Z. Phys. C 61, 449–452 (1994)

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Rare B-decays and heavy to light semileptonic transitions in the Isgur and Wise limit

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Received 10 May 1993

Abstract. From the experimental branching ratios for $B^- \to \rho^0 l^- \bar{\nu}_l$ and $D^+ \to \bar{K}^{*0}(\bar{K}^0) e^+ \nu_e$ one finds, in the heavy quark limit of HQET, $|V_{bu}| = (8.1 \pm 1.7) \times 10^{-3}$, larger but consistent with the actual quoted range $(2-7) \times 10^{-3}$. In the same framework one predicts for $R(B \to K^* \gamma) = (2 \pm 2)\, 10^{-2}$.

The study of the Cabibbo-Kobayashi-Maskawa [1] suppressed decays $B^- \to \rho^0 e^- \bar{\nu}_e$, interesting in itself for the determination of $|V_{bu}|$ [2], has been recently related by the spin-flavour symmetries of the HQET [3] (in the heavy quark limit, HQL) to the rare B-decays [4, 5]. In such a way the predictions for the branching ratio of the decay $B \to K^* \gamma$, which provide a test of the Standard Model [6], depend strongly on the value of $|V_{bu}|$, for which the experimental data about the $b \to u e \bar{\nu}_e$ (inclusive and exclusive) decays give an information depending also from the theoretical approach followed to evaluate the corresponding amplitudes.

Here we shall reach rather firm conclusions by following the suggestion of Isgur and Wise of relating the involved form factors by flavour symmetry [7].

The $\bar{B} \to \rho$ semileptonic decays are chosen rather than the decays with π in the final state, because in this last case the upper limit of the invariant mass of the final leptons is very near to the B^* resonance (which is expected equal to m_B in the HQL); as a consequence the pole of the B^* dominates the spectrum in that region so that the prediction of HQET fails near to the no-recoil point [8].

In the first section we review the spectrum $B \to V l \nu_l$ and give the involved form factors. In the second we get $|V_{bu}|$ by comparing experiment with the theoretical predictions. In the third are discussed the rare B decays in the HQL.

1

The spectrum in the invariant mass q^2 of the lepton pair for the semileptonic decays of a heavy meson with one vector meson in the final state ($H_j \to V_k l \nu_l$) is given by e.g. in [9] (neglecting lepton masses):

$$\begin{aligned}
&\frac{d\Gamma(H_j \to V_k e \nu_e)}{dq^2} \\
&= \frac{G_F^2 |V_{jk}|^2}{192 \pi^3 m_H^3} \sqrt{\lambda(m_H^2, m_V^2, q^2)} \\
&\quad \times \Bigg\{ |A_1^{(jk)}(q^2)|^2 \Bigg[2(m_H + m_V)^2 q^2 \\
&\quad + \frac{(m_H + m_V)^2}{4 m_V^2} (m_H^2 - m_V^2 - q^2)^2 \Bigg] \\
&\quad + |A_2^{(jk)}(q^2)|^2 \frac{\lambda^2(m_H^2, m_V^2, q^2)}{4 m_V^2 (m_H + m_V)^2} \\
&\quad - A_1^{(jk)}(q^2) A_2^{(jk)}(q^2)\, \lambda(m_H^2, m_V^2, q^2) \\
&\quad \times \frac{(m_H^2 - m_V^2 - q^2)}{2 m_V^2} \\
&\quad + |V^{(jk)}(q^2)|^2 \frac{2 q^2}{(m_H + m_V)^2} \lambda(m_H^2, m_V^2, q^2) \Bigg\}; \qquad (1)
\end{aligned}$$

where

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz), \qquad (2)$$

$$q^2 = (p_H - p_V)^2. \qquad (3)$$

The form factors of the weak currents for an initial $\bar{B}$ meson are:

