The Isgur-Wise Function from Charmed Decays of B-Mesons.

F. BUCCELLA [1], F. LOMBARDI [1][2] and P. SANTORELLI [1][2]

[1] *Dipartimento di Scienze Fisiche, Università di Napoli*
Mostra D'Oltremare pad. 19, I-80125 Napoli
[2] *INFN, Sezione di Napoli - Mostra D'Oltremare pad. 16, I- 80125 Napoli*

(ricevuto il 7 Gennaio 1992; approvato il 23 Aprile 1992)

Summary. — The semi-leptonic and two-body decays of B-meson with one charmed final state are described in the framework of Isgur and Wise theory by assuming $\xi(w^2) = 1 + bw^2 + cw^4$ and by taking for D and D* the physical masses rather than a common value m_c. With respect to the parametrization already proposed, pole or exponential, we get better predictions for the nonleptonic decays with $b = 0.95 \pm \pm 0.11$ and $c = 0.46 \pm 0.12$. By releasing the theoretical requirement $\xi(0) = 1$ we find $\xi(w^2) = (0.89 \pm 0.12) + (0.62 \pm 0.37)w^2 + (0.25 \pm 0.27)w^4$ with $\xi(0)$ smaller by one standard deviation from the theoretical value.

PACS 14.40.Jv – Charmed and other heavy mesons and meson resonances.
PACS 13.25 – Hadronic decays of mesons.

Introduction.

A new interesting class of symmetries for the hadrons containing one heavy quark (Q) has been recently discovered in the framework of QCD [1]. The decoupling of the spin of the heavy quark Q from the spin of the light quarks (q) and the gluons in the limit $\Lambda_{QCD} \ll M_Q$ brings to its conservation; in that limit with an appropriate transformation one may also get a Lagrangian where the dependence of the mass of the heavy quark is absent obtaining also the invariance with respect to $SU(N_Q)$, where N_Q is the number of heavy flavours. One has indeed invariance with respect to $SU(2N_Q) \times SU(2N_Q)$ where the two algebras act on the heavy quark and antiquark, respectively.

As a consequence of the transformation properties under these symmetries of the vector and axial currents we may obtain many relationships for their matrix elements between hadrons containing one heavy quark ($Q\bar{q}$, Qqq', heavy hadrons) and the vacuum or the light hadrons and between two heavy hadrons.

In particular, neglecting the corrections in Λ_{QCD}/M_Q, all the matrix elements between two heavy mesons ($Q\bar{q}$) may be obtained in terms of a universal function $\xi(w^2)$ of the velocity transfer between the two heavy quarks $w^2 = (v - v')^2$.

This prediction may be compared with the actual knowledge about the matrix elements between B and $D^{(*)}$ mesons obtained from the measured semi-leptonic decays:

(1) $$\overline{B}^0 \rightarrow D^{+(*)} e^- \bar{\nu}_e$$

and, within the factorization approximation, the branching ratios in the final states $D^{+(*)}\pi^-$, $D^{+(*)}\rho^-$.

Previous authors have faced this problem by assuming particular forms for $\xi(w^2)$ depending only on one parameter, the pole form [2]

(2) $$\xi(w^2) = \frac{1}{1 - w^2/w_0^2},$$

or the exponential form $\exp[\beta w^2]$ [3] (suggested by exponential q^2-dependence of form factors in GIWS model [4]). Although, expecially in the first case, the agreement with experiments is reasonable, in both cases the values found for the parameters are unsatisfactory: in fact the value found for w_0 ($w_0 = 0.98$) corresponds to a value for

(3) $$q^2 = m_B m_{D^{(*)}} w^2 + (m_B - m_{D^{(*)}})^2 \simeq (4.6\,\mathrm{GeV})^2$$

definitely lower than the B_c^* resonances, which are expected to be around 6.5 GeV [5]. The value found for $\beta = 0.76$ is also larger than the value expected (cf. [4]).

Here we want to disentangle the test of Isgur and Wise theory from specific assumptions on the form of the universal function $\xi(w^2)$; to this extent in the first section we write the matrix elements of the weak currents in Isgur and Wise theory in terms of the universal function $\xi(w^2)$, for which we consider different parametrizations and compare their predictions with the previous papers.

In the second section we shall concentrate our attention on the prediction of the theory $\xi(0) = 1$; we shall let $\xi(0)$ be a free parameter in the different parametrizations to compare the value found from the data with the theoretical one.

Finally we shall give our conclusions.

