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One-dimensional simulation of debris-flow inception and propagation

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Abstract

The paper describes the development and the assessment of a numerical model for the simulation of debris flow phenomena. Aiming at this, after having considered the causes of triggering, this work focuses on the simulation of propagation characteristics of the masses mobilized, taking into account both the actual internal dissipative processes characterizing the sediment-water mixture and the effects induced by the modification of the boundaries where the flow takes place. The propagation model implemented is based on the one-dimensional hyper-concentrated shallow flows equations, suitable to take into account also the solid and fluid mass exchanges with the bottom. This system of non-linear partial differential equations is integrated by means of a one-dimensional finite volume scheme, second-order accurate in space and time. In order to show the performance and capabilities of the model, the results of its application to a laboratory dam-break experiment are analysed, and finally the application to a real-world test-case is discussed.

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Keywords: debris flows; triggering; hyper-concentrated shallow flows; finite volume method; Riemann problem.

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1. Introduction

Debris flows are among the most dangerous natural phenomena, due to the detrimental consequences in terms of human injuries and economical losses related to the damage of buildings, industrial facilities and infrastructures. The study of such hazardous phenomena is motivated by a number of needs: the identification of the causes of inception, the identification of the pathways through which these large volumes of material can propagate, the evaluation of both the velocities that can be attained by these moving masses and the thrusts that may act on buildings and infrastructures, the evaluation of the run-out distances and of the areas susceptible to these form of instability, the evaluation of the risk for the human beings.

In the scientific literature, many numerical schemes for the evaluation of the debris flow run-out and the extension of the susceptible areas have been presented. These models have often supplied satisfactory results when applied to real-world cases[1,2,3,4].

In this framework, the purpose of this paper is presenting a new numerical tool for the simulation of one-dimensional debris flows phenomena taken as a whole, including the causes of triggering, the presence of potential failure surfaces along which the instability phenomenon can originate, and the propagation characteristics of the masses mobilized, taking into account not only the actual internal dissipative processes that develop within the sediment-water mixture but also the effects induced by the evolution of the boundaries where the propagation phenomena occur.

In particular, the model proposed consists of the following modules:

- an *infiltration/groundwater module*, that allows to define the position of the water table and the soil water content, supplying the pore pressure distribution in the ground;
- a *soil stability module*, which allows the evaluation of the most probable failure surface along the slope;
- a *propagation module*, which describes the complex phenomenon of debris flow propagation, evaluating both speeds and flow depths attained by the unstable masses.

The modules that reproduce the different mechanisms that concur to the development of the debris phenomena are described hereinafter. Finally, a section is devoted to the verification of the model, considering the application to a laboratory dam-break experiment on movable bed and a realistic test-case.

2. Infiltration and groundwater modules, stability module

Rainfalls may have a crucial role in the inception of debris flows triggering mechanisms[5,6], affecting the equilibrium of a slope in different ways: i) they may cause the rise of the piezometric surface and the gradual saturation from the top to the bottom, causing the increase of the pore water pressure and the decrease of the shearing resistance of the soil; ii) they induce the increase of the soil weight, with a consequent increase of destabilizing forces; iii) moreover, the infiltration process may cause the reduction of the contribution of soil suction to the soil mechanical resistance, which can be crucial for the equilibrium of the slope, such as in the case of partially saturated covers.

In the present work, the Horton's model, which is based on the assumptions that both the hydraulic conductivity and the hydraulic diffusivity are constant and independent of the moisture content, is used in order to evaluate the effects of the infiltration process[7].

The groundwater module, instead, through a finite difference numerical scheme, provides a steady flow analysis of the water table that may arise in a generic porous media due to the presence of a spring of assigned discharge. The groundwater module is based on the assumptions of steady flow in the vertical plane, incompressible fluid, homogeneous and isotropic porous media, no-deformable solid matrix and Darcian regime. In consequence of this schematization of the groundwater processes that occur in the lowest part of the coating layer, the equations on which this module is established are the same that are used for the analysis of free surface flows in open channels, with the exception made of the dissipative process occurring within the fluid, because the flow is laminar.

