *Schorlemmer and Woessner* 2008; *Mignan and Wössner* 2012] or to model the full MFD including the incomplete part [*Kijko and Smit* 2017; *Martinsson and Jonsson* 2018; *Mignan* 2019], but the exponential property of the MFD is rarely verified through canonical statistical tests.

Several factors can significantly alter MFDs and produce artifacts that bias any inferred estimate. It is generally well known that artificial changes in the reporting of magnitudes (e. g., due to recalibrations or changing seismic networks) may affect the homogeneity of catalogs

[*Habermann* 1987; *Zúñiga and Wyss* 1995; *Tormann et al.* 2010; *Kamer and Hiemer* 2015] and that major earthquakes lead to marked under-reporting of small events shortly after [*Kagan* 2004; *Helmstetter et al.* 2006; *Hainzl* 2016; de *Arcangelis et al.* 2018]. Moreover, a single uniform magnitude scale is not always possible for several practical reasons [*Kanamori* 1983], so that HR catalogs often mix different kinds of magnitudes, which may have different non-exponential MFDs. For instance, the local magnitude, M_{L}, has an exponential scaling only in a limited magnitude range: it begins to saturate for large magnitudes (above $M6$) [*Kanamori* 1983] due to the Wood–Anderson instrument response acting as a high-pass filter [*Bormann and Saul* 2009], and it breaks in scale around $M2{-}4$ due to the anelastic attenuation in the medium acting as a low-pass filter [*Bethmann et al.* 2011; *Deichmann* 2017; *Munafò et al.* 2016].

Here we take a closer look at MFDs of currently available HR catalogs that span more than six orders of magnitude, examining different (1) spatio-temporal scales (individual sequences vs. entire southern California), (2) temporal states (2019 Ridgcrest vs. 1992 Landers sequence), and (3) tectonic environments (southern California vs. Italy). We analyze if their MFDs agree with an exponential (T)GR distribution (referring to this agreement as "consistency" hereinafter) and explore whether inconsistencies can be detected through common-practice methods to estimate the parameters of the (T)GR distribution (M_{c} and b-value). Understanding in more detail if, how, and why MFDs of available HR catalogs are inconsistent is fundamental to use them correctly in statistical seismology.

2 Catalogs and Statistical Methods

For southern California, we consider the following earthquake catalogs (see Data and Resources for accessed repositories):

- southern California Seismic Network (SCSN) catalog [*Hutton et al.* 2010; *SCEDC* 2013];
- U.S. Geological Survey's Advanced National Seismic System (USGS-ANSS) ComCat;
- *Hauksson et al.* [2012], containing relatively relocated hypocenters of SCSN events;
- QTM [*Ross*, *Trugman*, *et al.* 2019], based on template matching (TM) using SCSN events as template set; and
- three dedicated catalogs for the Ridgecrest sequence [*Ross*, *Idini*, *et al.* 2019; *Shelly* 2020b; *Lee et al.* 2020] based on TM using SCSN events as templates.

For the 2009 L'Aquila (Italy) sequence, we use the HR catalog of *Valoroso et al.* [2013] (see Data and Resources). We only focus on the magnitude information contained in these catalogs. All catalogs have a magnitude discretization, or binning, of $\Delta M = 0.01$.

To analyze their MFDs, we calculate the most relevant parameters for an exponential distribution, i. e., M_{c} and the b-value. At first, we apply two common M_{c} estimation methods [*Mignan and Wössner* 2012]: (1) the maximum curvature method [*Wiemer and Wyss* 2000] that uses the mode of the MFD; we include a correction of +0.2 magnitude units [*Wössner and Wiemer* 2005], hereinafter referred to as $M_c^{MAXC}(+0.2)$; and (2) Median-Based Analysis of the Segment Slope method [*Amorèse* 2007] that detects a change point; we use the 2σ confidence interval (~95 %) of a 1000-sample bootstrap distribution as the final estimate, hereinafter referred to as $M_c^{MBASS}(+2\sigma)$. To enhance the stability of both methods, we apply them to magnitudes rounded to one decimal place. The b-value is estimated with a bias-free maximum-likelihood method [*Tinti and Mulargia* 1987; *Marzocchi and Sandri* 2003; *Marzocchi et al.* 2020], but only for sample sizes of 100 or larger.

At the same time, we assess whether the magnitude is exponentially distributed using the canonical goodness-of-fit test of *Lilliefors* [1969]. Only a goodness-of-fit test can indicate whether data follow a certain distribution [*Clauset et al.* 2009]. The Lilliefors test is a modification of the Kolmogorov–Smirnov (KS) one-sample test to be used when the parameters of the distribution are unknown and need to be estimated from the sample. (Note that this test is the same as the often termed "modified KS test" referring to either *Stephens* [1974] or *Pearson and Hartley* [1972], which are based on extensive and revised Monte Carlo (MC) simulations compared to *Lilliefors* [1969]; our test statistic is likewise based on an extensive MC simulation with 10 million replications (see Data and Resources).) Because the exponential distribution is a continuous probability distribution, the 0.01-binned magnitudes are transformed into a continuous random variable by adding uniformly sampled random noise $\mathcal{U}(-\frac{\Delta M}{2}, \frac{\Delta M}{2})$. Note that for 0.01-binned magnitudes, the added uniform noise does not affect significantly the exponentiality of the distribution up to sample sizes of at least 1 million, as confirmed by Lilliefors test simulations (not shown). The Lilliefors test is performed as a function of M_c for 50 initializations of the random noise, from which we obtain an average p-value, $\overline{p}_M$, at each magnitude bin. The p-value expresses the probability to observe the data sample assuming the null hypothesis is true (here: the exponential distribution). According to Ronald Fisher's original interpretation, it measures the strength of evidence against the null hypothesis. It is worth remarking that our application cannot be seen as a formal statistical test because of this recursive testing, but is used to highlight significant departures from the exponential GR relation. We use $\overline{p}_M$ with a significance level of $\alpha = 0.1$ to obtain the lowest magnitude level above which the MFD can be considered exponential, hereinafter referred to as the Lilliefors-based magnitude of completeness, $M_c^{Lilliefors}$. Note that choosing $\alpha = 0.1$ is conservative in a statistical sense [*Clauset et al.* 2009]; less conservative choices $\alpha < 0.1$ increase the probability to not reject models that have only a very small chance to follow an exponential distribution. To improve stability, $\overline{p}_M$ must exceed α for at least five consecutive magnitude bins, in which case the first exceedance, i. e., the lowest magnitude bin, yields the eventual $M_c^{Lilliefors}$.

Departures from the exponential distribution can occur either over a magnitude range or intermittently at various magnitude levels. To facilitate identifying and characterizing MFD inconsistencies, we determine the slope of the MFD (i. e., the b-value) as function of M_c. Although the b-value of non-exponential MFDs does not have a physical meaning, a systematic dependence on M_c provides clues on the kind of MFD inconsistency.

Note that we do not discuss the existence of various non-exponential MFDs for individual fault segments, such as the characteristic earthquake model [*Schwartz and Coppersmith* 1984], which should not present significant differences from an exponential distribution if the characteristic magnitude is much larger than M_c.