1. – Weak amplitudes for $\overline{B}^0$ decays with $D^{(*)+}$ in the final state.

Here we shall take into account the correction in Λ_{QCD}/m_Q which implies the use of the real mass of the concerned hadrons rather than its approximations in terms of the mass of the heavy quark contained and write:

(4) $$\langle D(v')|\bar{c}\gamma_\mu b|B(v)\rangle = C_{bc}\sqrt{m_B m_D}\,\xi(w^2)(v_\mu + v'_\mu),$$

(5) $$\langle D^*(v',\varepsilon)|\bar{c}\gamma_\mu\gamma_5 b|B(v)\rangle = C_{bc}\sqrt{m_B m_{D^*}}\,\xi(w^2)[\varepsilon_\mu^*(1 + v\cdot v') - (\varepsilon^*\cdot v)v'_\mu],$$

(6) $$\langle D^*(v',\varepsilon)|\bar{c}\gamma_\mu b|B(v)\rangle = -iC_{bc}\sqrt{m_B m_{D^*}}\,\xi(w^2)\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\nu}v^\alpha v'^\beta,$$

where, neglecting $v\cdot v'$ dependence [1],

(7) $$C_{bc} = \left(\frac{\alpha_S(m_b)}{\alpha_S(m_c)}\right)^{-6/25} \simeq 1.11\,.$$

TABLE I. – *Results obtained with the different parametrizations of the Isgur-Wise function.*

	Experimental data	$\left(1-\frac{w^2}{w_0^2}\right)^{-1}$	$\exp[\beta w^2]$	$1+bw^2+cw^4$
Fitted parameters		$w_0 = 0.98 \pm 0.04$	$\beta = 0.76 \pm 0.05$	$b = 0.95 \pm 0.11$ $c = 0.46 \pm 0.12$
$Br(\bar{B}^0 \to D^+ + e^- + \bar{\nu}_e)$	$(1.70 \pm 0.40)\%$ [2]	1.55	1.52	1.61
$Br(\bar{B}^0 \to D^{+*} + e^- + \bar{\nu}_e)$	$(4.76 \pm 0.61)\%$ [2]	5.17	5.42	5.03
$\frac{\Gamma(\bar{B}^0 \to D_L^{+*} + e^- + \bar{\nu}_e)}{\Gamma(\bar{B}^0 \to D_T^{+*} + e^- + \bar{\nu}_e)}$	(0.84 ± 0.27) [6]	1.00	0.98	1.00
$f_+(0)$	0.67 [6]	0.57	0.52	0.66
$Br(\bar{B}^0 \to D^+ + \pi^-)$	$(0.35 \pm 0.06 \pm 0.08)\%$ [7]	0.26	0.22	0.35
$Br(\bar{B}^0 \to D^{+*} + \pi^-)$	$(0.30 \pm 0.06 \pm 0.06)\%$ [7]	0.28	0.26	0.31
$Br(\bar{B}^0 \to D^+ + \rho^-)$	$(0.9 \pm 0.5 \pm 0.3)\%$ [3]	0.63	0.54	0.78
$Br(\bar{B}^0 \to D^{+*} + \rho^-)$	$(1.9 \pm 0.9 \pm 1.3)\%$ [3]	0.80	0.74	0.83
χ^2		0.54	0.91	0.49
$\Delta\chi^2_{\text{spectrum}}$		0.35	0.49	0.41

In table I we report the results of most significative fits to the three semileptonic data

(8) $$Br(\bar{B}^0 \to D^+ e^- \bar{\nu}_e) = (1.70 \pm 0.40)\% \, [2],$$

(9) $$Br(\bar{B}^0 \to D^{+*} e^- \bar{\nu}_e) = (4.76 \pm 0.61)\% \, [2],$$

(10) $$\frac{\Gamma(\bar{B}^0 \to D_L^{+*} e^- \bar{\nu}_e)}{\Gamma(\bar{B}^0 \to D_T^{+*} e^- \bar{\nu}_e)} = 0.84 \pm 0.27 \, [6],$$

to the spectrum [7] in the decay $\bar{B}^0 \to D^{+*} e^- \bar{\nu}_e$ and nonleptonic charmed two-body decays:

