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# Cohesive law identification of adhesive layers subject to shear load The Twice Notched Flexure Test

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## Abstract

By the means of a general analytic solution, suitable to identify the cohesive law that characterizes the behavior of adhesive layers subject to shear load and recently published by this author, a new test configuration is presented. This test configuration, named Twice Notched Flexure Test (TNF), is thought as alternative to the traditional End Notched Flexure (ENF) test or similar ones. It is expected to have several interesting improvement features.

In order evaluating the TNF identification methodology, a de-cohesion virtual test is performed by means of detailed FE simulation. The simulation shows that, within the limits of the present model, the cohesive law can be deduced with very high precision.

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## 1. Introduction

A very popular group of test methodologies, used for the adhesives' shear cohesive law identification, consists of imposing a flexural load to test specimens made of a couple of bars glued by an adhesive layer. During the load process, some experimental outcomes are stored, such as relative displacements between the adherends, imposed forces or reactions and so on. Finally, the relevant data are combined in the framework of a de-cohesion model and the cohesive law evaluation one were searching for is attained.

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By varying the boundary conditions applied to the test specimen to be subjected to the flexural load, various well-known test methodologies can be realized. They are the End Notch Flexure (ENF) test, ASTM (2014), the End Loaded Split (ELS), ISO (2014), the 100% mode II Mixed Mode Bending (MMB), ASTM (2013), the Four Point End Notch Flexure test (4ENF), Martin and Davidson (1999), to cite only the most popular.

Recently, a general formulation for the shear cohesive law identification has been proposed by this author in Cricri (2018). If the two adherends bars are equal to each other and of rectangular section, the cohesive law  $\tau(v)$  can be evaluated using the following formula:

$$\tau(v_a) = \frac{1}{B_l} \frac{d}{dv_a} Q(v_a) \tag{1}$$

Where  $B_l$  is the width of the adhesive layer, and  $Q(v_a)$  is given by:

$$Q(v_a) = Q(v_b) + \frac{3}{4EI} M_a^2 - \frac{3}{2} \frac{V_a}{h} (v_a - v_b) - \frac{h(N_{a2} - N_{a1})}{8EI} \left( M_a - \frac{h(N_{a2} - N_{a1})}{24} \right) - \frac{4}{3EI} [M_b - M_{bR}]^2 \tag{2a}$$

$$M_{bR} = \frac{1}{4} \left( M_a - (x_b - x_a) V_a + \frac{h(N_{a2} - N_{a1})}{4} \right) \tag{2b}$$

Where  $EI$  is the flexure stiffness of each adherends. Further, in figure 1, the part of the test specimen actually used for the identification is represented, together with the symbols used in the formulas above.

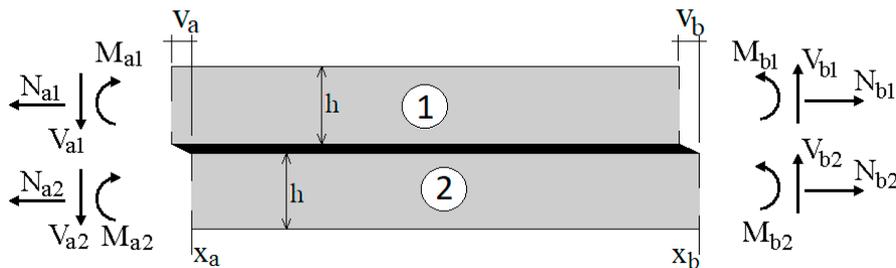


Fig. 1. Assembly of adherends with the adhesive layer

In equation (2) the unknown  $Q(v_a)$  is defined by the value that the same function  $Q$  assumes at the displacement  $v_b$ . Apparently, the calculation requires a priori knowledge of the cohesive law; In reality, the problem can be solved using an iterative algorithm, described in detail in Cricri (2018) and applied in the next section.