3 Results

3.1 Ridgecrest sequence

We first compare the magnitude statistics of the relocated *Hauksson et al.* [2012] catalog to the original SCSN catalog for the Ridgecrest sequence (see Fig. 1a–c) within the time range 2019-04-01–2019-12-31 and a distance of 100 km from the mainshock. The MFD of both catalogs is very similar (gray and yellow in Fig. 1a), because the *Hauksson et al.* [2012] catalog takes over the magnitudes of SCSN and is a subset thereof. Both MFDs feature a discontinuity around $M3.5$, which has a strong influence on the b-value (see the abrupt change for $M_c \geq 2.8$ in Fig. 1b), which peaks at $M3.44$ with a b-value of ~1.2. Below $M3.5$, the Lilliefors p-values (Fig. 1c) indicate a rejection of exponentiality for both catalogs. The composition of SCSN magnitude types in terms of their MFD (red, blue, and green in Fig. 1a) reveals that the discontinuity is caused by an improper merging of the local (M_L) and moment magnitude (M_w) scale (see Discussion). $M_c^{\text{Lilliefors}} = 3.54$ accounts for this discontinuity, whereas $M_c^{\text{MAXC}}(+0.2) = 1.10$ and $M_c^{\text{MBASS}}(+2\sigma) = 1.51$ do not and are much lower. The latter two do not comply with the assumed exponential distribution of the GR relation and lead to biased b-value estimates (Fig. 1b): The b-value below $M3.1$ (smaller than 1) is considerably different from the one between $M3.1$ and 3.6 (well above 1), and above $M3.6$ (around 1.05).

The dedicated catalogs for the Ridgecrest sequence with even higher resolution [*Ross*, *Idini*, *et al.* 2019; *Shelly* 2020b; *Lee et al.* 2020] are affected by the same inconsistency (Fig. 1d–f). Note that we restricted all three catalogs—and the SCSN catalog for comparison—to their common spatio-temporal window: 2019-07-04 15:35 (2 hr prior to the $M6.4$ foreshock) until 2019-07-17 (~11 days after the $M7.1$ mainshock), within a radius of 37 km from the coordinate 35.74°N, 117.54°W (approx. in the middle of both hypocenters). For the *Ross*, *Idini*, *et al.* [2019] catalog, the Lilliefors p-values (Fig. 1f) reveal that the exponentiality cannot be rejected around $M_c = 1.8$ and above $M_c = 3.5$, but it is rejected in between ($M_c = 2.0\text{–}3.5$). This indicates that this catalog has two different exponential distributions with distinct b-values: a first between ~$M1.8$ and $M3.5$ containing ~95 % of the data above ~$M1.8$, dominated by the M_L scaling (b-value about 0.8); and a second above $M3.5$ when the discontinuity is overcome, dominated by the scaling of M_w and M_{Lr} (b-value about 1.0). Due to the short-lived exponentiality (i. e., no persistent exponentiality with increasing M_c) far below the discontinuity, $M_c^{\text{Lilliefors}}$ could mislead as it does not relate to the exponential distribution of the largest events, but to a secondary one of the smaller events. The MFD of the *Shelly* [2020b] catalog shows a similar behavior with increased p-values around $M_c = 2.0$, albeit not exceeding the significance level; $M_c^{\text{Lilliefors}}$ is above the discontinuity and similar to the SCSN catalog. The *Lee et al.*

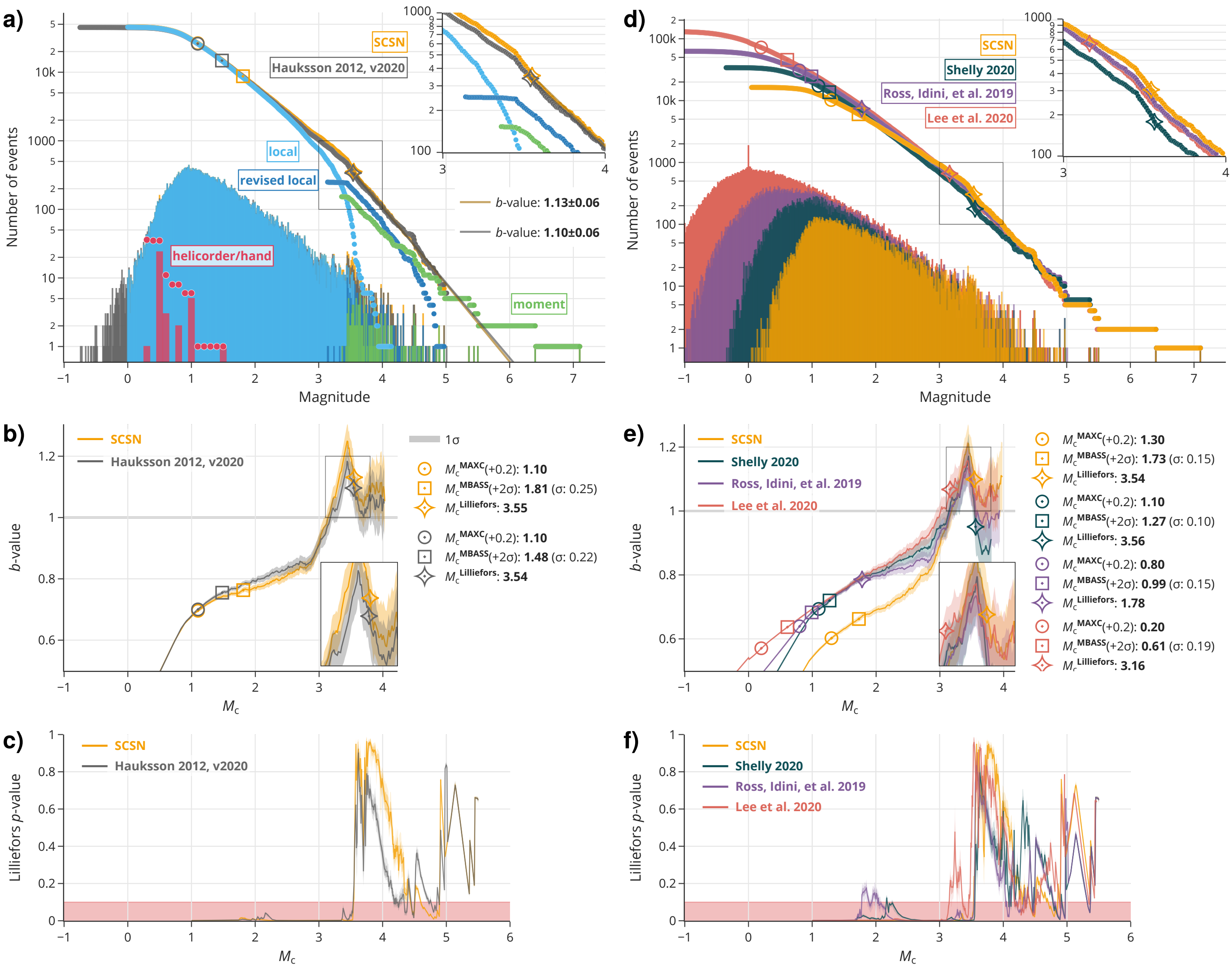

Figure 1: Magnitude statistics of the 2019 Ridgecrest sequence using various catalogs. The left column (a–c) relates to nine-month data extracts (see text) of the SCSN and *Hauksson et al.* [2012] catalog, including the underlying composition of magnitude types; the right column (d–f) relates to three template-matching-based (TM-based) catalogs [*Ross*, *Idini*, *et al.* 2019; *Shelly* 2020b; *Lee et al.* 2020] and the SCSN catalog in their common spatio-temporal window (see text). The top row (a, d) shows the catalogs in terms of their magnitude–frequency distribution. The second (b, e) and last row (c, f) show, as a function of lower magnitude cutoff, or magnitude of completeness, M_c, the *b*-value (the slope of the fitted Gutenberg–Richter relation) and the Lilliefors *p*-value (assuming an exponential distribution as null hypothesis), respectively. Different estimates of M_c are indicated in the first and second row (see legend and text). Each inset in those rows magnifies the enframed section of the plot area.