(11) $$Br(\bar{B}^0 \to D^+ \pi^-) = (0.35 \pm 0.06 \pm 0.08)\% \, [7],$$

(12) $$Br(\bar{B}^0 \to D^{*+} \pi^-) = (0.30 \pm 0.06 \pm 0.06)\% \, [7],$$

(13) $$Br(\bar{B}^0 \to D^+ \rho^-) = (0.9 \pm 0.5 \pm 0.3)\% \, [3],$$

(14) $$Br(\bar{B}^0 \to D^{*+} \rho^-) = (1.9 \pm 0.9 \pm 1.3)\% \, [3].$$

Dugan and Grinstein [8] have formulated a QCD basis factorization in the decays of heavy mesons (also supported by phenomenological analysis [7] using Bjorken test for $\bar{B}^0 \to D^{+*} \pi^-$ [9]). They show that the factorization holds in the Isgur and Wise limit (if we suppose that d and $\bar{u}$ in π and ρ are collinear) and corrections are suppressed by powers of Λ_{QCD}/m_b, Λ_{QCD}/m_c and Λ_{QCD}/E, where E is the total energy of the light quark pair produced from the virtual W:

$$E = \frac{m_b^2 - m_c^2}{2m_b}. \tag{15}$$

The rates concerned may be given in terms of the function $\xi(w^2)$ as:

$$\Gamma(\bar{B}^0 \to D^+ \pi^-) = \frac{G_F^2 a^2 f_\pi^2}{64\pi} |V_{bc}|^2 |V_{du}|^2 \frac{m_D}{m_B} \xi^2(w_\pi^2)(m_B - m_D)^2 (4 - w_\pi^2)^2 |\boldsymbol{p}_D|, \tag{16}$$

$$\Gamma(\bar{B}^0 \to D^{+*} \pi^-) = \frac{G_F^2 a^2 f_\pi^2}{16\pi} |V_{bc}|^2 |V_{du}|^2 \frac{m_B}{m_{D^*}} \xi^2(w_\pi^{*2})\left(1 + \frac{m_{D^*}}{m_B}\right)^2 |\boldsymbol{p}_{D^*}|^3, \tag{17}$$

$$\Gamma(\bar{B}^0 \to D^+ \rho^-) = \frac{G_F^2 a^2 f_\rho^2}{16\pi} |V_{bc}|^2 |V_{du}|^2 \frac{m_B}{m_D} \xi^2(w_\rho^2)\left(1 + \frac{m_D}{m_B}\right)^2 |\boldsymbol{p}_D|^3, \tag{18}$$

$$\Gamma(\bar{B}^0 \to D^{+*} \rho^-) = \frac{G_F^2 a^2 f_\rho^2}{64\pi} |V_{bc}|^2 |V_{du}|^2 \xi^2(w_\rho^{*2}) \frac{m_{D^*}}{m_B}(4 - w_\rho^{*2}) \cdot \tag{19}$$

$$\cdot [(4 - w_\rho^{*2})(m_B - m_{D^*})^2 + 4m_\rho^2(2 - w_\rho^{*2})] |\boldsymbol{p}_{D^*}|,$$

where

$$w_{\pi(\rho)}^{(*)2} = \frac{m_{\pi(\rho)}^2 - (m_B - m_{D^{(*)}})^2}{m_B m_{D^{(*)}}}, \tag{20}$$

$|V_{bc}| = 0.044$, $a = 1.15$ is a QCD correction factor [8], $f_\pi = 0.132\,\text{GeV}$ and $f_\rho = 0.208\,\text{GeV}$.

We report also the prediction for

$$f_+(q^2 = 0) = \frac{C_{bc}}{2} \frac{m_B + m_D}{\sqrt{m_B m_D}} \xi(w^2(q^2 = 0)). \tag{21}$$

Different parametrizations give rise to different predictions for $f_+(0)$ and, in the framework of the factorization hypothesis, for the two-body nonleptonic decays, expecially into $D(D^*)\pi$ where the transfer concerned is rather small (m_π^2).

The exponential and pole parametrizations give smaller values of $f_+(0)$ than the fitted one (0.67) [6] to semi-leptonic decays in BSW [5] model and so predict systematically smaller values for nonleptonic rates.

One now looks to a more general form for $\xi(w^2)$, namely a two-pole form

$$\xi(w^2) = \frac{\alpha}{1 - w^2/w_1^2} + \frac{1 - \alpha}{1 - w^2/w_2^2} \tag{22}$$

or a pole + dipole form [10]:

$$\xi(w^2) = \frac{\alpha}{1 - w^2/w_1^2} + \frac{1 - \alpha}{[1 - w^2/w_2^2]^2} \tag{23}$$

respectively; in both cases we fix $w_1 = 1.69$, which corresponds to $q^2 = (6.3\,\text{GeV})^2$, the expected location of the lower B_c^* resonance.