If the infiltration front reaches the groundwater level, the coupling between the 1D vertical infiltration module and the 1D sub-horizontal groundwater module is made considering a mass source term along the groundwater flow.

In this paper, in order to model both the failure stage of a debris avalanche and the possible inception of debris flows during the propagation phenomenon, a limit equilibrium analysis is carried out according to Bishop's method[8]. This method assumes that the border of the failure domain is defined by an arc and the method for the evaluation of the safety coefficient is iterative, because the non-linearity of the constitutive law does not allow an explicit solution of the system. The soil stability analysis is carried out every l time intervals, with l value decided a priori and the failure domain, which turns into the bed over which the debris flow propagates, is obtained as an envelope of all the critical surfaces for which a safety factor lower than one has been found.

3. Propagation module

3.1. Description of the mathematical model

The mathematical model used to describe the propagation of hyper-concentrated one-dimensional shallow flows, in presence of sediment transport and erodible bed, is the following[9]:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}(\boldsymbol{U})}{\partial x} + \boldsymbol{H}(\boldsymbol{U})\frac{\partial \boldsymbol{U}}{\partial x} = \boldsymbol{S}(\boldsymbol{U}) \qquad (1)$$

where

$$\boldsymbol{U} = \begin{bmatrix} h \\ \rho h \\ \rho h u \\ z \end{bmatrix} \; ; \; \boldsymbol{F}(\boldsymbol{U}) = \begin{bmatrix} hu \\ \rho h u \\ p + \rho h u^2 \\ 0 \end{bmatrix} \; ; \; \boldsymbol{H}(\boldsymbol{U}) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho g h \\ 0 & 0 & 0 & 0 \end{bmatrix} \; ; \; \boldsymbol{S}(\boldsymbol{U}) = \begin{bmatrix} \frac{N_b}{c_b} \\ \frac{\rho_b N_b}{c_b} \\ -\rho g h S_f \\ -\frac{N_b}{c_b} \end{bmatrix} \qquad (2)$$

In Eqs. (1) and (2), x and t are the space and time independent variables, respectively; **U** is the vector of the conserved variables; **F**(**U**) is the vector of the fluxes; h is the mixture depth; u is the vertically averaged mixture velocity; ρ is the vertically averaged mixture density; p is the hydrostatic thrust, defined as $p = 1/2\rho g h^2$; g is the gravity acceleration; z is the bed elevation; N_b is the net volume of sediment transferred from the erodible bed to the flowing water-sediment mixture per unit time and unit bed surface area; S_f is the friction slope; c_b is the sediment concentration in the saturated bed; ρ_b is the density of the saturated bed. These equations form a non-linear system of hyperbolic partial differential equations, which can develop discontinuous solutions also starting from continuous initial conditions. Furthermore, in addition to the discontinuities of the flow field (hydraulic jumps, propagating bores), the bed elevation z can be discontinuous for the presence of artificial and natural bed sills, trenches and deep excavations[10,11].

The net flux of sediment transferred from the erodible bed to the flowing water-sediment mixture is defined as:

$$N_b = E - D \qquad (3)$$

where E and D are the sediment entrainment and deposition fluxes, respectively. These fluxes are evaluated by means of [12]:

$$E = \begin{cases} \beta(\vartheta - \vartheta_c)|u|h^{-1}d^{-0.2} \\ 0, \quad \vartheta \le \vartheta_c \end{cases} \quad ; \quad D = \alpha\, c\, \omega_0\, (1 - \alpha\, c)^m \qquad (4)$$

In Eq. (4), ϑ is the Shields parameter; ϑ_c is a threshold value of the Shields parameter; β is the erosion parameter; d is the sediment equivalent diameter; ω_o is the settling velocity; m is the coefficient of the hindered settling; α is a coefficient for the evaluation of the sediment concentration near the bed; c is the sediment concentration in the flowing mixture; $c_b = 1 - \nu$ is the sediment concentration in the saturated bed, and ν is the bed porosity. The density ρ of the flowing mixture, the density ρ_w of the water, the density ρ_s of the sediments and the sediment concentration c are related by $\rho = \rho_w + c(\rho_s - \rho_w)$, while the density ρ_b of the saturated bed and the concentration c_b of the sediment in the saturated bed are related by $\rho_b = \rho_w + c_b(\rho_s - \rho_w)$.

