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RESEARCH

Geometry and the Restoration of Ancient Sundials: Camera Obscura Sundials in Cava de’ Tirreni and Pizzofalcone

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Abstract Today restoring ancient “camera obscura sundials” by drilling holes in building façades appears as an overly intrusive intervention in historical architecture. For this reason, our study proposes an innovative, low-cost gnomonic instrument, capable of adapting to any type of relationship between the façade where the original gnomonic hole was located) and the sundial on the floor. The tool that we have designed allows incoming sunlight to be caught by a reflection system of flat mirrors, appropriately tilted, thus producing a solar ray that exits the instrument with a different inclination. We created new angular relationships between the gnomonic hole and the astronomic data engraved along the sundial in two case studies of historic sundials that are now inactive and abandoned. The research was conducted weaving astronomy and gnomonics with geometry and mathematics, to create a 3D model to verify, plan and execute the restoration of historic sundials.

Keywords Camera obscura sundials · Gnomons · Catoptric sundials · Cooper formula · Solar declination · Rocco Bovi

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Introduction

Sundials are ancient timekeeping devices which indicate the time by means of the shadow cast by a gnomon. Meridian lines, also called "camera obscura sundials", are a special kind of indoor solar calendars that work exclusively at noon each day when the sun is in the south, thus becoming a perpetual calendar indicating the date, rather than time. Meridian lines can be either set horizontally (on the floor) or vertically (on a wall), and consist of marble, brass or bronze strips carved with calendrical, astronomical or geographic data and symbols. For vertical meridian lines a necessary element is the gnomon—a rod or even an obelisk, in the case of meridian lines of considerable length—the tip of whose shadow indicates the information to be read. Horizontal meridian lines are traditionally housed inside monumental works of architecture and they are defined by two elements, in strict geometrical correspondence between each other: the graduated strip marking the north–south direction on the floor; and the gnomonic hole, a small circular opening in the building's façade to admit sunrays, thus projecting onto the graduated strip a small luminous ellipse (the "macula" or the "Sun's image") that indicates the moment, each day of the year, when sun crosses the local meridian. The name camera obscura sundials derives from the condition of darkness required to see the Sun's image (Zannotti 1779); because of their calendrical and astronomical functions, these valuable gnomonic devices were usually intended for use by scholars and scientists, unlike the more common outdoor sundials, which indicated the time in order to let the population better plan their trading, fishing, farming activities.

Sundials are graphic evidence of the complex astronomical events constantly occurring in the celestial sphere. To set them, there was frequently a convergence of various doctrines, from mathematics to physics, geography, cartography, from geometry to art: this multidisciplinary convergence is indicative of the fervent cultural climate that arose in Europe between the fifteenth and nineteenth centuries. In this rich intellectual mileu, several sundials of great artistic and scientific value were built in Italy, in particular in the region of Campania. In the second half of the eighteenth century, the multifaceted scientist Rocco Bovi made a name for himself there as a designer and constructor of four important meridian lines (two of them can still be visited), including the one located in the quarter of the Prior of the Charterhouse of San Martino in Naples (Fig. 1).

The Two Case Studies in the Present Research

The Camera Obscura Sundial at Cava de' Tirreni

Our first case study is the last of the four designed by Bovi, and is found in the SS. Trinity Benedictine Abbey, in Cava de' Tirreni. The meridian line lies on the second floor of the south-facing building, exactly in front of the former archive, which today houses the library (Fig. 2).

Fig. 1 Rocco Bovi's camera obscura sundial (1771–1772) in the Carthusian monastery of San Martino, Naples. Photo: Alessandra Pagliano

Fig. 2 Location of the camera obscura sundial in the SS. Trinity Benedictine Abbey, Cava de' Tirreni (province of Salerno). Photo: Alessandra Pagliano

The exemplary work executed by Bovi is also recorded in the archives of the Abbey. Paul Guillaume writes: "… a humble monk, Don Rocco Bovi, whose mathematical and astronomical knowledge was quite considerable, drew, right in front of the archives on the floor of a large corridor, a meridian line of a remakable perfection. This was in 1783, as can be read on one end of this sundial" (Guillaume 1877, our trans.).

On the marble strip we find the inscription D. ROCO BOVIO, A.D. MDCCLXXXIII engraved near an unusual drawing of two oxen, presumably related to the author's last name (Fig. 3). Earlier, in the meridian line of San Martino, he had engraved boats alongside his signature, recalling his origin (he came from Scilla) and presumably symbolizing the long sea voyage that brought him to the Neapolitan coasts.

The meridian in Cava de' Tirreni consists of a white marble strip 8.20 m long and 22 cm thick set in the floor just in front of the library (Silvano De Stefano Abate 1908). It is oriented, as usual, in a north–south direction and is thus not perfectly aligned with the walls of the adjacent corridors, which originally hosted monk cells (Fig. 4).

Today the sundial is inactive and abandoned. The meridian line is set just in the middle of the marble strip. Zodiac signs are arranged on both sides, according to the traditional pairings: Leo–Gemini, Virgo–Taurus, Libra–Aries, Scorpio–Pisces and Sagittarius–Aquarius. Cancer and Capricorn are instead engraved separately, because they coincide, respectively, with the beginning of the meridian line (Fig. 5), indicating the summer solstice, and with the end of the line, in the position farthest from the gnomonic hole, indicating the winter solstice (Fig. 6).

Fig. 3 The beginning of the marble meridian line in Cava de' Tirreni. Photo: Alessandra Pagliano

Fig. 4 North-south axis of the sundial, not aligned with the direction of the adjacent corridors. Photo: Alessandra Pagliano

Unlike the more complex meridian line in San Martino (Pagliano et al. 2014), that of Cava de' Tirreni provides scant numerical, angular and geographical data. Another important difference is the absence of any engraved ellipse to outline the projected solar disk on the principal days of the year, which would have made it easier to read. The only pair of engraved "sun fingerprints" coincides with the solstices, as shown in Figs. 5 and 6. Also worthy of note is that where Sagittarius should be, we find only the arrow, weakly carved, while the entire area appears to present an attempt to erase the image and polish the marble strip. It may be that Bovi had second thoughts about the artistic representation, but the change was not due to a gnomonic correction, because we geometrically verified the position along the meridian line and it perfectly coincides with the date of 21 November.