In this work, a new test configuration is presented, able to evaluate the shear cohesive law that characterizes the adhesive layer. This test configuration, named Twice Notched Flexure Test (TNF), is thought as alternative to the traditional End Notched Flexure (ENF) test or similar ones. Actually, the TNF configuration is obtainable with a very simple modification of the ENF test specimen, and using the same test setup.

In order to generate data to be used to evaluate the TNF identification methodology, a by shear de-cohesion virtual test is performed by means of detailed FE simulation. The simulation shows that, within the limits of the model, the cohesive law can be deduced with very high precision.

## 2. Cohesive law identification via the TNF test

The new test configuration proposed here is obtainable with a simple modification of the ENF test specimen; the setup is described in Figure 2. Unlike the standard tests cited in the previous paragraph (ENF, ELS, MMB, 4ENF), the TNF test has the following improvement features:

- 1) It presents, in the ideal case, an ever increasing load curve and is therefore always stable;
- 2) It is suitable for an exact identification using the formula (2) without any strain measurement.

A test configuration with similar stability characteristics has already been described in Davies et al. (1999), where it is called Over Notched Flexure test (ONF). Actually, the TNF test proposed here differs from the ONF test because the support on the right is placed outside the glued area. This fact, apparently marginal, allows for the exact calculation of the bending moment  $M_b$  solely based on equilibrium considerations; therefore it makes the exact identification method (2) applicable in a simple and immediate way. Finally, note that the TNF test requires the same simple setup (i.e. a three point bending device) of the standard ENF test.

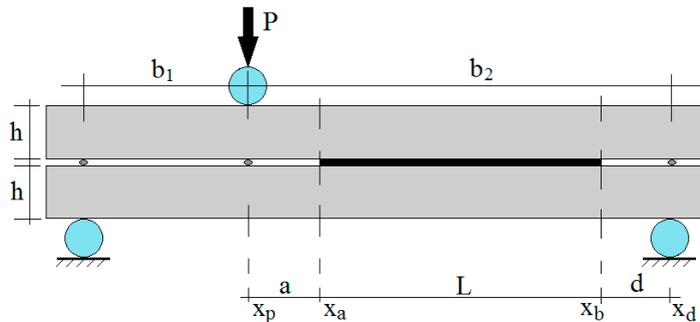


Fig. 2. Twice Notched Flexure test load scheme

With reference to Fig. 2, the internal forces that determine the cohesive law identification are the following:

$$\begin{aligned}
 N_{a2} - N_{a2} &= 0; \\
 M_a &= R_2(L + d) / 2; \\
 V_a &= R_2 / 2; \\
 M_b &= R_2 d / 2; \\
 R_2 &= P b_1 / (b_1 + b_2) .
 \end{aligned}$$

By putting these values in the general formula (2), one obtains:

$$Q(v_a) = Q(v_b) + \frac{3}{16EI} \{ [R_2(L + d)]^2 - (R_2 d)^2 \} - \frac{3 R_2}{4 h} (v_a - v_b) \tag{3}$$

From the above equation it is clear that, in order to calculate the function  $Q(v)$ , it is necessary to measure, in addition to the load curve  $P(\lambda)$  – where  $\lambda$  is a time-ordinal parameter which indicates the experimental points, only the relative displacements of the adherends’ glued surfaces  $v_a(\lambda)$  and  $v_b(\lambda)$ . These latter quantities can be measured using single point displacement sensors, as well as the Digital Image Correlation (DIC) technique; the accuracy level is related to the length of the actual sliding  $v$ , which the adhesive layer is subjected to during the de-cohesion process.