[2020] catalog shows a short-lived exponentiality just below the discontinuity as indicated by $M_c^{\text{Lilliefors}} = 3.16$, which means that the discontinuity around $M3.5$ is reduced compared to the other catalogs—but it is unclear why. The b-value at $M_c^{\text{Lilliefors}} = 3.16$ (about 1.05) is comparable to the one for $M_c > 3.6$ (Fig. 1e)—a coincidence because it remains anomalous in between. For all catalogs, $M_c^{\text{MAXC}}(+0.2)$ and $M_c^{\text{MBASS}}(+2\sigma)$ again yield underestimated (i. e., over-confident) completeness magnitudes which do not comply with the exponential assumption.

3.2 Landers sequence

Inspecting the Landers sequence as an example for an older catalog period shows that it is composed of even more magnitude types, each having different non-exponential MFDs (Fig. 2a). Most notably, the coda amplitude-based magnitude, M_{coda}, is over-represented below $M3.0$, whereas the helicorder/hand magnitude, M_h, mixes differently binned magnitudes (0.5, 0.1, and 0.01). These two magnitude types considerably affect the estimated b-value of the overall catalog as function of M_c (Fig. 2b): M_{coda} raises the b-value for $M_c = 2.0–3.0$ compared to $M_c > 3.0$, whereas M_h results in distinct b-value jumps due to the irregular binning, especially in 0.5 magnitude steps noticeable up to $M_c = 4.0$. The 0.5 binning of M_h causes the overall MFD to be non-exponential for $M_c = 3.2–3.5$ and again briefly for $M_c = 4.0$ (Fig. 2c). Below $M3.0$, the MFD is not exponential anymore due to the combined effects of M_h and M_{coda} and the b-value is overestimated. The common completeness estimation methods with $M_c^{\text{MAXC}}(+0.2) = 1.70$ and $M_c^{\text{MBASS}}(+2\sigma) = 2.33$ would result in such a biased b-value because they again do not comply with the exponential assumption.

3.3 Regional catalog of southern California

We further investigate whether the MFD of a regional seismic catalog is inconsistent (Fig. 3). We compare the 10-year QTM catalog [*Ross*, *Trugman*, *et al.* 2019] to the SCSN catalog for 2008–2017 to obtain the contributing magnitude types. M_L and M_w merge around $M4.4$ (Fig. 3a), but its impact on the b-value is too uncertain because of too few data. Yet, the merging might be the reason for the p-value decrease around $M_c \approx 4.4$ (Fig. 3c). The M_h magnitude—especially its 0.1 binning—causes some irregularities and fluctuations in the b-value below $M3.5$.

The QTM catalog has the same $M_c^{\text{Lilliefors}}$ as SCSN ($M3.24$), which implies that QTM does not comply with an exponential distribution to lower magnitudes. This finding is similar to the Ridgecrest sequence (Fig. 1d–f), but without the lower magnitudes having a distinct secondary exponential distribution. Below $M_c^{\text{Lilliefors}}$, the MFD gradually curves toward low magnitudes, accompanied by continuously decreasing b-values. $M_c^{\text{MAXC}}(+0.2)$ and $M_c^{\text{MBASS}}(+2\sigma)$ again do not comply with the exponential assumption and differ much more from $M_c^{\text{Lilliefors}}$ than in the catalogs for the individual sequences.

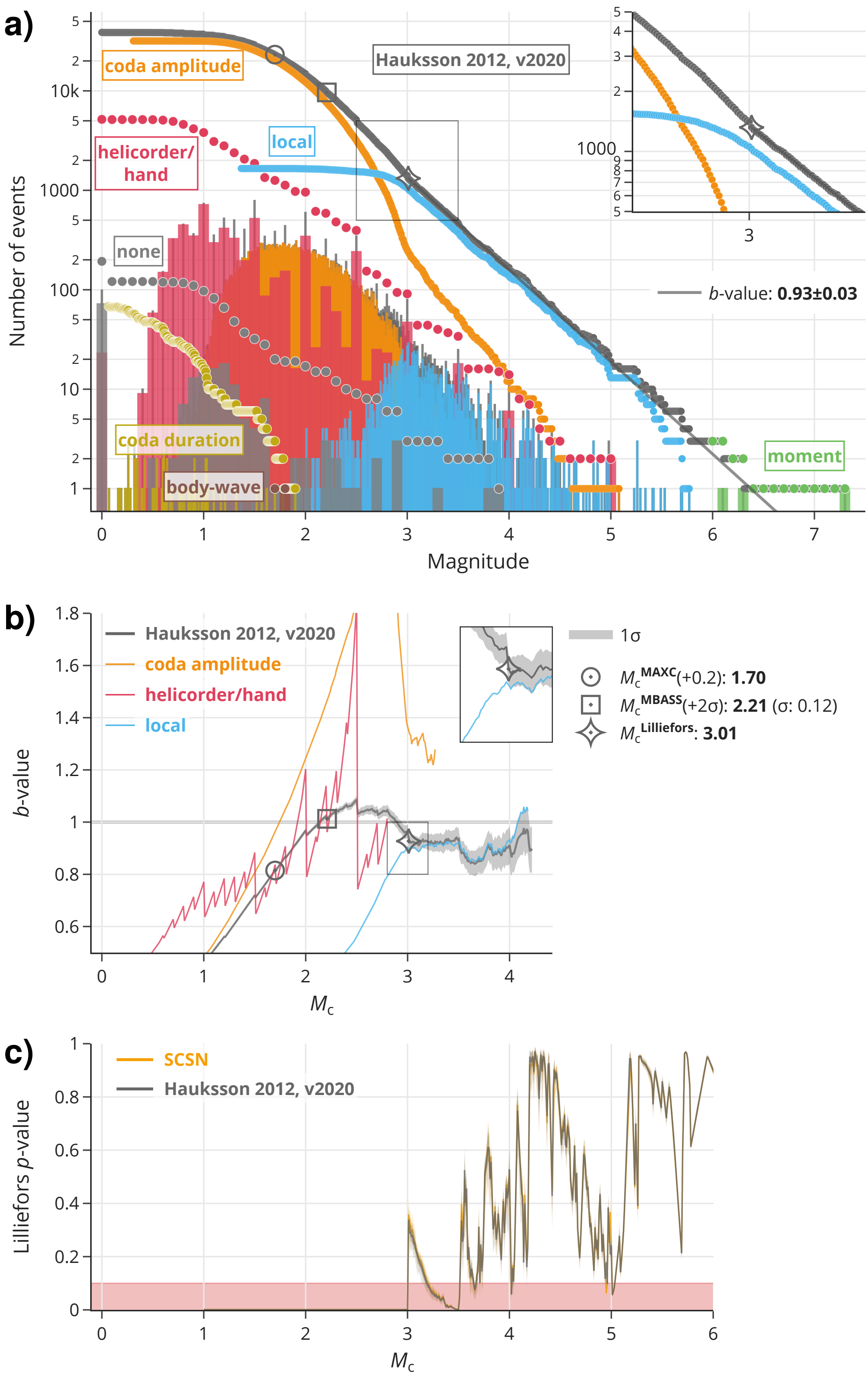

Figure 2: Analogous to the left column of Fig. 1, but for the 1992 Landers sequence using one year of data (1992-03-01 until 1993-02-28, within 100 km from the mainshock). In addition, b) shows the b-value as function of M_c for three dominating magnitude types (see legend).