As mentioned, the sundial is now longer in use. It is lacking the original gnomonic hole that would make it accurate. On the night of 25 October 1954, Cava de' Tirreni was devastated by floods, causing considerable damages to the Abbey, destroying the building of the former diocesan seminary and the upper corridors, including the library and the meridian line room. Restoration began quickly, and was completed in 1956, with a large areas of damaged floors replaced by reinforced concrete beams. The restoration of the façades consisted of the replacement of the original wooden trusses with steel structures. The meridian line was carefully removed and then replaced in the original position, however, surrounded today by a modern tiled floor. On the façade, just above the window, the original gnomonic hole was plastered over: today, an irregular square shaped slot is visible from below,

Fig. 5 The beginning of the meridian line, noon on 21 June, with Cancer represented by the image of a crab, its elliptical form marking the 'fingerprint' of the sun. Photo: Alessandra Pagliano

carved out from the rainwater gutter. The wall presents a slight splay to allow solar rays to be easily admitted into the room. The attempt to restore the gnomonic functionality has failed, however, because the ray doesn't fall on the meridian line at noon (Figs. 7, 8).

Consequently, our research focuses on a geometric and historic analysis of this valuable scientific instrument in order to provide a project for its reactivation and hence make it better known and appreciated. Based on several on-site surveys, we determined the correct size of the circular gnomonic hole, whose diameter corresponds to the minor axis of the marble ellipses engraved on both solstices, $\sim$2.5 cm. The gnomonic hole is placed along the vertical line passing through the point denoted by the Latin incision CENTRUM ERECTUM at the beginning of the marble stripe (Fig. 9).

The restoration of the hole in the geometrically correct position would unfortunately necessitate some difficult masonry work on the wall above the window frame (Fig. 10): that is, the gnomonic hole could be restored only by removing a small portion of the roof at the eaves, whose continuity would be therefore interrupted.

Today, a new gnomonic hole in the correct position with respect to the marble sundial be considered detrimental to the image of the façade, even though it would be a restoration of the original condition.

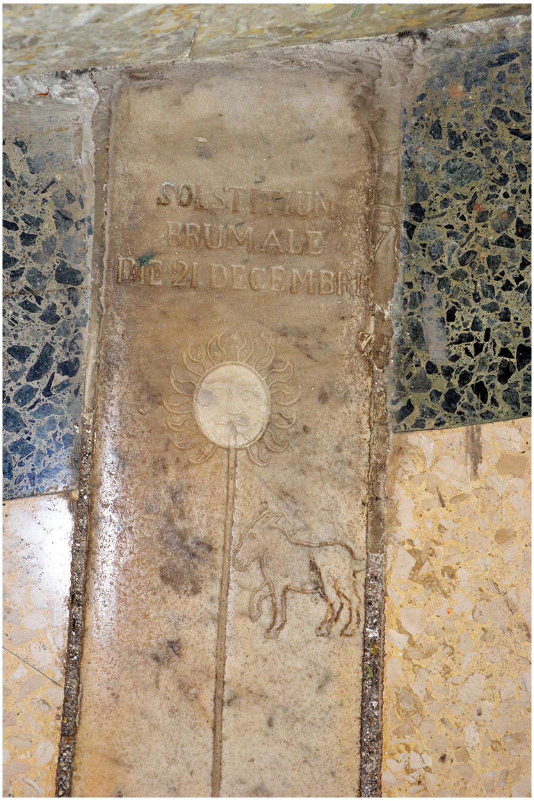

Fig. 6 The end of the meridian line, noon on 21 December, coinciding with the sign of Capricorn, the goat. Note that that the goat representing Capricorn has two hind legs, instead of the more common coiled fish tail. Photo: Alessandra Pagliano

Fig. 7 The innaccurate luminous Sun's image that today appears on the floor (some minutes before noon), in consequence of the erroneous attempt to restore the original functionality of the meridian line. Photo: Alessandra Pagliano

Fig. 8 The irregular square-shaped hole carved out from the gutter in the attempt to create a new gnomonic hole. Photo: Alessandra Pagliano

The Camera Obscura Sundial at Pizzofalcone

Another inactive ancient meridian line is found in the suppressed Theatine monastery of Santa Maria degli Angeli at Pizzofalcone, in Naples. The building is currently home to a branch of the Military Prosecutor of the Republic (Figs. 11, 12).

The sundial is located in the prosecutor's room, and is therefore inaccessible to the public; this condemned the sundial to a long period of oblivion, excluding from the more detailed surveys, until its recent re-discovery and dissemination by Nicola Severino (2003). The sundial is engraved on a marble strip 7.40 m long, and is in fairly good condition. A dividing wall that had been set on top of it has recently been removed.

Over the centuries, repeated restoration works on the exterior façade led to the filling of the small gnomonic hole, and thus destroying the sundial's function. No information has emerged from our bibliographic and historical research for the name of the designer or manufacturer; we must assume that it was made by a member of the monastic community skilled in astronomy, geometry and geography. The lack of information about the designer might be due to the anonymity required by the Theatine order (Vezzosi 1780).