With the aim of taking into account the actual dissipative processes within the flowing mixture, the rheological model considered in this work is the Visco-Plastic-Collisional model[13]. This model includes yield, viscous, collision, and turbulent stress components, and the corresponding friction slope S_f can be evaluated as follows:

$$S_f = \frac{\tau_y}{\gamma_m h} + \frac{K\eta u}{8\gamma_m h^2} + \frac{n_M^2 u^2}{h^{4/3}} \quad (5)$$

where n_M is the Manning coefficient; K is the viscous resistance parameter; γ_m is the specific weight of the sediment mixture; η is the fluid viscosity; τ_y is the yield shear stress.

3.2. Proposed numerical procedure

In this work, we consider a second-order extension of the numerical model described in[9], which generalizes to movable beds the geometric source terms treatment procedure introduced in[14]. The second-order accuracy in space and time is achieved using a MUSCL-Hancock type scheme[15,16]. First, the conserved variables are reconstructed linearly in each cell, and limited. Then, the variables are extrapolated in time using an half time step, and finally a Riemann problem is solved at the interfaces between cells with initial data consisting of the evolved boundary extrapolated values. In order to avoid the expected spurious oscillations, a TVD constraint is enforced in the data reconstruction step by limiting the slopes of the linear reconstructions.

The Riemann solver used for the evaluation of the advective fluxes is the HLLC-type solver introduced by[9], where the geometric source terms are taken as a part of the hyperbolic problem. In order to take into account the friction and erosion-deposition source terms, the inhomogeneous problem is solved by means of the classic *Strang Splitting Method*, which is second order accurate:

$$\boldsymbol{U}^{n+1} = S^{\left(\frac{\Delta t}{2}\right)} A^{(\Delta t)} S^{\left(\frac{\Delta t}{2}\right)} (\boldsymbol{U}^n) \quad (6)$$

In Eq. (6), S and A are second-order accurate operators which are solved separately for the source and the advective terms, respectively.

4. Numerical tests

In order to test the suitability of the proposed model to analyse the debris flow initiation, propagation and arrest, several tests have been carried out by the authors. For the sake of brevity, in this work only two tests are shown: an experimental laboratory dam-break, and a realistic debris-flow phenomenon that could occur in the Coroglio Area (Napoli, Italy) as a result of prolonged rains.

4.1. Dam-break on movable bed

This test is devoted to verify the capability of the model to cope with movable beds in realistic cases, and for this reason it is applied to a laboratory experiment described in Capart and Young[17], where the experimental data has been obtained by digitizing the plots represented in Evangelista et al.[18].The test is run considering a horizontal rectangular channel of length L = 1.2 m, width b = 0.2 m, and with the vertical gate located at x_0 = 0.6 m. The bottom is covered on the entire length with a saturated layer of light artificial pearls of uniform size, with equivalent diameter d = 6.1 mm, density ρ_s = 1048 kg/m^3, and porosity ν = 0.6. At the beginning of the experiment, a layer of tranquil water with depth h_0 = 0.1 m is present upstream the gate, and the sudden removal of the gate causes the formation of a dam-break wave that erodes the movable bed.

In order to take into account the bed friction and the effects of the erosion and deposition of the bed material, the complete Eq. (1) is considered, with source terms defined in Eq. (2). In particular, the bed friction has been evaluated by means of the Manning formula, with $n_M = 0.02$ s m$^{-1/3}$. The values assumed by the parameters figuring in Eq. (4) are, respectively: $\theta_c = 0.045$; $\beta = 0.02$ m$^{1.2}$, obtained after a brief calibration; $\omega_o = 0.076$ m/s; $m = 2.0$; $\alpha = min\{c_b/c\,;2\}$.