The two poles and the pole + dipole form for $\xi(w^2)$ have both the unpleasant feature that the location of the second pole, or of the dipole, corresponds to a value of q^2 smaller than the position of the B_c^* resonances.

By releasing the condition that one of the two poles be in the zone of the $B_c^*(1^-)$ resonances the two-pole fit tends to the one-pole fit with a small correction due to a pole at small w_2 ($w_2 \simeq (0.0 \div 0.4)$), the pole + dipole fit also tends to the one-pole fit.

Note that the range in q^2 available for the comparison with experiment ($[0, (m_B - m_{D^{(*)}})^2]$) is substantially lower than the threshold of B_c^* resonances; we thus expect a smooth q^2 dependence of the form factors in this range, and hence of $\xi(w^2)$ in the corresponding range ($[-0.6, 0]$ for m_D, and $[-0.5, 0]$ for m_{D^*}). Moreover, from (22) and $f_+(0) = 0.67$ [6] one would obtain $\xi(w^2(q^2 \simeq 0)) \simeq 0.5$, which implies for a nondecreasing $\xi(w^2)$ to lie in the narrow range $\approx 0.5 \div 1.0$. With these motivations we choose the form

$$\xi(w^2) = 1 + bw^2 + cw^4 . \tag{24}$$

By comparing the χ^2 values for the various fits we find that both the one-parameter fits give satisfactory predictions to the data, as well as the two-parameter fit.

By comparing the predictions for the various parametrizations of the Isgur-Wise function $\xi(w^2)$, one sees that the best predictions are obtained in the case of the parabolic fit proposed by us, which provides a larger value of $f_+(q^2)$ at small q^2.

In table I we also report the contribution ($\Delta\chi^2_{\text{spectrum}}$) of the spectra data to the χ^2. We notice that the different parametrizations, with the only exception of the exponential form, better predict branching ratios than spectrum. In fig. 1 we give the

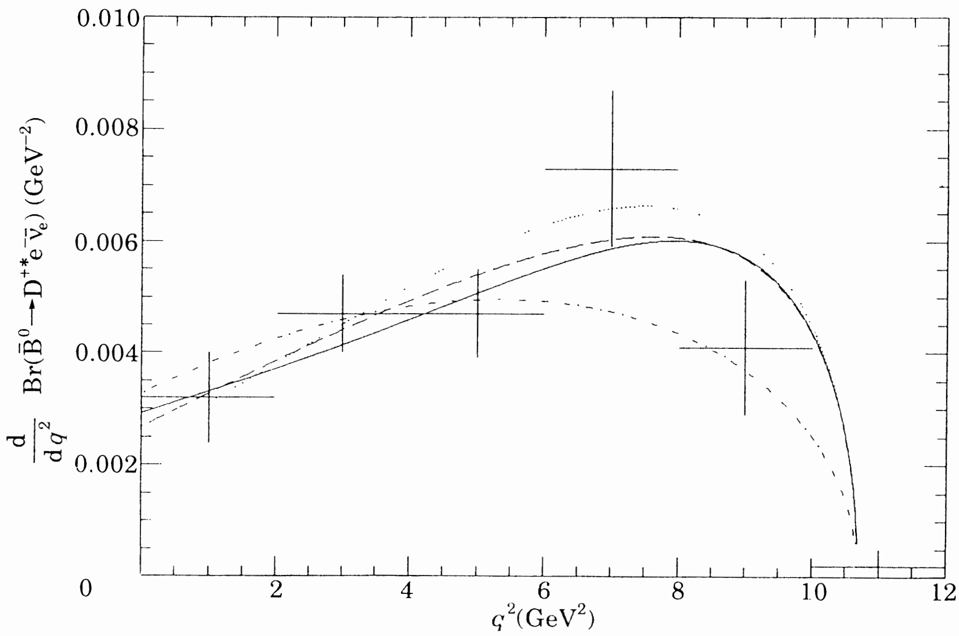

Fig. 1. – Comparison of theoretical and experimental spectra in $\bar{B}^0 \to D^{+*} e^- \bar{\nu}_e$. The full curve corresponds to the spectrum predicted by eq. (24), the dashed one by eq. (1), the dotted one by exponential form, the dash-dotted by a simple pole with $w_0 = 1.69$. Notice that $\xi(w^2(q^2 = 0)) \neq \xi(w^{*2}(q^2 = 0))$.

graph of the spectrum predicted by the different parametrizations of the Isgur-Wise function.