In order to evaluate  $Q(v_a(\lambda))$  from equation (3), an iterative algorithm is used, such that the unknown quantity  $Q(v_b(\lambda))$  is approximately calculated using the current approximate evaluation of  $Q(v_a(\lambda))$  itself. In detail, at each  $\lambda$ , it is possible to define the relation:

$$Q(v_b(\lambda)) = Q(v_a(\omega(\lambda))), \text{ where } \omega(\lambda): |v_a(\omega)| = |v_b(\lambda)| \tag{4}$$

Using the above equation, one can rewrite the equation (3) as follows:

$$Q(v_a(\lambda)) = Q(v_a(\omega(\lambda))) + Q_0(v_a(\lambda)) \tag{5a}$$

$$Q_0(v_a(\lambda)) = \frac{3}{16EI} \{ [R_2(\lambda)(L + d)]^2 - (R_2(\lambda)d)^2 \} - \frac{3 R_2(\lambda)}{4 h} (v_a(\lambda) - v_b(\lambda)) \tag{5b}$$

Equation (5) is suitable for an iterative evaluation of  $Q(v_a(\lambda))$  with the following algorithm, that is rapidly convergent for all tested cases:

$$Q_N(v_a(\lambda)) = Q_{N-1}(v_a(\omega(\lambda))) + Q_0(v_a(\lambda)) \rightarrow Q(v_a(\lambda)) \tag{6}$$

Finally, the cohesive law is evaluated by equation (1), where the derivative is approximated with the following finite increments ratio:

$$\tau(v_a(\lambda)) \simeq \frac{1}{B_l} \frac{Q(v_a(\lambda+1)) - Q(v_a(\lambda))}{v_a(\lambda+1) - v_a(\lambda)} \tag{7}$$

### 3. Methodology validation via virtual test

In this section, a finite elements (FE) simulation of TNF test is performed to the scope to validate the identification methodology presented in this work.

With reference again to Figure 2, the geometrical and constitutive characteristics of the adherends' FE model, used here for analytical-numerical comparisons, are indicated in Table 1.

Table 1. Adherends data used in the FE model of the TNF test.

$h$	$b_l$	$b_2$	$a$	$L$	$d$	$B_l$	$E$	$G$
10 mm	70 mm	70 mm	10 mm	50 mm	10 mm	1 mm	60 GPa	23 GPa

The FE model used here is two-dimensional. The beams are modelled with four-node isoparametric elements, all having exactly a square shape and side length equal to 0.25 mm.

The adhesive layer is modelled by using cohesive elements with an exponential law, as was firstly introduced by Xu and Needelman (1994) for the mixed mode cohesive tractions. The analytical expression of this law, particularized for the mode II condition, is the following:

$$\tau(v) = \sqrt{2e} \tau_{max} \left( \frac{v}{v_{sc}} \right) e^{-\left( \frac{v}{v_{sc}} \right)^2} \tag{8}$$

In Table 2 the cohesive model data are reported. Of course, the cohesive elements length is equal to the beams' elements side length, *i.e.* 0.25 mm. In order to guarantee the displacement continuity, these elements have four nodes and linear shape functions.

Table II. Cohesive model used for all the FE analyses

$\tau_{max}$	$v_{sc} = \frac{2J_c}{\tau_{max}\sqrt{2e}}$	$J_c = \int_0^\infty \tau(x)dx$
20 MPa	0.21444 mm	5 MPa mm

As to the boundary conditions, first, in order to equally share the machine loads between the two beams, an internal condition, prescribing equal  $y$ -displacement of the nodes belonging to the two surfaces in contact, was imposed in correspondence of the three loading points. Moreover, the machine load  $P$  was applied via imposing the  $y$ -

displacement at the central loading point.

To the aim obtaining a sufficiently detailed-in-time solution, since the cohesive elements are affected by material nonlinearity, the external load was split into 300 sub-steps and the solution for each of them is calculated using a Newton-Raphson scheme. The analyses were performed with the ANSYS code.

The overall response of the simulation is shown in the following figures. In particular, in Figs. 3-4 the displacements and stresses in the *x*-direction of the adherends are shown, as resulting from the simulation at an intermediate step of the loading process.