The numerical solutions at $t = 0.4$ s and $t = 0.5$ s are obtained, considering $N = 240$ finite volumes ($\Delta x = 0.005$ m; $\Delta t = 0.0001$ s). In order to take into account the dry cells, the limit depth is set to $\varepsilon_h = 10^{-4}$ m.

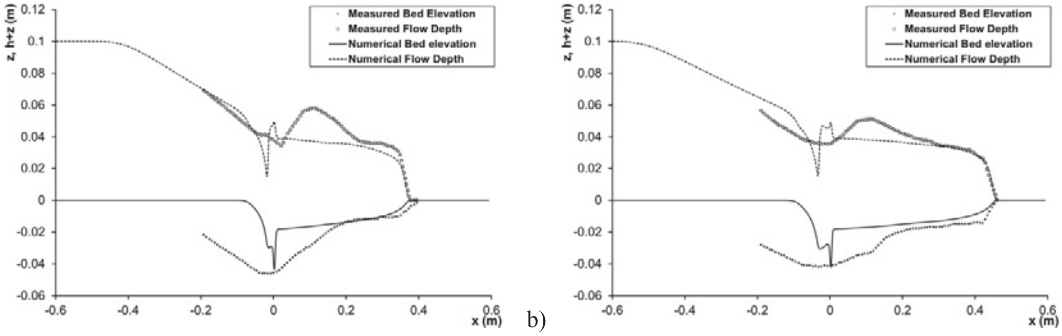


Fig. 1. Comparison between numeric and laboratory results at instants a) t = 0.4 s; b) t = 0.5 s.

In Fig. 1, the comparison between the numeric and experimental results shows a satisfactory agreement between the wave-front celerity positions and an overall agreement between free-surfaces profiles and bottom profiles. A V shaped free-surface profile is present close to the dam location due to the change in the sign of the bottom slope downstream the point of maximum excavation, representing a hydraulic jump that is present also in the laboratory experiment by Capart and Young[17]. No *ad hoc* numerical trick, as small initial water depth in dry cells, has been used in order to tackle the propagation over dry bed: nonetheless, no negative water depth appears because the Riemann solver used is depth-positivity preserving.

4.2. Case study - debris flow formation and propagation

The case presented here refers to a slope-parallel transect of the Posillipo hill in Napoli (Italy), in the district of Fuorigrotta (Coroglio Area). The versant in exam is very steep, with slopes ranging from 45% to 80% with respect to the horizontal. Regarding the stratigraphy of the slope, it consists in a surface layer of loose pyroclastic soil, resting on a tuff formation that is assumed as a non-erodible bedrock, not affected by instability phenomena during the simulation.

4.2.1. Case study parameters

In order to evaluate the triggering of the instability phenomenon, the storm has been deduced from the following expression of the IDF curve, assuming that the basin in exam falls within the pluviometric sub-area "A1-Litoranea", as identified in the hydrologic report of the *Piano Stralcio per l'Assetto Idrogeologico* of the Autorità di Bacino regionale della Campania Centrale:

$$i_{\delta,T} = \mu_{i_\delta} K_T = \frac{I_0}{\left(1+\frac{\delta}{d_c}\right)^{C+DZ}} \cdot K_T \tag{7}$$

where $i_{\delta,T}$, (*mm/h*), is the annual maximum of the rainfall intensity averaged over the duration δ, (*h*), and return period *T*, (*Years*); $\mu_{i_\delta} = \mu_{i_\delta}(\delta)$, (*mm/h*), is the law with which the expected value of i_δ, the annual maximum of the rainfall intensity averaged over the given duration δ, varies with the duration itself; $K_T = 3.06$, is the growing factor for the assigned return period, T = 100 Years; $I_0 = 89.447\ mm/h$, is the average of the annual maximum of the instantaneous rainfall intensity; $d_c = 0.2842\ h$, is the "characteristic" duration; Z = 100 m, is the average altitude of the basin on the mean sea level; C = 0.758 and D = - 0.000145 m^{-1}, are model parameters, calibrated by a regionalization technique.