$$\begin{aligned}
&\langle V_j(\varepsilon, p_j) | (A^\mu)_j^b | \bar{B}(p) \rangle \\
&= (m_B + m_V) A_1^{(bj)}(q^2) \left(\varepsilon^{*\mu} - \frac{\varepsilon^* \cdot q}{q^2} q^\mu \right)
\end{aligned}$$

$$- A_2^{(bj)}(q^2)\frac{\varepsilon^*\cdot q}{m_V+m_B}\left(p_j^\mu+p^\mu-\frac{m_B^2-m_V^2}{q^2}q^\mu\right)$$

$$+2m_V A_0^{(bj)}(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu, \qquad (4)$$

$$\langle V_j(\varepsilon,p_j)|(V^\mu)_j^b|\bar{B}(p)\rangle$$

$$=2iV^{(bj)}(q^2)\frac{\varepsilon^\mu_{\nu\rho\sigma}p^\nu p_j^\rho\varepsilon^{*\sigma}}{m_B+m_V}, \qquad (5)$$

and are related, by HQET (leading order), to the corresponding ones for the process $D\to Vl\nu_l$ if V is a light vector meson.

Isgur and Wise [7] found as a consequence of flavour symmetry the relations between the weak form factors for $\bar{B}\to K^*$ and $D\to K^*$, which, for the our parameterization, imply:

$$A_1^{(bj)}(q_B^2)=C_{bc}\left(\frac{m_D+m_V}{m_B+m_V}\right)\sqrt{\frac{m_B}{m_D}}\,A_1^{(cj)}(q_D^2), \qquad (6)$$

$$A_2^{(bj)}(q_B^2)=\frac{C_{bc}}{2}\sqrt{\left(\frac{m_D}{m_B}\right)^3}\left(\frac{m_B+m_V}{q_D^2}\right)$$
$$\times\{y_2(m_D+m_V)A_1^{(cj)}(q_D^2)$$
$$+A_2^{(cj)}(q_D^2)\left[\frac{y_1q_D^2}{m_D+m_V}-y_2(m_D-m_V)\right]$$
$$-2m_Vy_2A_0^{(cj)}(q_D^2)\Big\}, \qquad (7)$$

$$A_0^{(bj)}(q_B^2)=\frac{C_{bc}}{4m_V}\sqrt{\left(\frac{m_D}{m_B}\right)^3}\left\{\frac{2m_Vx_2}{q_D^2}A_0^{(cj)}(q_D^2)\right.$$
$$+\left[2\left(\frac{m_B}{m_D}\right)^2-\frac{x_2}{q_D^2}\right](m_D+m_V)A_1^{(cj)}(q_D^2)$$
$$\left.+\left[(m_D-m_V)\frac{x_2}{q_D^2}-\frac{x_1}{m_D+m_V}\right]A_2^{(cj)}(q_D^2)\right\}, \qquad (8)$$

$$V^{(bj)}(q_B^2)=C_{bc}\frac{m_B+m_V}{m_D+m_V}\sqrt{\frac{m_D}{m_B}}\,V^{(cj)}(q_D^2), \qquad (9)$$

where

$$q_B^2=(m_Bv-p_V)^2\quad q_D^2=(m_Dv-p_V)^2\quad v^2=1,$$
$$y_1=\frac{(m_B+m_D)}{m_D}\quad y_2=\frac{(m_D-m_B)}{m_D}, \qquad (10)$$
$$x_1=y_2q_B^2+y_1(m_B^2-m_V^2)\quad x_2=y_1q_B^2+y_2(m_B^2-m_V^2),$$

$$C_{bc}\cong\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25}\cong 1.1.$$

It is worth noticing that the q_D^2 values corresponding to the physical region for q_B^2 are not located in the q^2 allowed range for $D\to K^*$ semileptonic decays.

The symmetry breaking corrections to (6)-(9) are proportional in the full kinematical range to $\alpha_s m_L/m_H$, where α_s is the coupling constant of QCD and m_L is the mass of light quark coupled to b by the weak current; for the case considered here $V\equiv\rho$, $m_L=m_u$ and therefore we may safely neglect these corrections [10].