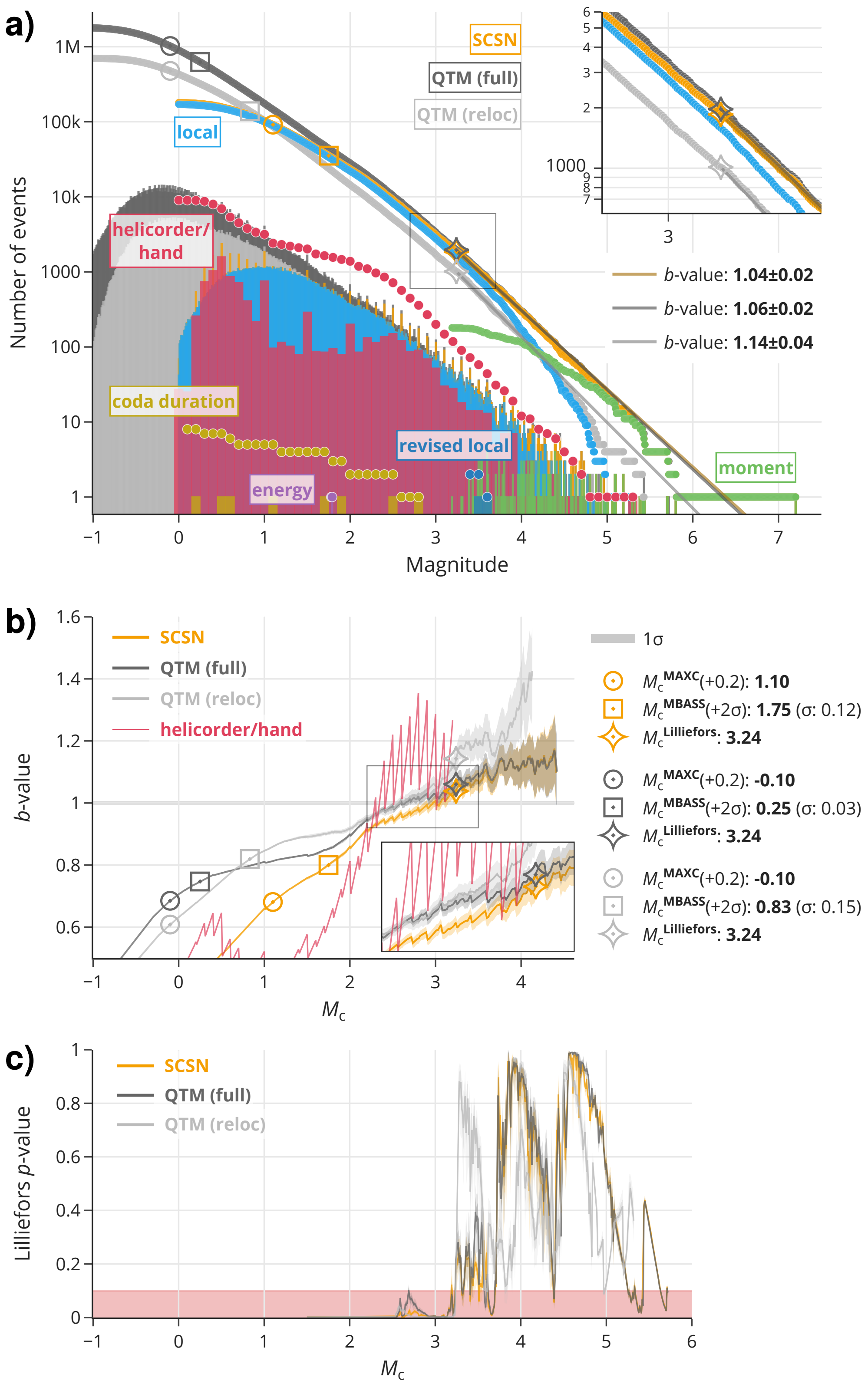

Figure 3: Analogous to Figs. 1 and 2, but for catalog data of entire southern California in 2008–2017, the time period of the QTM catalog [*Ross*, *Trugman*, *et al.* 2019] ('QTM (full)', dark gray). Its subset of only relocated events ('QTM (reloc)') is shown in light gray.

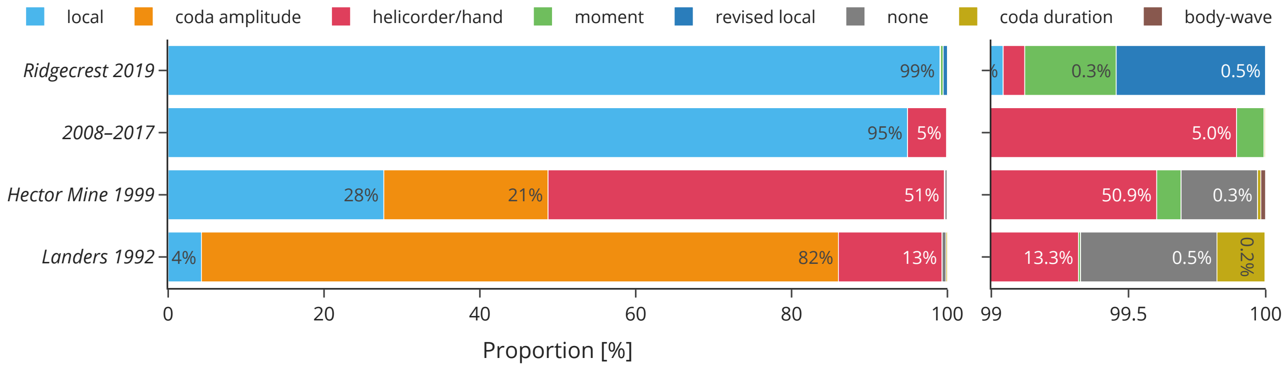

Figure 4: Proportion of magnitude types for different temporal periods of the SCSN catalog. Magnitude types are sorted by the order given in the legend. The right-hand subfigure zooms into the 99–100 % range.

Particular attention must be paid to the 'reloc' subset of the QTM catalog; it excludes many events (such as the 2010 $M7.2$ Baja California/Sierra El Mayor–Cucapah event). As a consequence, the b-value diverges from the SCSN catalog for $M_c > 3$. This subset should be used with great care for statistical analyses because its MFD is apparently not a good representation of the actual MFD.