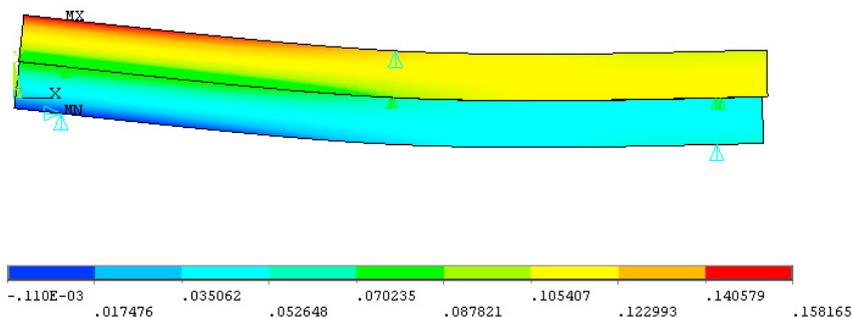


Fig. 3. FE virtual test output – x displacements

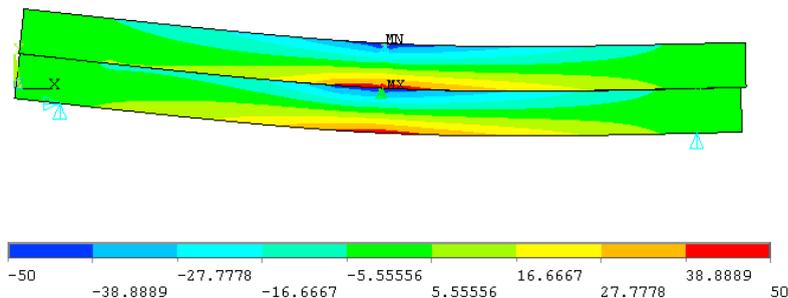


Fig. 4. FE virtual test output – x normal stress

Figure 5 shows the results of the simulation relevant to the cohesive law identification. Note that the load curve  $P(\lambda)$  is strictly increasing, as previously written. In this regard, it is observed that the sequence of the load curves resulting from the ENF, 4ENF and TNF test simulations has increasing stability properties. This characteristic is attributed to the fact that the total bending moment applied at the beginning of the glued zone (i.e. at point  $x_a$ ) is, respectively, for the three tests, increasing, constant and decreasing in the direction of the crack length increase (i.e. the direction of increasing  $x$ ). A similar explanation to justify the stability of the ONF test is also invoked in Wang and Vu-Khanh (1996).

In order to evaluate the correctness and efficiency of the proposed TNF test, the data obtained from the virtual test were used to calculate  $Q(v)$ . The calculation has been performed by means of the iterative algorithm described in the previous section; subsequently, the interface shear stress  $\tau(v)$  has been calculated by numerical differentiation. It is noted that the entire identification process was performed using a simple spreadsheet.

As shown in Figure 6, the result is practically indistinguishable from the integral of the cohesive law (8), imposed in the FE model and reported below in closed form:

$$Q(v) = \int_0^v \tau(x)dx = J_c \left( 1 - e^{-\left(\frac{v}{v_{sc}}\right)^2} \right) \tag{9}$$

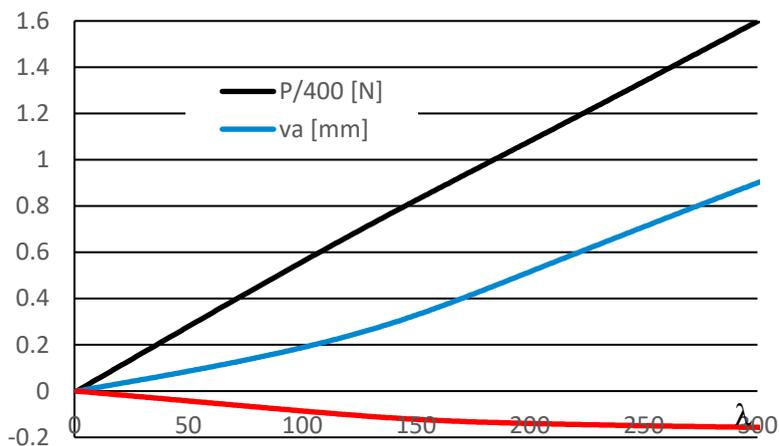


Fig. 5. Sequence of outcomes measured by TNF virtual test.