The parameters appearing in the Horton's model have been evaluated referring to the soil classification provided by the Soil Conservation Service[19]. In particular, it has been assumed that the soil in exam falls in the B category, "*soil with moderately low runoff*". In this condition, the infiltration parameters are, respectively: $f_0 = 200\ mm/h$; $f_\infty = 12.7\ mm/h$; $k = 2\ h^{-1}$.

With reference to the the groundwater model, a value of the permeability coefficient consistent with the type of soil found along the slope has been estimated, putting it equal to 10^{-5} m/s. The geotechnical parameters assumed for the stability analysis, referring to the surface layer of loose pyroclastic soil, are: $\gamma_{natural}$ = 14.42 kN/m^3, is the specific weight in natural conditions; γ_{dry} = 12.75 kN/m^3, is the specific weight in dry conditions; $\gamma_{saturated}$ = 17.71 kN/m^3, is the specific weight in saturated conditions; c' = 12.70 kPa, is the cohesion; φ = 34°, is the friction angle; W = 0.131, is the water content; n = 0.506, is the porosity; e = 1.03, is the void ratio; Sr = 0.331, is the saturation degree. These geotechnical parameters have been determined at the former Geothechnical Laboratory of the University of Naples Federico II, Department of Hydraulic, Geothecnical and Environmental Engineering (now Department of Civil, Architectural and Environmental Engineering), using numerous samples directly taken in the course of inspections.

4.2.2. Stability analysis

In order to evaluate the triggering of the instability phenomenon, several storms of different durations have been considered and, for each storm, the progress of the moistening front has been evaluated. With reference to the groundwater table calculation, a natural wellspring, with discharge equal to 0.05 l/s, has been considered in a fixed point of the versant, thus generating a flow through the porous media.

After having evaluated, for each storm duration, both the depth of the moistening front and the groundwater table, a stability analysis has been performed, according to the Bishop's method, and a storm duration $\delta = 36\ h$ critical for the inception onset has been found. This duration falls in the interval (32-40) h, and it is consistent with other evaluations carried out for other debris flow phenomena occurred in similar geologic contexts[20].

4.2.3. Debris flow propagation

The propagation and run-out analysis has been performed by means of the second-order accurate finite volume scheme described previously, after having defined the failure domain and the volumes susceptible to propagate downstream. The parameters appearing in the model have been assumed as it follows:

- O'Brien & Julien formula: n_M = 0.03 $m^{-1/3}$ s; K = 24; η = 0.574 Pa · s; τ_y = 71.69 N/m^2;
- Cao sediment transport model: β = 0.0001 $m^{1.2}$; ρ_s = 2680 kg/m^3; d = 1.82 mm; $\vartheta_c = 0.045$; $\omega_0 = 0.095\ m/s$; m = 2; $\nu = 0.506$.

At the beginning of the simulation, the masses are at rest, with solid concentration equal to c_b = 0.494. The analysis has been performed by assuming time step of Δt = 0.001 s, with a spatial grid of N = 3200 cells of width Δx = 0.454 m.