We also investigate temporal changes in the proportion of magnitude types for southern California. Fig. 4 summarizes these proportions for the periods of the SCSN catalog analyzed above. The 1999 $M7.1$ Hector Mine sequence was added as an intermediate temporal sample. (For the sake of completeness, we have also applied the MFD analysis to the Hector Mine sequence see Fig. S3 and Text S2 in the supplemental material.) The most apparent change over time is the gradual replacement of both M_{coda} and M_{h} by M_{L}. Simultaneously, M_{L} gets replaced by M_{w} as can be seen from the individual MFDs (Figs. 2a, S3a, 3a, 1d). As a consequence of the changing magnitude proportions over time, the magnitude types merge at different magnitude levels, which may cause one or more discontinuities as shown above. We summarize those in Text S1 for the four analyzed periods of the SCSN/*Hauksson et al.* [2012] catalog.

Fig. 5 shows that the time dependence of the discontinuities propagates to $M_{\mathrm{c}}^{\mathrm{Lilliefors}}$, making it time-dependent as well. $M_{\mathrm{c}}^{\mathrm{Lilliefors}}$ is elevated during 1985–1995 and again from 2010 onward compared to 1980–1985 and 1995–2010. Fig. S1 shows the same analysis for two-year time intervals in which $M_{\mathrm{c}}^{\mathrm{Lilliefors}}$ fluctuates more strongly; its highest estimates typically dominate the respective five-year interval in Fig. 5. These estimates match $M_{\mathrm{c}}^{\mathrm{Lilliefors}}$ found for the individual sequences and entire southern California in 2008–2017 (filled symbols in Fig. 5). For comparison, $M_{\mathrm{c}}^{\mathrm{MAXC}}$ and $M_{\mathrm{c}}^{\mathrm{MBASS}}$ generally decrease over time, reflecting that smaller events are increasingly being added to the catalog. In the last five-year catalog period, $M_{\mathrm{c}}^{\mathrm{Lilliefors}}$ differs from both $M_{\mathrm{c}}^{\mathrm{MAXC}}$ and $M_{\mathrm{c}}^{\mathrm{MBASS}}$ by ~2.5 magnitude units, highlighting again their discrepancy already found for the Ridgecrest sequence.

It should be noted that the USGS-ANSS ComCat, which depends on SCSN as a regional

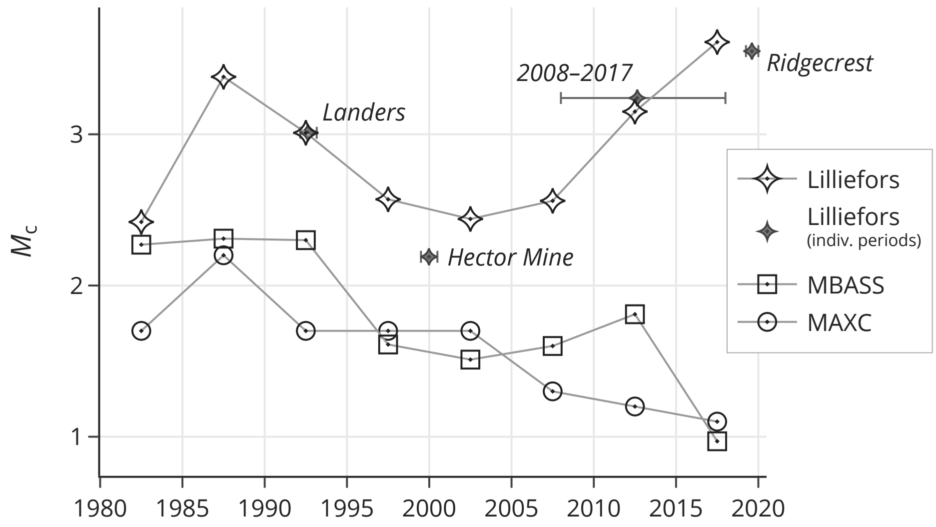

Figure 5: Estimates of M_c using three different methods for five-year intervals of the [*Hauksson et al.* 2012] catalog. The symbols are aligned with the midpoint of each time interval. For comparison, $M_c^{\text{Lilliefors}}$ estimated for four individual periods of this catalog (Figs. 1–3 and S3) are indicated with filled symbols; the horizontal whiskers relate to their time span.

seismic network, inherits the same magnitude composition (see Fig. S2) and therefore the same inconsistencies outlined above.

3.4 L'Aquila sequence

The HR catalog of *Valoroso et al.* [2013] for the L'Aquila sequence suffers from a similar apparent over-representation of low-magnitude events as observed for the Landers sequence. Here, it is the single cause for the rejection of the exponential distribution below $M_c^{\text{Lilliefors}} = 1.81$ (see Fig. 6, black curve). We investigated the catalog within consecutive non-overlapping time windows (indicated in Fig S4) and found a time-dependent degree of over-representation (Fig. 6a, colored MFDs): It is not evident in the first four days (red) but starts to appear in the subsequent week (yellow) and is more dominant for the following time periods after about 1.5 weeks after the mainshock (green, blue, and dark blue). Accordingly, the b-value below $M_c^{\text{Lilliefors}} = 1.81$ becomes exceptionally over-estimated at these later times (Fig. 6b). The MFD of the whole catalog (black curve) below $M_c^{\text{Lilliefors}}$ mixes the MFD behavior in the individual time windows: the overall over-representation is weaker than in the last periods, but greater than in the initial periods. This compensation effect also applies to the b-value below $M_c^{\text{Lilliefors}}$ (Fig. 6b).

The over-confident $M_c^{\text{MAXC}}(+0.2)$ and $M_c^{\text{MBASS}}(+2\sigma)$ point to a completeness magnitude where the b-value is maximally biased, whereas $M_c^{\text{Lilliefors}}$ yields a b-value of about 1.0. Although the b-values differ among the individual time periods at this magnitude level (see inset of Fig. S4b),