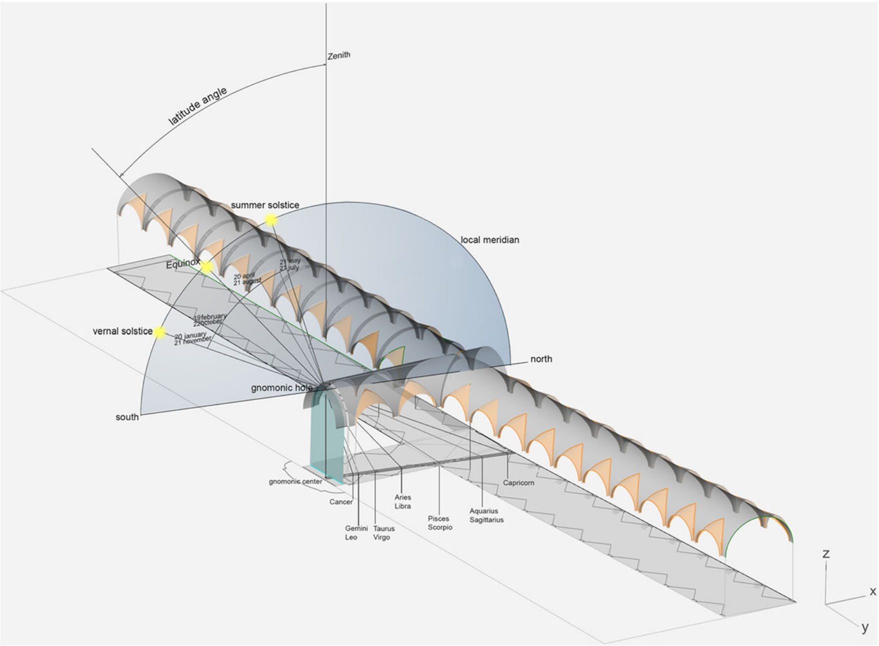

Fig. 9 Geometric method used to determine the correct position for the gnomonic hole; axonometric view. Image: Alessandra Pagliano

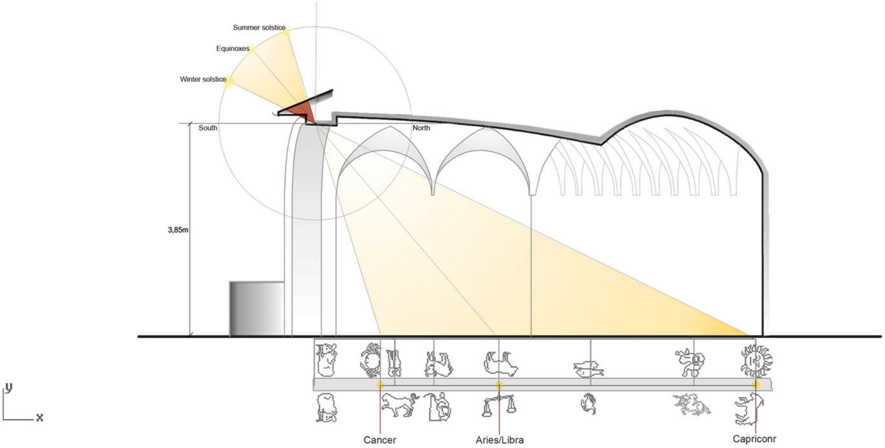

Fig. 10 Cross-section showing the geometrically correct placement of the gnomonic hole. Image: Alessandra Pagliano

The meridian line of Santa Maria degli Angeli is inside a room provided with two balconies, respectively facing north and south. This suggests the hypothesis of an unfinished work: in fact, both windows would have been fundamental for taking measures of the ecliptic, pointing to Polaris in the north, and southwards,

Fig. 11 The meridian line dated 1794 in the suppressed Theatine monastery of Santa Maria degli Angeli at Pizzofalcone (Naples). Photo: Antonio Stefanelli

to record when sun and stars pass on the local meridian. Unlike Bovi's sundial in Cava de' Terreni, the Pizzofalcone meridian line is characterized by numerous calendrical indicators, divided into two parallel bands where a pair of parentheses indicates the final day of each month. The astronomical information concerns the dates when the sun enters in the constellations of the zodiac and the daily degree of solar declination. Our geometric method, based on solar declination data, determined the original location of the hole with respect to the marble strip and, subsequently, its placement along the exterior façade. A small hole, 2 cm in diameter, would have allowed the projected solar disk fit inside the engraved symbols (Fig. 13).

Correctly reconstructed geometrically, the hole would be cut in the wall with a small offset from the window's vertical axis of symmetry, due to the fact that the room, which was built before the sundial, is not perfectly aligned with the marble strip (which must necessarily be directed north–south) (Fig. 14). The creation of a new hole, aimed at restoring the proper functioning of the meridian line, however, would involve a partial disruption of the current decorations on the façade, especially the plaster frame above the windows, whose shape has changed over the centuries.

Fig. 12 The beginning of the meridian line. Photo: Antonio Stefanelli

A New Gnomonic Hole for Inactive Meridian Lines

Restoring the original gnomonic hole by drilling the façade might appear to be an intrusive alteration of historical architectures, but it would actually be only a restoration of the original condition established to adapt the façade to the inclination of the incoming solar rays. Meridian lines and the architectonic spaces that housed them shared an intimate link of shape, proportions and size. However, the main problem we observed is that institutions do not understand the true value of these instruments, which today are often considered exclusively as artistic decorations, and thus they do not invest in their restoration. To overcome this, we propose an innovative, low-cost solution: a gnomonic device capable of adapting to any type of relationship between the meridian line and the façade on which the gnomonic hole is to be re-created (Fig. 15). Regardless of the original position of the hole, our tool allows the incoming sunlight to be caught by a system of reflections between appropriately tilted flat mirrors, thus causing sunrays to come out of the device with an altered inclination, allowing the solar disk to be projected correctly on the meridian line (Fig. 16). In this way it is possible to create new angular relationships between the gnomonic hole and the astronomic data engraved along the sundial, all to the advantage of the architecture, which is thus not subjected to any variation of the façade. However, our project must also address the constant variability of the