Hereinafter, for the sake of brevity, only few significant snapshots of the numerical results are shown:

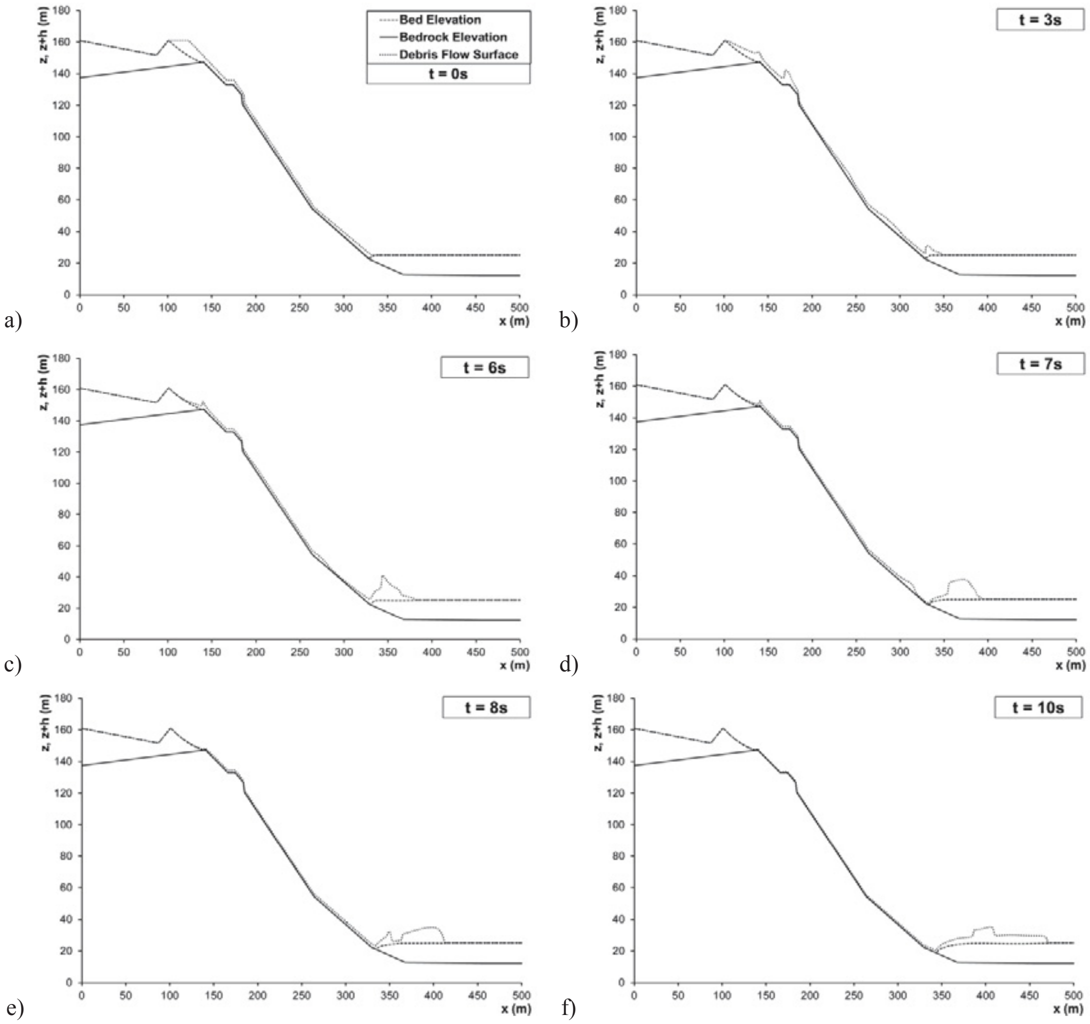
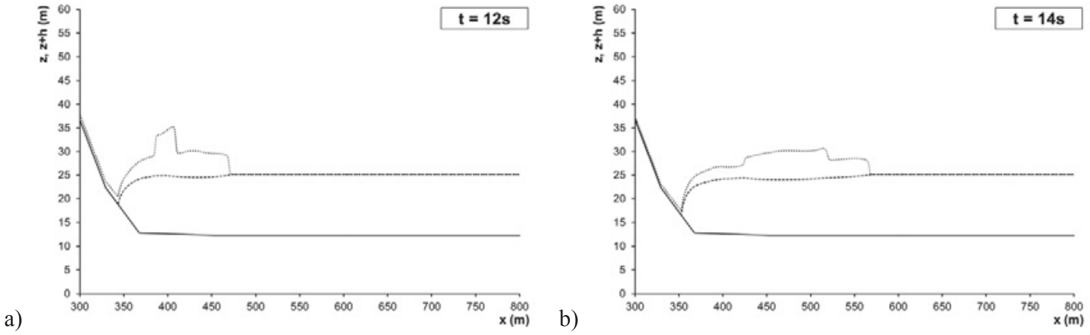