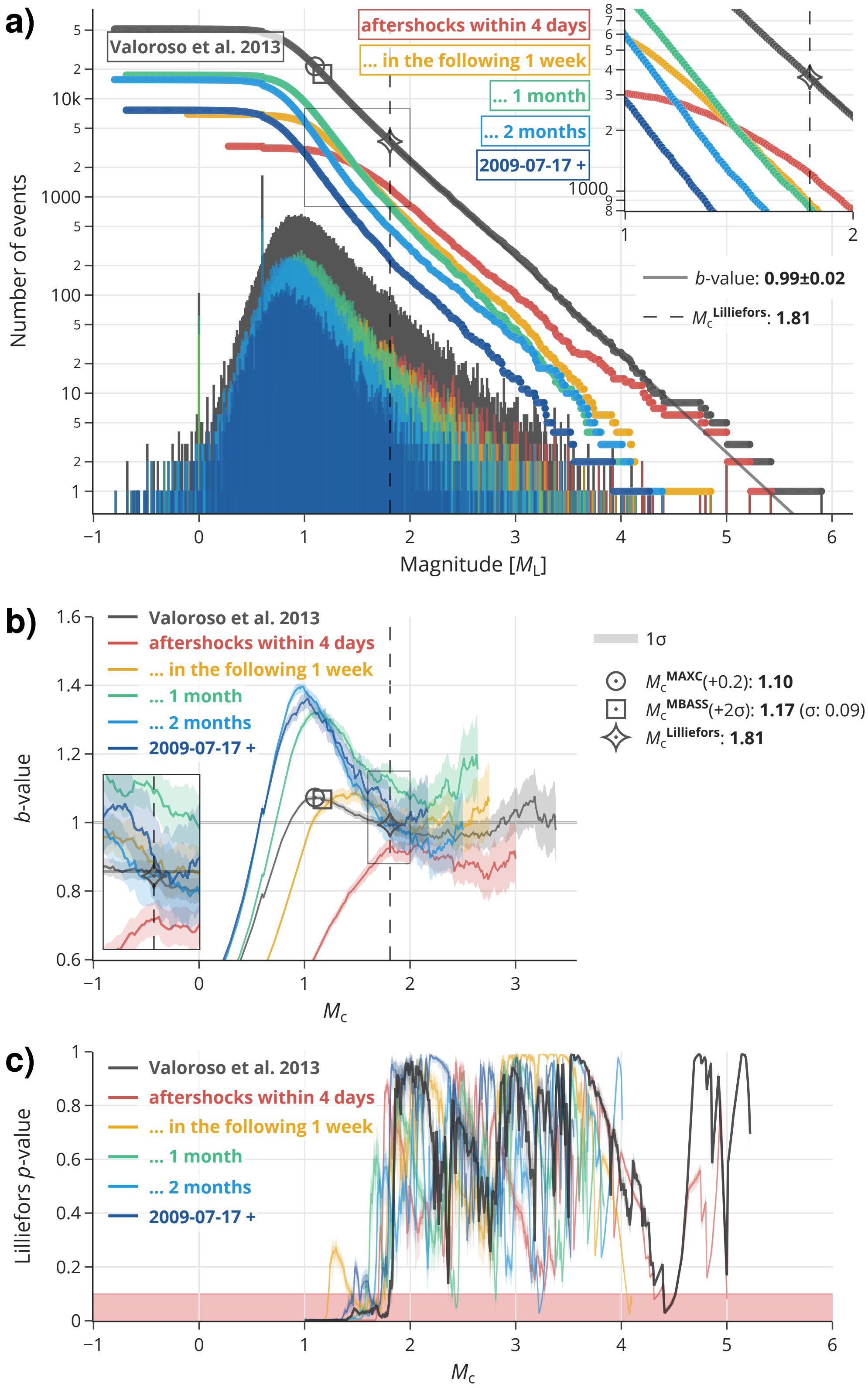

Figure 6: Analogous to Figs. 1, 2, and 3 but using the catalog of the 2009 L'Aquila, Italy sequence [*Valoroso et al.* 2013]. The analysis focuses on time-dependent magnitude statistics in non-overlapping time windows after the mainshock (see legend and Fig. S3). Accordingly, the last time window starts about 3.3 months after the mainshock.

the uncertainty estimates indicate that they are not significantly different from 1.0. (Note that only a 1σ, i. e., 68 %, confidence interval is shown.)

For the sake of completeness, we performed the time-window analysis for the Ridgecrest sequence using the catalog of *Hauksson et al.* [2012] (Figs. S5 and S6) and *Ross*, *Idini*, *et al.* [2019] (Figs. S7 and S8), and for the Landers sequence (Figs. S9 and S10). The analyses of both catalogs for the Ridgecrest sequence show that MFDs of the earliest, mostly incomplete, period are much more curved than MFDs of later periods. The MFDs of later periods appear exponential down to $M1.5$ mostly because they are more complete, but partly because the discontinuity around $M3.4$ is undersampled and barely noticeable. Yet, the Lilliefors test still detects the discontinuity and results in short-lived rejections below it (Figs. S6c and S8c). The earliest period is the most active and its MFD shape dominates the overall MFD (e. g., it contains more $M > 2.5$ events than all other periods combined). For the Landers sequence, the individual MFDs behave similar to the L'Aquila sequence: the early period tends to a lower b-value, whereas the later periods increase in b-value with decreasing M_c below $M_c^{\text{Lilliefors}}$.

4 Discussion

In all inspected HR catalogs, the exponential distribution does not hold toward low-magnitude ranges. This undesired shortcoming applies to both sequence-based and regional catalogs. Noteworthy, common completeness estimation methods (such as MAXC and MBASS) cannot capture these departures, leading to (1) M_c that do not comply with the exponential distribution and (2) biased b-values that do not describe the magnitude distribution of the largest events adequately. It is already known that the MAXC method may underestimate M_c [*Wössner and Wiemer* 2005; *Mignan et al.* 2011] unless spatio-temporally confined samples are used (not the focus of our study). The MBASS method is considered to estimate M_c more conservatively [*Mignan and Wössner* 2012]. Here we showed that these two methods provide much lower M_c estimates than the Lilliefors test in every investigated HR catalog, which strongly affects the estimated b-value. For TM-based catalogs in particular, $M_c^{\text{Lilliefors}}$ is either equal to the one of the network-based catalog (Fig. 3), or it indicates that the exponential MFD of small events differs from the one of larger events (Fig. 1d–f). Both cases imply that adding more small earthquakes with advanced detection methods does not preserve the exponential shape of the MFD.

According to our observations, HR catalogs should be used with caution for estimating any property of the MFD. In the following we discuss the different kinds of MFD inconsistencies, when they compensate, and how to possibly overcome them.

4.1 The different kinds of MFD inconsistency and their origin

Our findings show that MFD inconsistencies have different origins and may be divided into three categories.

The first category contains MFDs with abrupt discontinuities; we observed those specifically in catalogs for the Ridgecrest sequence and entire southern California. These discontinuities are due to the mixture of different magnitude types. Since the magnitude composition changes over time for southern California, one or more discontinuities can occur at different magnitude levels (especially between M_L vs. M_w for more recent catalogs, and between M_coda vs. M_L for older catalog periods). The revised local magnitude (M_Lr) introduced by SCSN for 2016 onward to bring both M_L and M_w "into closer agreement" [*SCEDC* 2016] apparently does not solve the issue or is not sufficient. It remains to be seen if TM-based catalogs could become exponential down to $M2.0$ without this discontinuity. For older catalog periods, a culprit for discontinuities is the irregular and mostly coarse binning of M_h.

Besides abrupt changes, MFDs can change gradually in slope toward low magnitudes. The second category is composed of MFDs characterized by an over-representation of low magnitudes with respect to an exponential distribution, as observed for Landers and L'Aquila. For Landers it could be attributed to the M_coda scale. Although we cannot inspect the origin of this kind of inconsistency, we suspect that it is caused by data recording or processing issues leading to inappropriate magnitude estimates. Over-representation results in an increasing b-value with decreasing M_c. Like many catalog inconsistencies, this effect can induce fake time variations of the b-value (Figs. 6b and S9b), especially when obtained with common completeness estimation methods. The observed time-dependence of this inconsistency can be explained with the improved completeness over time: Since low magnitudes are over-represented compared to higher ones, their increasing contribution to the catalog over time makes the inconsistency more prevalent, changing the b-value.