2

From the experimental data on the semileptonic $D^+\to\bar{K}^0(\bar{K}^{0*})e^+\nu_e$ decays, within the pole approximation for the form factors, the E691 Collaboration [11] found the following values for their residua:

$$A_0^{(cs)}(0)=0.71\pm0.16,\quad A_2^{(cs)}(0)=0.00\pm0.22,$$
$$V^{(cs)}(0)=0.90\pm0.32,$$
$$A_1^{(cs)}(0)\equiv\frac{1}{m_D+m_{K^*}} \qquad (11)$$
$$\times\{2m_{K^*}A_0^{(cs)}(0)+(m_D-m_{K^*})A_2^{(cs)}(0)\}.$$

From $SU(3)$ invariance we may identify the residua for $D^0\to K^{*-}$ with ones for $D^0\to\rho^-$ weak form factors and from (6)-(9) and the values given in (11) one may predict the form of the spectrum for

$$\frac{d\Gamma(B\to\rho e\nu_e)}{dq^2},$$

and the rate

$$\frac{1}{|V_{bu}|^2}\Gamma(B^-\to\rho^0l^-\bar{\nu}_l)=0.80\times10^{-11}\,\mathrm{GeV}. \qquad (12)$$

From the measured branching ratio $\mathrm{BR}(B^-\to\rho^0l^-\bar{\nu}_l)=(10.3\pm3.6\pm2.5)\times10^{-4}$ [12] one obtains (neglecting the errors on values of the form factors in (11))

$$|V_{bu}|=\sqrt{\frac{\mathrm{BR}(B^-\to\rho^0l^-\bar{\nu}_l)|_{\mathrm{exp.}}}{\Gamma(B^-\to\rho^0l^-\bar{\nu}_l)/\tau_B}}$$
$$=(8.1\pm1.7)\times10^{-3} \qquad (13)$$

larger but still consistent, within the experimental uncertainties, with the value found from the experimental information on $|V_{bu}/V_{bc}|=0.10\pm0.03$ [2, 13] and for the value $|V_{bc}|=0.043\pm0.003$ obtained from the study of semileptonic B-decays in the heavy quark limit [14]:

$$|V_{bu}|=(4.3\pm1.3)\times10^{-3}. \qquad (14)$$

The spectrum predicted is described in Fig. 1*.

3

A high precision prediction about the rate of the rare decay $B\to K^*\gamma$ [16], which is induced at one loop in the Standard Model, is very important, since the discrepancy

* If we ignore the location of B^* resonance and calculate the spectrum and the rate of $B\to\pi$ semileptonic decay we obtain a linear dependance of $d\Gamma/dq^2$ by the q^2. The result is very similar to the one obtained by Körner and Schuler [15]

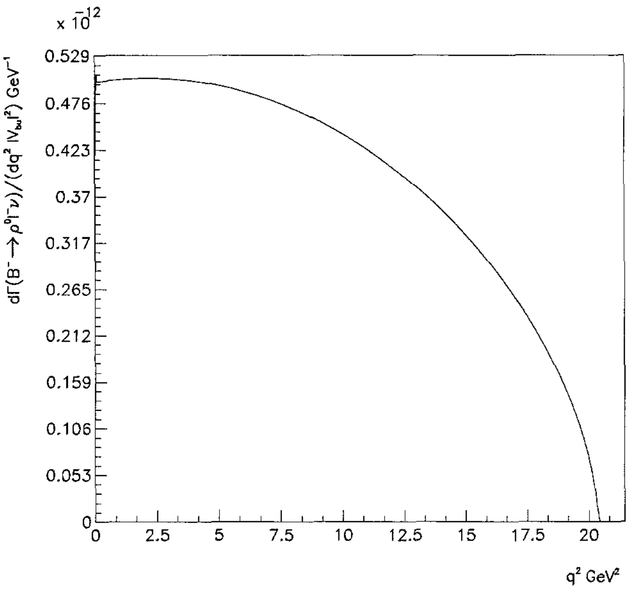

Fig. 1. We report the spectrum predicted for the decay $B^- \to \rho^0 l^- \bar{\nu}_l$

between theory and experiment would be indirect evidence of new physics.