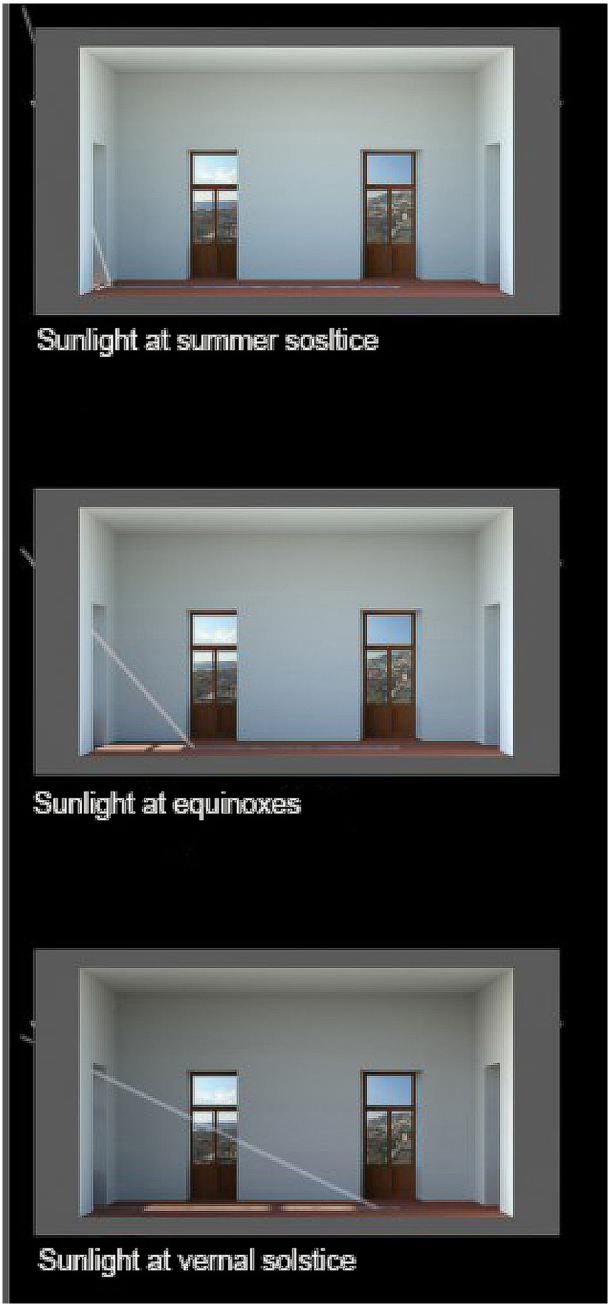

Fig. 13 The original position of the gnomonic hole with respect to the marble strip. Drawing: Antonio Stefanelli

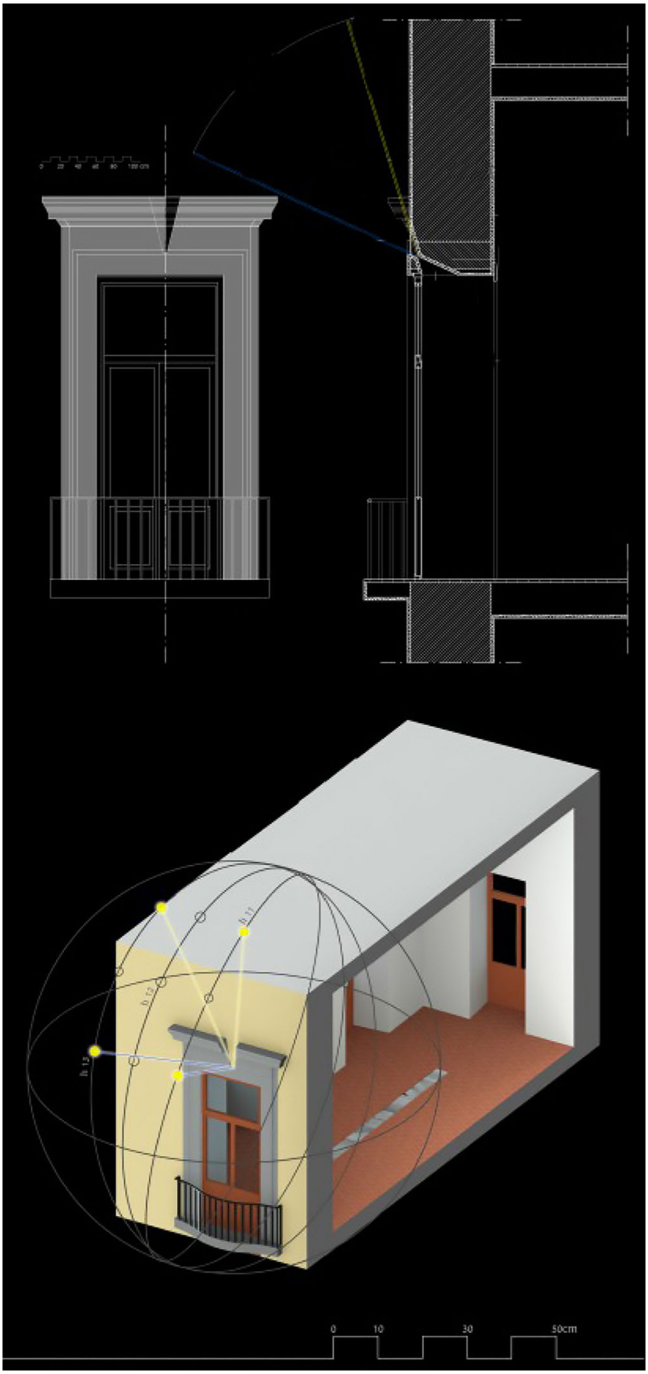

Fig. 14 The restoration project to reactivate the meridian line with the position of a new gnomonic hole to be cut in the wall. Drawing: Antonio Stefanelli