Fig. 2. Debris flow propagation. Snapshots at a) t = 0 s; b) t = 3 s; c) t = 6 s; d) t = 7 s; e) t = 8 s; f) t = 10 s.



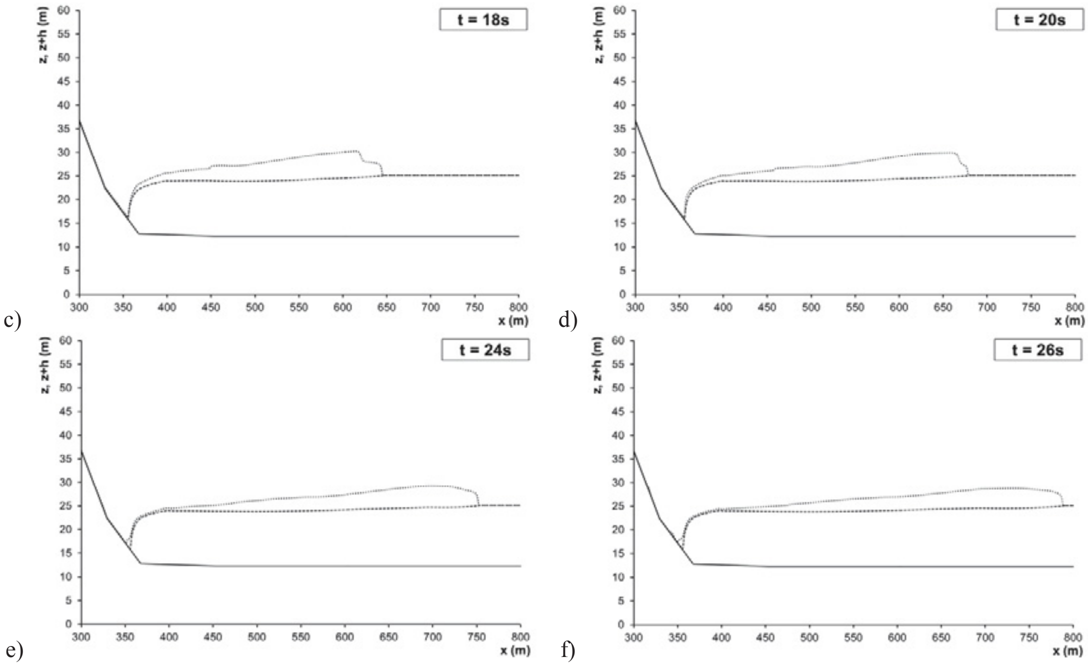


Fig. 3. Debris flow propagation. Snapshots at a) t = 12 s; b) t = 14 s; c) t = 18 s; d) t = 20 s; e) t = 24 s; f) t = 26 s.

With reference to the run-out process, in Fig. 3 we observe the stretching of the flow body, the reduction of flow depth, velocity, and of the steepness of the front, which appear to be a clear symptom of the incipient arrest of the debris flow.