2. – Comparison with experiment of the prediction $\xi(0) = 1$.

As a consequence of invariance with respect to $SU(N_Q)$ and of CVC for the flavour conserving charges one has

$$\xi(0) = 1 . \tag{25}$$

The value zero for the quadrivelocity transfer between the hadrons correspond at that extreme kinematical situation where the q^2 to the lepton pair is maximum and therefore the differential cross-section corresponding to the value goes to zero for kinematical reasons: this makes a direct measure of $\xi(0)$ difficult. For this reason we release the condition $\xi(0) = 1$ and report in table II the values obtained for $\xi(0)$ by the data for the pole (with $w_0 = 1.69$ or as a free parameter), the exponential, the linear and the parabolic approximation.

We find in general a value less than 1, within one standard deviation for the parabolic fit, one and half for the pole fit with free w_0 and higher deviations for the others. In particular for the pole fit with w_0 fixed to be in the zone of $B_c^*(1^-)$ resonance ($w_0 = 1.69$) one finds $\xi(0) = 0.73 \pm 0.02$.

The results found sound as a favourable response for the Isgur and Wise theory and also for the parabolic parametrization here proposed, which give the nearest value to 1 for $\xi(0)$.

3. – Conclusions.

1) For the parametrization with two poles or a pole + dipole, once compared with data, either one recovers the one-pole prediction or one finds a solution with $w_1 = 1.69$ and a pole at $w_2 = 0.48$ or a dipole at $w_2 = 0.67$ which are physically unsatisfactory.

2) The correction of order Λ_{QCD}/m_Q of putting the physical masses for D and D* (instead of m_c) is welcome to improve the agreement with the data.

3) The test for the Isgur and Wise condition $\xi(w^2 = 0) = 1$ is rather satisfactory since the fitted values for $\xi(w^2 = 0)$ for the three parametrizations (pole, exponential and parabolic) are in the range ($0.83 \div 0.89$) and the corresponding fits are not really better than the ones obtained with $\xi(w^2 = 0) = 1$. In particular the parabolic fit gives the nearest value to 1 (0.89 ± 0.12).

One obtains also a good fit to the data with a simple pole with $w_0 = 1.69$ (the value corresponding to the position of the B_c resonance) and $\xi(w^2 = 0) = 0.73 \pm 0.02$.

In fig. 2 we compare the graph of the Isgur-Wise function in the most significant parametrizations.

TABLE II. – *Results obtained with the different parametrizations of the Isgur-Wise function by releasing the condition* $\xi(0) = 1$.

	Experimental data	$\xi(0)\left(1-\frac{w^2}{w_0^2}\right)^{-1}$	$\xi(0)\left(1-\frac{w^2}{1.69^2}\right)^{-1}$	$\xi(0)\exp[\beta w^2]$	$\xi(0)+\frac{\rho^2}{2}w^2$	$\xi(0)+bw^2+cw^4$
Fitted parameters		$\xi(0) = 0.87 \pm 0.11$ $w_0 = 1.2^{+0.3}_{-0.2}$	$\xi(0) = 0.73 \pm 0.02$	$\xi(0) = 0.83 \pm 0.07$ $\beta = 0.48 \pm 0.13$	$\xi(0) = 0.79 \pm 0.06$ $\rho = 0.75^{+0.10}_{-0.11}$	$\xi(0) = 0.89 \pm 0.12$ $b = 0.6 \pm 0.4$ $c = 0.2 \pm 0.3$
$Br(\overline{B}^0 \to D^+ + e^- + \bar{\nu}_e)$	$(1.70 \pm 0.40)\%$ [2]	1.60	1.68	1.60	1.60	1.61
$Br(\overline{B}^0 \to D^{+*} + e^- + \bar{\nu}_e)$	$(4.76 \pm 0.61)\%$ [2]	4.84	4.50	4.82	4.78	4.84
$\frac{\Gamma(\overline{B}^0 \to D_L^{+*} + e^- + \bar{\nu}_e)}{\Gamma(\overline{B}^0 \to D_T^{+*} + e^- + \bar{\nu}_e)}$	(0.84 ± 0.27) [6]	1.06	1.13	1.06	1.08	1.05
$f_+(0)$	0.67 [6]	0.60	0.65	0.59	0.58	0.63
$Br(\overline{B}^0 \to D^+ + \pi^-)$	$(0.35 \pm 0.06 \pm 0.08)\%$ [7]	0.30	0.35	0.29	0.28	0.32
$Br(\overline{B}^0 \to D^{+*} + \pi^-)$	$(0.30 \pm 0.06 \pm 0.06)\%$ [7]	0.31	0.35	0.30	0.31	0.31
$Br(\overline{B}^0 \to D^+ + \rho^-)$	$(0.9 \pm 0.5 \pm 0.3)\%$ [3]	0.71	0.82	0.69	0.67	0.75
$Br(\overline{B}^0 \to D^{+*} + \rho^-)$	$(1.9 \pm 0.9 \pm 1.3)\%$ [3]	0.86	0.95	0.86	0.86	0.85
χ^2		0.42	0.56	0.44	0.48	0.43