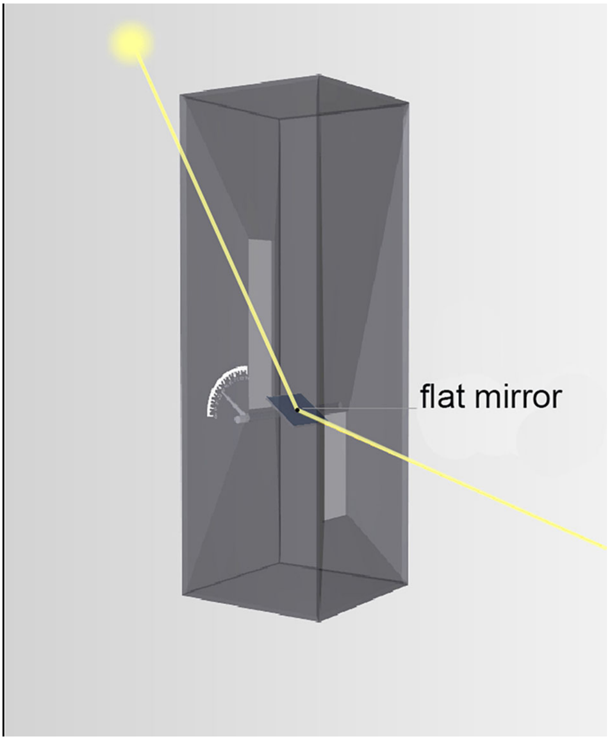

Fig. 15 A new gnomonic instrument to activate ancient meridian lines. Image: Angelo Triggianese

angle of incidence and we have therefore had to adapt the instrument according to the daily solar declination.

Parametric Design for a New Solar Device

All those minds that relish the curiosity of seeing the various types of sundials that have been invented up to now, usually admire, among others, those that are called reflected, which are painted in a place where the sun never strikes, that is, in the inner vaults of the rooms and loggias; they show us the hours by means of the reflection that the sun's rays make through a small glass that is located over a surface of a window, or a column capital, or a beam placed between the columns of a loggia, or in another convenient location. So, beyond their ingeniusness, which appears marvelous by regularly describing them on surfaces so irregular, they also bring the convenience of being able (without taking the person to the sun, being indoors) to see the hours, which are in fact seen in a closed room, where is no other opening except the one where the sunray passes (Colomboni 1669, pp. 635–636; our trans.).

As Italian mathematician Angelo Maria Colomboni wrote in his treatise in 1669, since ancient times the mirrors have been the favorite tool used by gnomonists to

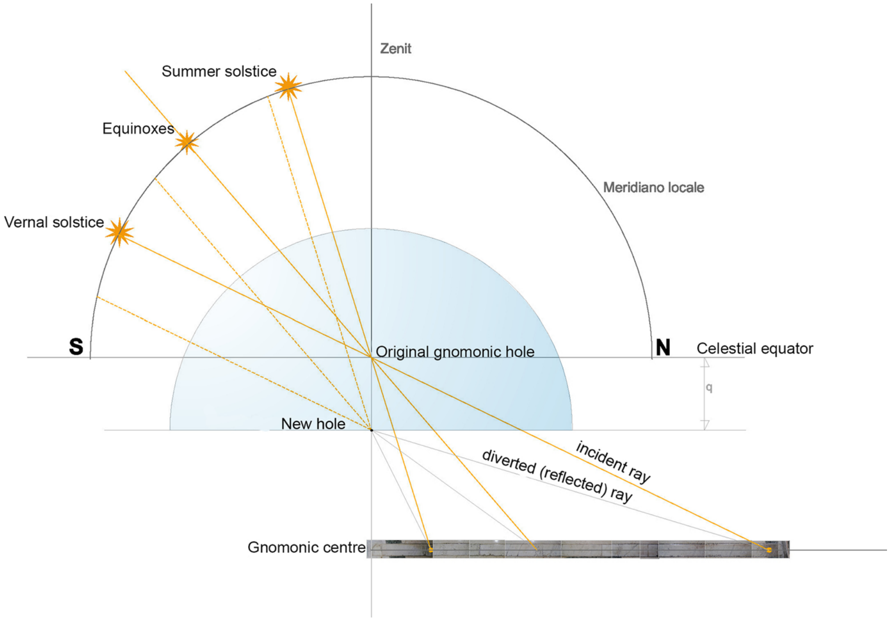

Fig. 16 New angular relationships between the gnomonic hole and the astronomic data engraved along the meridian line. Image: Alessandra Pagliano

bring light where it cannot enter naturally. Such sundials built with mirrors are called "catoptric".

In our two case studies we propose to reactivate the meridian lines adopting the same methodology used for centuries for catoptric sundials, based on geometry and optics whose laws describe the phenomenon of the reflection of light. If the mirror, regardless of its inclination, is placed in a position such that the line perpendicular to the surface belongs to the vertical plane of the local meridian (called π in Fig. 17, determined by the circle on the celestial sphere defined as the locus of points through which the sun passes at noon), the reflected ray also belongs to the same plane. The normal line divides the angle between the incident ray and the reflected ray into two equal angles. This law allows us to operate with the diverter in the same vertical plane to which the sun's rays belong every day at noon. In the two case studies presented here, the "natural" alignment between the sun, the gnomonic hole and the point on the sundial corresponding to the date, is interfered with by the obstruction of the gnomonic hole along the outer façade. It is therefore necessary to locate on plane π a new point, called the "reflection gnomic center" (point C), which will be the foot of the perpendicular to the plane of reflection of the sun's ray, and whose function will be similar to the role of the original gnomonic hole (point F). The elevation above the floor and the distance from the original gnomonic hole can be fixed depending on the characteristics of the intervention site (Fig. 17).