By relating $B \to K^* \gamma$ to the semileptonic $\bar{B} \to \rho$ decay, O'Donnell and Tung [5] obtained for the ratio

$$R(B \to K^* \gamma) = \frac{\Gamma(B \to K^* \gamma)}{\Gamma(b \to s \gamma)} = \frac{m_b^3}{(m_b^2 - m_s^2)^3} \frac{(m_B^2 - m_{K^*}^2)^3}{m_B^3} \times \tfrac{1}{2}\{|F_1(0)|^2 + 4|F_2(0)|^2\} \tag{15}$$

the prediction, which differs from the one deduced by us*

$$R(B \to K^* \gamma) \left(\left. \frac{d\Gamma(\bar{B} \to \rho l \bar{\nu}_l)}{dq^2} \right|_{q^2=0} \right)^{-1} = \frac{192\pi^3}{G_F^2} \frac{1}{|V_{bu}|^2} \frac{(m_B^2 - m_{K^*}^2)^3}{(m_B^2 - m_\rho^2)^3} \frac{m_b^3}{(m_b^2 - m_s^2)^3} |\mathcal{T}|^2, \tag{16}$$

because we invoke $SU(3)_{u,d,s}$ symmetry for $V^{(bj)}(0)$ rather than for $T_1^{B \to V_j}(0) = V^{(bj)}(0)/(m_B + m_{V_j})$ as in [5].

Note that $|\mathcal{T}| = 1$ in the HQL and the corrections to the prediction of HQET in relating the form factor in (4)–(5) (with $j = u$ or s) to the one of the matrix element $\langle V_j | \bar{q}_j \sigma_{\mu\nu} q^\nu b_R | \bar{B} \rangle$ are expected to be small (cf. [5]) and we neglect them.

In effect the conclusions of the paper of O'Donnell and Tung are that the corrections to the assumption that K^* and ρ are heavy mesons are negligible (at least to relate the form factors of the matrix elements of the weak currents and the $\bar{s}\sigma_{\mu\nu} b_R$ and $\bar{u}\sigma_{\mu\nu} b_R$ operators respectively at $q^2 = 0$). The universality of the Isgur-Wise function, in principle, allow us to give, without relating the ratio R to the spectrum of $B \to \rho l \nu_l$, the $R(B \to K^* \gamma)$ in terms of the Isgur-Wise function $\xi(w^2)$ extracted, for example, by the experimental data on charmed semileptonic B-decays (cf. for example [14]). In such case the following relations hold*

$$F_1(0) = 2F_2(0), \qquad F_1(q^2) = \frac{(m_B + m_{K^*})}{2\sqrt{m_B m_{K^*}}} \xi(w^2(q^2)). \tag{17}$$

But the value

$$w^2(q^2=0) \equiv \left. \left(\frac{p_B}{m_B} - \frac{p_{K^*}}{m_{K^*}} \right)^2 \right|_{q^2=0} = -\frac{(m_B - m_{K^*})^2}{m_B m_{K^*}} \tag{18}$$

is too far from the physical range of $\bar{B} \to D^{(*)}$ semileptonic decays and the predictions depend strongly on the behaviour assumed for $\xi(w^2)$**.

A more reliable prediction for the ratio R from (16) is obtained by relating the spectra of $B \to \rho l \nu_l$ and $D \to K^*$ semileptonic decays.