The third category is composed of MFDs characterized by an under-representation of low magnitudes with respect to an exponential distribution. Under-representation (i. e., a gradual curvature in the MFD [*Mignan* 2012]) results in a continuously decreasing b-value with decreasing M_c. This effect is very dominant in more recent catalog periods for southern California including the TM-based catalogs. The explanation of under-representation is probably more challenging. We argue that the most likely origin is the mixture of spatio-temporally inhomogeneous (in)completeness. As shown for the sequences (Figs. 6, S4–S10), the effect is dominant immediately after a large earthquake and vanishes over time, which is commonly known as short-term incompleteness [STAI, *Kagan* 2004; *Helmstetter et al.* 2006; *Hainzl* 2016; de *Arcangelis et al.* 2018]. For instance, *Kagan* [2004] estimated that up to 28 000 early aftershocks after the Landers mainshock are missing (or two thirds of $M2$ events). Our observations corroborate the hypothesis of under-reporting low-magnitude events. As other scientists have pointed out [*Wiemer and Wyss* 2000; *Wössner and Wiemer* 2005; *Mignan et al.* 2011], a gradual curvature in regional catalogs (Fig. 3) can additionally arise from the spatial inhomogeneity of completeness due to the varying seismic network density. The contribution of the temporal evolution of the network is maybe a weaker factor, because $M_c^{MAXC}(+0.2)$, which is related to the strongest curvature in the MFD, decreases only marginally in the period from 2005–2010 to 2015–2020 (see Fig. 5).

In our observations, even TM-based methods can apparently not sufficiently improve the under-reporting (Figs. 1 and 3)—possibly due to their selectiveness, i. e., strong dependence on events in the network-based catalog. To investigate the influence of temporal incompleteness, we removed the period in which STAI is most evident— (until 4 days after the $M7.1$ Ridgecrest mainshock, see Figs. S11 and S12). Accordingly, a strong gradual curvature remains in the MFDs and $M_c^{\text{Lilliefors}}$ of the TM-based catalogs are very close to the one of the network-based catalog at $M_c \approx 1.65$ (except for the *Shelly* [2020b] catalog). This proximity indicates that (1) the abundance of small earthquakes from advanced detection methods does not necessarily make the MFD more exponential toward low magnitudes even in a more complete period; and (2) the under-representation in HR catalogs may last well beyond the short-term incompleteness. Moreover these $M_c^{\text{Lilliefors}}$ relate only to short-lived exponentiality (Fig. S12c). When removing the first ~9 days after the $M7.1$ mainshock (see Figs. S13 and S14), $M_c^{\text{Lilliefors}}$ improved for the SCSN and *Ross*, *Idini*, *et al.* [2019] catalog to 1.34 and 0.90, respectively. The former change indicates an improved completeness and the latter relates again to a short-lived exponentiality (Fig. S14c). Worthy of note, incompleteness is not detected by the common methods to estimate M_c; even after removing STAI, their estimates are lower than $M_c^{\text{Lilliefors}}$ especially for the TM-based catalogs, leading to strongly biased b-values (Fig. S14b).

An additional explanation for the apparent under-representation of low magnitudes (which does not preclude the previous ones) is the scaling break of the (amplitude-based) local magnitude M_L, which, as shown by several studies [e. g., *Bakun* 1984; *Hanks and Boore* 1984; *Ben-Zion and Zhu* 2002; *Edwards et al.* 2010; *Zollo et al.* 2014; *Staudenmaier et al.* 2018; *Lanzoni et al.* 2019], scales differently with M_w below $M2$–4 (with $M_L \propto 1.5M_w$) due to the attenuation of the higher frequency content in the medium (i. e., their corner frequencies remain constant) [*Bethmann et al.* 2011; *Munafò et al.* 2016; *Deichmann* 2017]. Anti-aliasing in the digital sampling process (an additional low-pass filter) can contribute to the scaling break [*Uchide and Imanishi* 2018]. Finally, we argue that even when accounting for this scaling break, a gradually curved MFD at very low magnitudes may remain, e. g., as observed for induced seismicity [*Herrmann et al.* 2019]. Under-representation may therefore be further related to underlying physical processes such as a minimum rupture size [see also *Ellsworth* 2019].

4.2 Apparent compensation of inconsistencies

Sometimes, the over- and under-representation can cancel out and lead to an apparently (and unknowingly) wider exponential MFD when choosing an unfortunate time window so that both effects are approximately in balance. For the entire southern California region, such a compensation can happen for instance in the period 1992–2018 (Fig. 7a–c). (Note that we did not include the period after 2019 due to the inconsistency at $M3.5$ outlined for the Ridgecrest sequence.) The compensation gives the impression of an apparent validity of the exponential