If we consider a generic day of the year, we uniquely identify on the meridian line a point (point M), highlighted by the luminous macula; this point is determined by the intersection of the floor and the line m (passing through the gnomonic hole

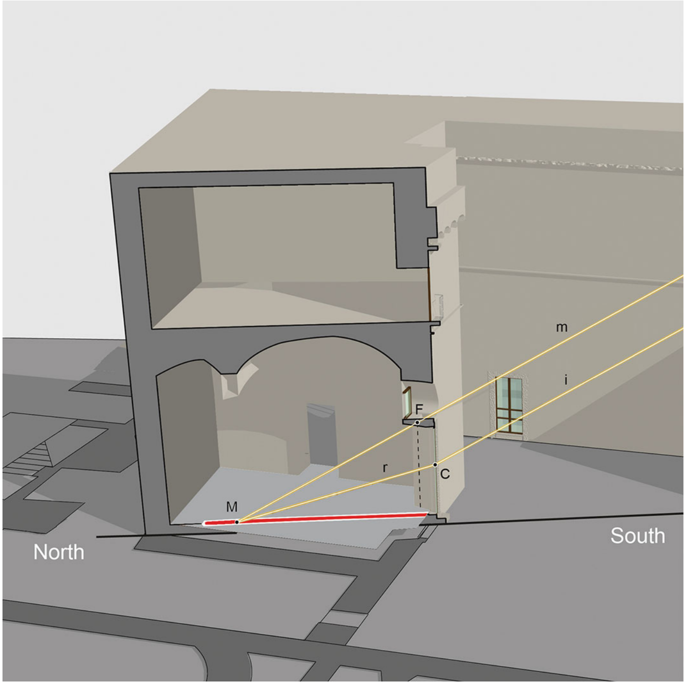

Fig. 17 Perspective section (with plane π) of a hypothetical room with a meridian line on the floor, showing: point F, gnomonic hole; point M, luminous macula on a generic day of the year; point C, reflection gnomic center; lines m and line I, solar rays on a generic day of the year; line r, reflected ray. Image: Angelo Triggianese

F), whose slope is related to the local latitude and the solar declination. The daily value of solar declination can be calculated using Cooper's formula, which will be discussed in greater depth in the following section. The position of point M is therefore necessary to determine the inclination of the mirror so that the incoming light beam (intercepted at a different height from that of the original gnomonic hole) can assume the reflected direction coinciding with the line r in Fig. 17. For noon each day of the year, point M can easily be extrapolated from data engraved on the meridian line, line m is determined by the local solar declination, and point C can be chosen arbitrarily according to the most suitable location for the solar diverter. Line r is the line joining two fixed points (point C, the reflection gnomic center, and point M along the meridian line); the inclination of the reflection plane is the bisector of the angles between the lines r and I (line I is parallel to line m).[1] First we must choose the position of the diverter tool; then, due to the continuous variable inclination of incoming solar rays, a specific daily adjustment of the reflection plane is required. To simplify these calculations and to be able to control such operations in the planning stage, we took advantage of a parametric modeling software, such as

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[1] According to the first law of reflection the incident ray, the reflected ray and the perpendicular to the plane lie on the same plane; for the second law of reflection the angle of incidence and the angle of reflection are equal.

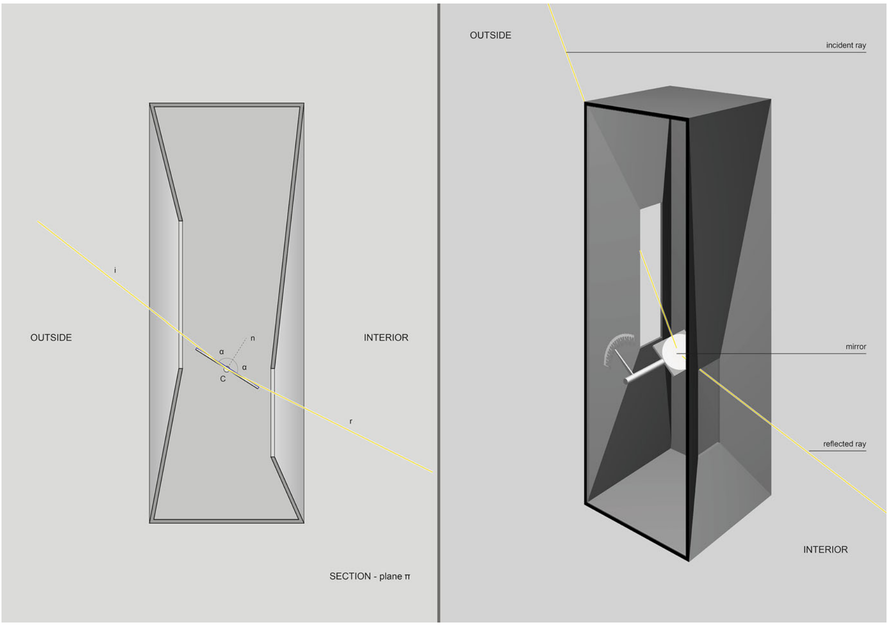

Fig. 18 *Vertical* section and perspective section of the gnomonic diverter with one mirror inside. Image: Angelo Triggianese

Grasshopper®, a graphical editor of algorithms tightly integrated with graphical tools of virtual drawing. This tool, in addition to being the basis of modern parametric design, allows a significant reduction of the design/drawing operations of a correct design process because it is possible, in a fast way, to repeat the same graphics operations by setting the parameters in order to obtain the correct configuration in function of their modification. In this way, we can obtain variable three-dimensional configurations starting from selected parameters.

Day by day, starting from the inclination of the sun's rays and the characteristics of the sundial, the software can automatically generate the geometric configurations described above in order to identify the inclination of the mirror in the desired position. Setting the date as the parameter that regulates the graphics operations, we can immediately see the three-dimensional model as a benefit of a direct control of the design. At the same time, changing the elevation and the distance from the original gnomonic hole to the reflection gnomonic center, we can test the best position of the instrument in relation to its operation and in function of the site's architectural features (Fig. 18).