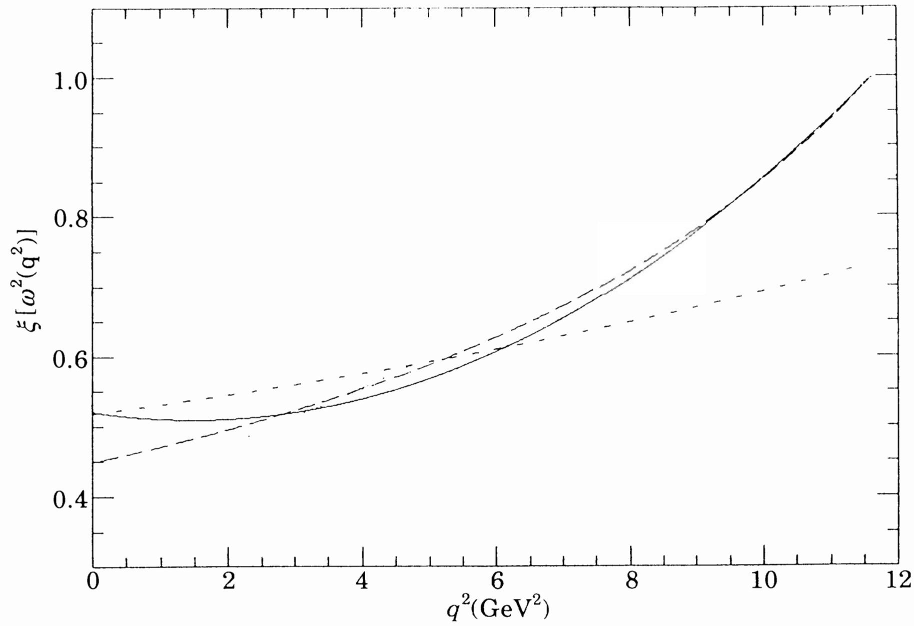

Fig. 2. – Comparison of graph of the Isgur-Wise function in different parametrization. The correspondence of the curves to the various parametrizations is the same as in fig. 1.

4) While the three parametrizations are satisfactory to describe the amplitudes for the semi-leptonic channels, it is the parabolic form which better predicts, in the framework of the factorization hypothesis, the rate for the nonleptonic decays.

REFERENCES

[1] N. Isgur and M. B. Wise: *Phys. Lett. B*, **237** 527 (1990); A. F. Falk, H. Georgi, B. Grinstein and M. B. Wise: *Nucl. Phys. B*, **339**, 253 (1990).
[2] J. L. Rosner: *Phys. Rev. D*, 42, 3732 (1990).
[3] T. Mannel, W. Roberts and Z. Ryzak: *Phys. Lett. B*, **259**, 359 (1991).
[4] N. Isgur, D. Scora, B. Grinstein and M. B. Wise: *Phys. Rev. D*, **39**, 799 (1989); T. Altomari and L. Wolfenstein: *Phys. Rev. D*, **37**, 681 (1989).
[5] M. Bauer, B. Stech and M. Wirbel: *Z. Phys. C*, **29**, 637 (1985).
[6] J. G. Körner, K. Schilcher, M. Wirbel and J. L. Wu: *Z. Phys. C*, 48, 663 (1990).
[7] D. Bortoletto and S. Stone: *Phys. Rev. Lett.*, **65**, 2951 (1990).
[8] M. J. Dugan and B. Grinstein: *Phys. Lett. B*, **255**, 583 (1991).
[9] D. Bjorken: *Nucl. Phys. Proc. Suppl.*, **11**, 325 (1989).
[10] J. G. Körner and G. A. Schuler: *Z. Phys. C*, **38**, 511 (1988).