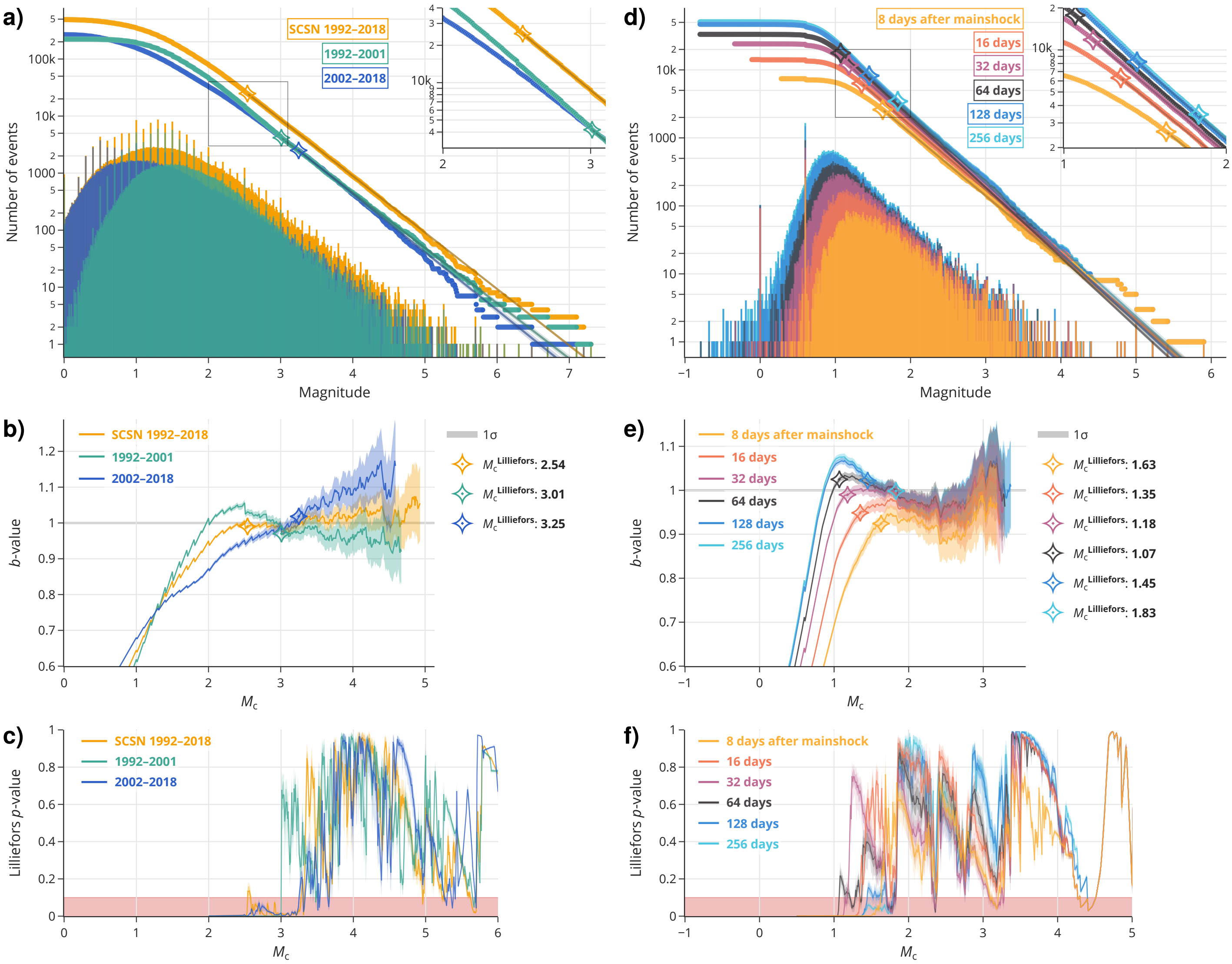

Figure 7: Demonstrating for two catalogs that the cancellation of over- and under-representation of low magnitudes can lead to an apparently wider exponential MFD. Left (a–c): regional catalog of southern California for 1992–2018, in which $M_c^{\text{Lilliefors}}$ is lower (yellow) than in the two individual time periods (blue and green, see legend). Right (d–f): catalog of the L'Aquila sequence, in which $M_c^{\text{Lilliefors}}$ depends on the time window length of the considered aftershock sequence.

GR relation at $M_c \approx 2.5$, although $M_c^{\text{Lilliefors}}$ of the two individual time periods is higher (~3.0 and 3.25, respectively). A similar compensation may happen for the sequence-specific catalogs of L'Aquila and Landers, when the under-representation at early times after the mainshock due to STAI cancels out with the later over-representation. For L'Aquila, $M_c^{\text{Lilliefors}}$ reaches its lowest level for a time window of 64 days after the mainshock (gray in Fig. 7d–f). For shorter or longer time windows of the aftershock sequence, the under- and over-representation dominates, respectively.

These compensation effects are an unfortunate consequence of the mixture of different non-exponential MFDs. It remains uncertain whether they can correct for the low-magnitude inconsistencies. Possibly, previous studies inferred lower completeness levels than reasonable.

4.3 A possible way to reduce MFD inconsistencies

Estimating magnitudes that produce a consistent MFD over a wide magnitude range appears to be a major challenge, and currently a limiting factor to exploit high-resolution catalogs in terms of magnitude statistics. Hence, magnitude estimation will require a different treatment than what is currently common practice. A possible remedy is to directly estimate moment magnitudes (M_w) for every earthquake. The estimation of M_w is well developed, robust with small uncertainty, and in principle consistent over the entire magnitude range [*Deichmann* 2018]. It has therefore established as the standard magnitude scale, such as by the International Seismological Center [*Di Giacomo et al.* 2015]. M_w has the further benefit that it is directly related to earthquake source physics (e. g., seismic moment) and is therefore seismologically and physically well defined. Several studies have demonstrated for natural microearthquakes that a direct estimation of M_w (i. e., without magnitude regressions) is feasible for event sizes approaching M_w0.0 [*Atkinson et al.* 2014; *Ross et al.* 2016; *Moratto et al.* 2017; *Staudenmaier et al.* 2018; *Uchide and Imanishi* 2018; *Butcher et al.* 2020]. However, also the estimation of M_w and source parameters might practically not be free of biases and limitations, e. g., due to near-surface amplifications at low frequencies especially for small events [*Abercrombie and Leary* 1993].

5 Conclusions

Our study highlighted that HR catalogs usually do not preserve the exponential MFD that characterizes ordinary catalogs, and that common methods to estimate the completeness magnitude, and consequently the b-value, underestimate severely the magnitude level below which the MFD departs from an exponential distribution. Moreover, MFDs of both HR catalogs based on the seismic network and on advanced detection methods depart from exponentiality at a similar magnitude level. These departures are mostly due to an improper mixing of different magnitude types, spatio-temporal incompleteness, or recording/processing issues. Another possible explanation is the intrinsic scaling break toward low magnitudes, such as for M_L.

Observed inconsistencies make it necessary to set a considerably higher completeness level than often anticipated, for instance by using Lilliefors' goodness-of-fit test with the exponential distribution as we did here.

Our findings have implications for both HR catalog producers and modelers that use MFDs of such catalogs. Modelers should be cautious when using HR catalogs that are composed of different magnitude types, span several orders of magnitude (especially below $\sim M3$), and cover wide spatio-temporal scales. The different kinds of inconsistencies outlined in our study for a selection of catalogs are usually not detected by common methods to estimate M_c, leading to strongly biased b-values and, as a consequence, to an inappropriate extrapolation of the rate of large earthquakes from low-magnitude events. This deficiency calls into question b-value-related studies that used those catalogs without a proper check of exponentiality. Moreover, the time-dependence of inconsistencies introduces spurious b-value variations in time.

There is no doubt that the advent of HR catalogs brought great benefits in many aspects, but the results reported here may encourage HR catalog producers to evaluate carefully the homogeneity of the magnitude scales in their catalog (so that the MFD becomes consistent). Since it is not trivial to merge different magnitude scales into one consistent MFD, a possible solution may be to establish the estimation of M_w for each earthquake as common practice. Such an effort could reduce the observed and outlined inconsistencies and make the MFD more physically interpretable.

MFDs conceal inconsistencies more than it seems at first glance. Although they can be revealed and accounted for with deliberate methods like the one presented here, it may be more rewarding to make MFDs themselves more consistent, which would provide greater opportunities for the statistical analysis of existing and future catalogs.

Data and Resources

The southern California catalogs were downloaded from these repositories: SCSN [*SCEDC* 2013, last accessed June 2020], *Hauksson et al.* [2012] (`https://scedc.caltech.edu/research-tools/alt-2011-dd-hauksson-yang-shearer.html`, version '1981–2019', last accessed June 2020), USGS-ANSS ComCat (`https://earthquake.usgs.gov/data/comcat`, last accessed June 2020), QTM of *Ross*, *Trugman*, *et al.* [2019] (`https://scedc.caltech.edu/research-tools/QTMcatalog.html`, last accessed June 2020), *Ross*, *Idini*, *et al.* [2019] (`https://scedc.caltech.edu/research-tools/QTM-ridgecrest.html`, last accessed June 2020), *Shelly* [2020b] (data release: *Shelly* [2020a]), *Lee et al.* [2020] (`http://bit.ly/2WswZQk`, last accessed June 2020). The catalog of *Valoroso et al.* [2013] was provided by L. Chiaraluce (personal communication, November 2019). For the Lilliefors test, we used the implementation of `statsmodels v0.11.1` [*Seabold and Perktold* 2010]. Supplemental material for this article includes further information and results referred to in the text, such as a summary of the MFD inconsistencies for the SCSN catalog, MFD analyses in time windows during the aftershock sequence of Ridgecrest and Landers (as done for L'Aquila), and MFD analyses of the TM-based Ridgecrest catalogs excluding the evident short-term incompleteness period. Our method to calculate $M_c^{\text{Lilliefors}}$ is available as a *Python* class and demonstrated for an example catalog at `https://doi.org/10.5281/zenodo.4162497`.

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