Thanks to the parametric design, the scheme described becomes a simple methodology that can also be applied in other contexts with similar problems. In fact, by changing only the parameters that depend on the environment (such as the latitude or the geometrical-formal aspects of the existing space), the tool is immediately adaptable to any site, capable of repeating all geometric operations required to obtain the new spatial configuration. We can manually regulate the daily adjustment of the mirror inclination, but we can also design an automated mechanism so that, without any manual operation, mirrors rotate, day by day, by the necessary angle to function correctly. This second design step tries, in a growing trend, not only to experience virtuality in an abstract way, but to generate a dynamic relationship between the digital space and the real world outside. It can be a source of input capable of influencing the calculations of the processor both of the place in which the digital processes and are implemented and then in the concretization as output. In order to link the virtual environment to the real world, in addition to the tools programmed ad hoc by computer developers, there are special plug-ins of modeling software that transcribe graphics operations into computer language, so as to send commands to a processors and, consequently, to manage the movement of the actuators. In our case, the mirror, in function of the day, will assume its proper inclination based on the calculations of the software that parametrically modifies the situations derived from the context by acting directly on the three-dimensional model in a virtual environment. Regarding the design of the movement system of the mirror, we therefore consider it appropriate to perform the required operations by actuators electronically controlled by the Arduino platform (https://www.arduino.cc/). This tool, used to build interactive objects, cyclically executes the commands provided by the programming language to operate the devices connected to it. At the same time, thanks to sensors, it is able to receive input from the external physical world to fit and respond to stimuli. Among the endless options that electronics makes available to designers, we prefer to use Arduino not only for its ease of use but also thanks to the close link that it can establish with the three-dimensional design software, making a direct relationship between the technological capabilities and the tools of architectural drawing. Arduino can be managed using plug-ins of the same software used for the design of the sundial with which we are dealing. In this way, we can easily program the board that, receiving commands via a direct connection to a PC, stores the pieces of information without requiring any additional external computer connections. Thanks to the use of a small battery, the board can cyclically repeat the commands of movement to the actuators that will properly rotate the mirror. Consequently, the tool does not require any external connection and, accordingly, no external wire will be needed: the presence of cables would hinder its insertion in a monumental context in proximity to a window with openable wings. Thanks to the small size of the Arduino board (about 68.6 mm × 53.3 mm × 10 mm) and of the motion actuators, they can be placed within the designed object without increasing its size.

The methodology just described ensures the correct refunctionalization of any meridian line because, by exploiting the simple laws that regulate the principles of the reflection between mirrors, it allows diversion of the inclination of sunrays in a controlled and accurate way. Although from a theoretical point of view the diverter can be placed any position, our experience showed that the greater the amount incoming light that grazes the mirror, the less bright the solar disk will appear on the floor. The reflected beam, although different from the line m, tends to graze the surface of the mirror much more as the reflection gnomonic center C draws closer to the original hole F. The elliptical luminous macula on the meridian line, albeit in a correct position, assumes a more elongated shape and is characterized by a feeble brightness, with the consequent difficulty in reading the time. Thus we considered modifying the deflector tool, maintaining the principles of its operation but adding a further mirror: in this way, the incoming light beam will undergo a double deviation in order to assume the required inclination outgoing. As a result, the macula obtained on the floor has a shape close to the original one and, above all, its brightness is not compromised, allowing an immediate and effective reading. We added an extra variable value, that is, the distance (d) between the two centers of reflection C_1 and C_2 (Fig. 19).

We want to emphasize that this change does not alter in any way the geometric process described above. Parameterization, calibrated inserting the new elements added in this configuration, allows us to instantly calculate the inclination of both mirrors as a function of the same parameters of the initial solution (date, position of

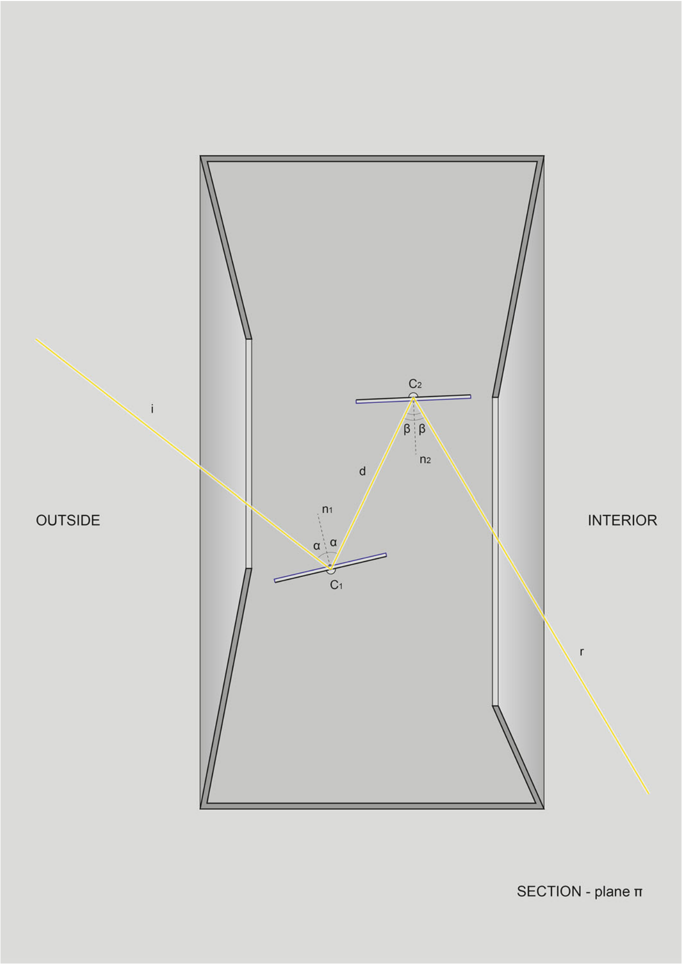

Fig. 19 *Vertical* section of the gnomonic diverter with two mirrors inside. Image: Angelo Triggianese