In Figs. 4 and 5, the velocities and the thrusts attained by the flowing mixture are shown at times t = 3, 7, 10, 12, 20 and 26 s :

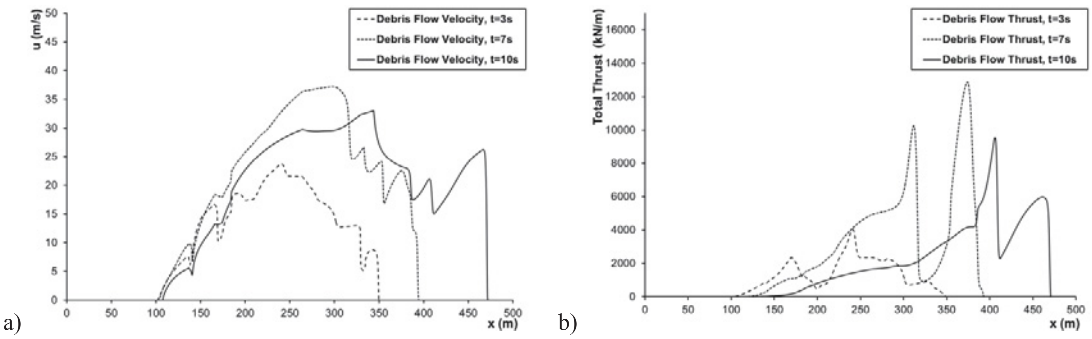


Fig. 4. a) Mixture velocity; b)Total thrust. Snapshots at t = 3 s, t = 7 s, t = 10 s.

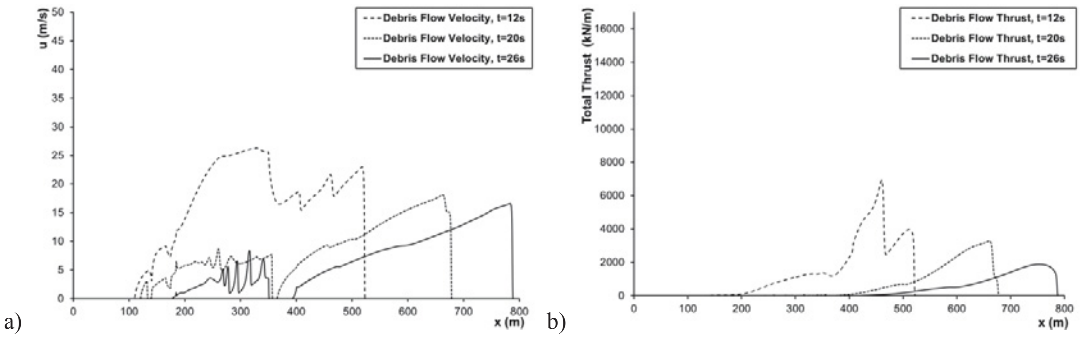


Fig. 5. a) Mixture velocity; b)Total thrust. Snapshots at t = 12 s, t = 20 s, t = 26 s.

The great destructiveness of a debris flow is apparent especially at the beginning of the propagation phenomenon, Fig. 4, the velocities range from 36 m/s on the steepest section of the slope to 25 m/s in the plane downstream, while total thrusts range from 14000 kN/m to 7000 kN/m, respectively. Instead, during the run-out phenomenon, Fig. 5, there is a considerable reduction of both velocities and thrusts, but not enough to avoid serious damages to people and buildings.

As shown in Figs. 2-3-4-5, the model proves to be capable to capture the typical features of debris flows phenomena[21], such as:

- the non-Newtonian internal dissipative processes;
- the propagation of the flow over a dry bed;
- the presence of steep fronts of propagations;
- the presence of rapid and sudden subsequent waves;
- the interaction between the flow and its solid boundaries;
- the formation of genuine shocks and contact discontinuities due to the strong sediment concentration gradients and the presence of geometric discontinuities of the bed;
- the run-out process and the deposition of the moving masses.

5. Conclusions

The proposed numerical model shows promising results, proving itself as tool for the analysis of debris flows phenomena. Owing to the lack of field data, the effectiveness of the model to describe the whole process has been demonstrated only from a qualitative point of view. In order to apply this model to real-world case-studies, a calibration of the parameters is needed. This is achievable only with a proper and wise geotechnical characterization of the properties of the soils in exam, as well as of the rheological properties of the volumes that can propagate downstream.

Further object of the future development is the extension of this model along three-dimensional slopes, in order to better characterize the phenomenon in the case of topographies closer to the real ones.

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