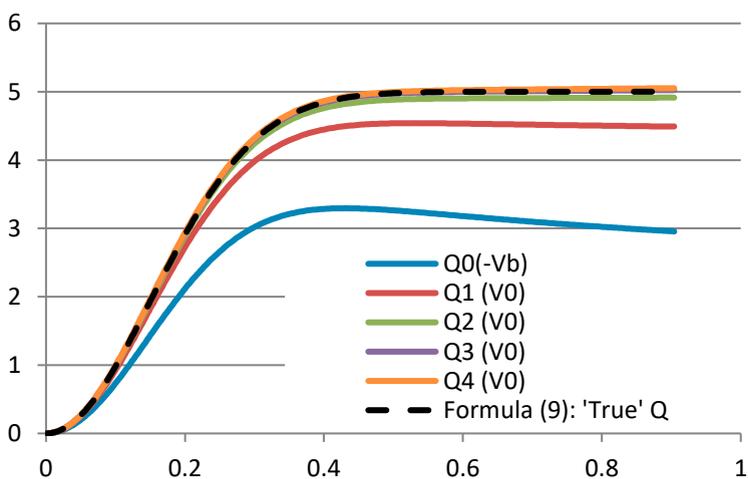


Fig. 6. Sequence of approximations of the function  $Q(v)$  in comparison with the 'true'  $Q(v)$ .

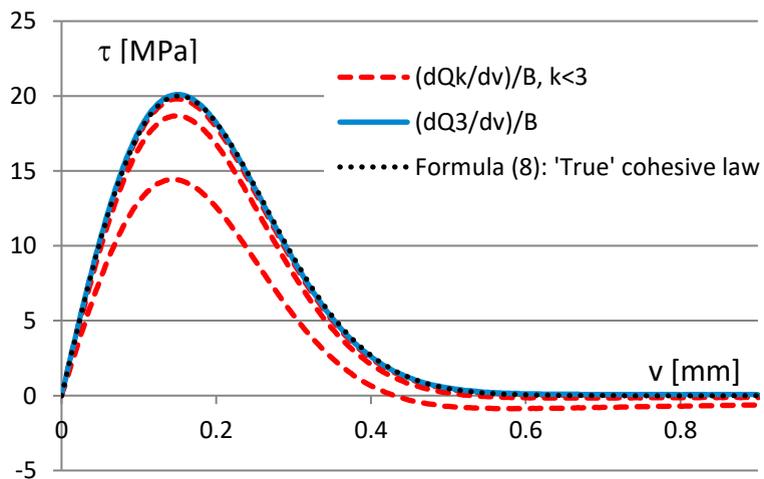


Fig. 7. TNF test: sequence of cohesive law approximations in comparison with the 'true'  $\tau(v)$ .

Finally, in Figure 7 the derivatives of the various approximations of  $Q(v)$  are shown. As expected, the sequence converges to the cohesive law (8) imposed for the virtual test without any appreciable difference.

#### 4. Concluding remarks

The new Twice Notched Flexure configuration, presented in this work, is derived from a general solution of the flexure problem of a couple of beams glued with a constant-in-thickness adhesive layer. In the ideal case, if the new TNF configuration is used for a shear cohesive law identification, some important advantages arise, compared to the standard tests like ENF or ELS. First, the TNF test load-displacement curve is stable; second, it is not necessary to perform any strain measurements and the cohesive law identification is exact within the limits of the theory; finally, the test can be carried out with the standard three-point bending device also used for the ENF test.

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