Following the hypothesis of Sect. 2 about the residua of the involved form factors, $SU(2)$ flavour symmetry and (cf. (1))

$$\left. \frac{d\Gamma(\bar{B} \to \rho l \bar{\nu}_l)}{dq^2} \right|_{q^2=0} = \frac{G_F^2 |V_{bu}|^2}{192\pi^3 m_B^3} \frac{(m_B^2 - m_\rho^2)^3}{4m_\rho^2} \times |(m_B + m_\rho) A_1^{(bu)}(0) - (m_B - m_\rho) A_2^{(bu)}(0)|^2 = \frac{G_F^2 |V_{bu}|^2}{192\pi^3} \frac{(m_B^2 - m_\rho^2)^3}{m_B^3} \cdot |A_0^{(bu)}(0)|^2, \tag{19}$$

from (16), (8) (or (6) and (7)) and (11) we derive

$$R(B \to K^* \gamma) = \frac{m_b^3}{(m_b^2 - m_s^2)^3} \frac{(m_B^2 - m_{K^*}^2)^3}{m_B^3} \cdot |A_0^{(bu)}(0)|^2 \tag{20}$$

giving ($m_b = 5$ GeV and $m_s = 0.55$ GeV)

$$R(B \to K^* \gamma) = (35 \pm 28) \times 10^{-2}. \tag{21}$$

The central value is very near to the one given in [17] in the polar approximation for $\xi(w^2)$. The error quoted in (21) depends on the large error in the determination of $A_2^{(cs)}(0)$.

* I am indebted to Patrick O'Donnell and Humphrey Tung for a clarifying communication

* The first relation is more general. For example it is necessary to avoid an unphysical pole in $q^2 = 0$ for the $h(q^2)$ and $g_-(q^2)$ form factors introduced in [7]

** From the data on $D \to K(K^*) l^- \bar{\nu}_l$ decays Ali and Mannel [17] extracted the values $w_0 = 1.8$ and $\beta = 0.25$ respectively for the pole and exponential parameterization for $\xi(w^2)$

Obviously one expects the assumption that K^* and ρ are heavy less reliable than HQL for b and c quarks; thus the previous result can be modified by taking $\mathcal{T}$ from [5]:

$$R(B\to K^*\gamma)=(35\pm28)\times10^{-2}\cdot|\mathcal{T}|^2$$
$$=\begin{cases}(42\pm33)\times10^{-2} & \text{for} \quad \mathcal{T}=1.09\\ (49\pm39)\times10^{-2} & \text{for} \quad \mathcal{T}=1.18.\end{cases} \tag{22}$$

It is worth recalling that the static limit for $b\,(\gamma_0 b=b)$ and $SU(2)_{bc}$ heavy flavour symmetry imply

$$R(B\to K^*\gamma)=\frac{C_{bc}^2 m_b^3}{(m_b^2-m_s^2)^3}\,\frac{(m_B^2-m_{K^*}^2)^3}{4m_B^4 m_D}$$
$$\times\left|(m_D+m_{K^*})A_1^{(cs)}(q_D^2)\right.$$
$$\left.-\left(\frac{m_B^2-m_{K^*}^2}{m_D+m_{K^*}}\right)\frac{m_D}{m_B}\,V^{(cs)}(q_D^2)\right|^2$$
$$=(2\pm2)\cdot10^{-2}. \tag{23}$$

The result is substantially equivalent to the one dictated by HQL for b, c and s and a wave function model for the Isgur-Wise function [18].

We derived, in the HQL for b and c, the spectrum predicted for $B\to\rho l\nu_l$ and, comparing theory and experiment, we give $|V_{bu}|$.

Also relating $R(B\to K^*\gamma)$ to the $\dfrac{d\Gamma(B\to\rho l\nu_l)}{dq^2}$ at $q^2=0$ we obtained the ratio R in terms of $A_1^{(bu)}(0)$ and $A_2^{(bu)}(0)$ (or $A_0^{(bu)}(0)$), estimated by extrapolation from the corresponding form factors for $D\to K^*$ semileptonic decay.

We are more confident on the prediction for R coming from the static limit for b and $SU(2)_{bc}$ heavy flavour symmetry.

A more precise determination of $c\to s$ weak form factors is needed for a more precise evaluation for $R(B\to K^*\gamma)$.

Acknowledgement. I am indebted to Professor Franco Buccella for many enlightening discussions.

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