the gnomonic hole, position of the reflective gnomonic center) with the addition of the distance between the axes of rotation of the two mirrors. From a geometrical and gnomonic point of view, nothing contradicts the success of refunctionalization. However, because the meridian line is closely linked to the architectural context, we cannot accept a solution that does not consider the need to safeguard and respect the historical and architectural value of the hosting monument. It is thus essential to check that the adopted solutions do not lead to an excessive alteration of the status quo. We do not want to fool observers into believing that the sun directly strikes the gnomonic hole, and so we prefer to make visible the technological apparatus, which unmistakably declares its modernity with respect to the original decorations. A small parallelepiped surrounds the mirrors and the movement mechanisms. The two opposite faces are slanted inward and characterized by two slots: the first admits inside solar rays, while the other allows reflected rays to go towards the meridian line. In the two case studies presented here the diverter could be used just because there is an existing, openable window on the surface to which the gnomonic hole belongs (or belonged): we can fix the small box to the glass surface of the window. To see the macula on the floor there is no need to darken the room, because the box casts its shadow along the meridian line and, by subtraction, the luminous macula (whose elliptical shape stems from the fact that the mirror surface is circular) is therefore easily visible within the shadow.

Given the small size (20 cm × 20 cm × 60 cm), it is important to emphasize the barely-invasive character of the addition of this object in a historic context. In addition, we can stress the complete reversibility of the intervention because it does not require any construction work to alter the existing building. Indeed, on a case-by-case basis, it is possible to envisage the use of the diverter only on the indispensable days, especially when the refunctionalization concerns only a portion of the meridian line. In fact, the diverter can be as easily moved as an ordinary piece of furniture because the device does not require any external connection cable and can work in total autonomy through simple internal batteries (Figs. 20, 21).

Fig. 20 Photomontage of the diverter object on the transparent surface of the window of the Abbey of Cava de' Tirreni. Image: Angelo Triggianese

Fig. 21 Photomontage of the diverter on the window frames of Pizzofalcone in Naples. The *box* casts its shadow on the floor, making visible the luminous macula inside. Image: Angelo Triggianese

The Cooper Formula and the Sundials

The geometrical method used to set a meridian line consists of projecting all solar declinations on the azimuth horizontal plane, usually coinciding with the floor. This strong connection to geometry allowed us to propose a mirror-based system, set according to the Cooper formula to restore the functionality ancient meridian lines. This mathematical solar formula determines, for each latitude, the daily solar declinations. To do this, we need a formula with the information on axial tilt, because it is responsible for the periodicity of the seasons and thus solstices and equinoxes.

The Cooper formula is valid for the construction of post reform sundials:

$$\delta = 23.45\sin[360(284 + Dn)/365] \text{ (Cooper 1969).}$$

As we can see, the solar declination δ depends on earth's axial inclination of 23.45°, the first multiplication factor, and the sine function, which takes into account the periodicity of seasons and solstices (the maximum and the minimum value of the function). The function argument depends on Dn, the number of a specific day in relation to the total number of days in the year. This empirical formula reproduces all solar declinations during the year, with a negligible change for leap year (simply obtained by changing the total number of days in the year from 365 to 366), regardless of all the other motions of the earth around the sun due to gravitational forces with the moon. The estimated error in the determined degree value is about $3'$, which corresponds to a few millimeters on the meridian line. The number 284 is a calibration in order to have the winter solstice on day 355 (21 December), corresponding to a solar declination of $-23.45°$; on the day 173 of the year (21 June) the Cooper formula calculates the solar declination as 23.45°, corresponding to the summer solstice.

There are other equations used to calculate the angle of solar declination, such as the much more complex Spencer formula:

$$\delta = 0.006918 - 0.399912\cos\Gamma + 0.07257\sin\Gamma - 0.006758\cos 2\Gamma + 0.000907\sin\Gamma - 0.002697\cos 3\Gamma + 0.00148\sin 3\Gamma \text{ (Spencer 1971).}$$

This is given essentially by a series expansion, in order to have sum addends of a different order. The factor Γ is equal to $2\pi\ (n - 1)/365$ and n is the day of the year, equivalent to Dn in the Cooper formula. In comparison to that of Cooper, the Spencer formula allows a reduction of the error in leap years; the error committed is less than $3'$. However, this numeric complexity, useful in other areas to calculate the energy of sunlight (for example on solar panels), is too sophisticated for the purposes discussed in this work. Furthermore, the simplicity of the Cooper formula is particularly suitable for implementing and managing the automated system of mirrors discussed above.

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References

Colomboni, Angelo Maria. 1669. *Prattica gnomonica, o vero, Tauole, con le quali ciascuno ageuolmente può far da sè gli Horologi da Sole orizontali, verticali e riflessi di qualsiuoglia grandezza.* Bologna.

Cooper, P. I. 1969. *The absorption of radiation in solar stills. Solar Energy* 12: 333-346.

Guillaume, Paul. 1877. *Essai Historique sur L'Abbaye de Cava*, Cava dei Tirreni.

D. Silvano De Stefano Abate. 1908. *La Badia della S.S. Trinità dei PP. Benedettini in Cava De' Tirreni, Guida storica illustrata*, Badia di Cava, and Forgotten Books 2013.

Pagliano, Alessandra. Murolo, Roberta. Santoro, Luca. 2014. *Disegnare il tempo: Rocco Bovi e gli orologi solari della Certosa di San Martino.* Rome.

Severino, Nicola. 2003. *Le meridiane a camera oscura di Pizzofalcone a Napoli e di Piedimonte Matese.* In Gnomonica italiana, anno II/5. Milan.

Vezzosi, Antonio F. 1780. *I scrittori de' Cherici Regolari detti Teatini*, vol.1 - 2, Roma.

Zanotti, Eustachio. 1779. *La meridiana del tempio di San Petronio rinnovata l'anno 1776*, Bologna.

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