THE EUROPEAN
PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

Neutrino phenomenology from leptogenesis

Franco Buccella[1,a], **Damiano F. G. Fiorillo**[1,2,b], **Gennaro Miele**[1,2,c], **Stefano Morisi**[1,2,d], **Ofelia Pisanti**[1,2,e], **Pietro Santorelli**[1,2,f]

[1] INFN, Sezione di Napoli, Complesso Univ. Monte S. Angelo, 80126 Naples, Italy
[2] Dipartimento di Fisica "Ettore Pancini", Università degli studi di Napoli "Federico II", Complesso Univ. Monte S. Angelo, 80126 Naples, Italy

Abstract Assuming a type-I seesaw mechanism for neutrino mass generation and invoking a baryogenesis *via* leptogenesis scenario, we consider a reasonable hierarchical structure for Dirac neutrino mass matrix, similar to up-type quark mass matrix. These hypotheses imply a relevant connection between high scale CP violation and low energy one. By requiring a compact heavy neutrino mass spectrum, which allows to circumvent Davidson–Ibarra limit, one can obtain an efficient leptogenesis restricting the allowed region for low energy neutrino parameters. Once the oscillating parameters are taken inside a 3σ range, through the numerical resolution of the leptogenesis Boltzmann equations one gets the following allowed intervals for the lightest neutrino mass and the Dirac CP phase: $-0.90\pi < \delta < -0.75\pi$ and $m_1 \sim (0.002 - 0.004)$ eV.

1 Introduction

As well known, neutrino oscillations imply non vanishing neutrino masses that require an extension of the electroweak Standard Model (SM). In this scheme one gets for free a possible mechanism to explain the baryon asymmetry of the universe (BAU). In this paper we consider the particular case in which SM is extended by adding three right-handed Standard Model singlets N_i with Yukawa interactions

$$\mathcal{L} = -\sum_{\alpha\beta} \left(Y^\ell\right)_{\alpha\beta} \overline{L}_{L\alpha} H \ell_{R\beta} - \sum_{\alpha i} \left(Y^\nu\right)_{\alpha i} \overline{L}_{L\alpha} \tilde{H} N_i +$$
$$-\frac{1}{2} \sum_{ij} (M_R)_{ij} \overline{N_i^c} N_j, \qquad (1)$$

[a] e-mail: buccella@na.infn.it
[b] e-mail: damianofg@gmail.com
[c] e-mail: miele@na.infn.it
[d] e-mail: stefano.morisi@gmail.com
[e] e-mail: pisanti@na.infn.it
[f] e-mail: Pietro.Santorelli@na.infn.it

where $\alpha, \beta, i, j = 1, 2, 3$ represent family indexes. In the basis where charged leptons are diagonal the three active light neutrinos get a Majorana mass from the type-I seesaw mechanism [1–4] that in the family space reads,

$$M_\nu = -M_D (M_R)^{-1} M_D^T, \qquad (2)$$

where the matrix $M_D \equiv Y^\nu \langle H^0 \rangle$, and neutrino mixing parameters and masses are fitted from the observations. In this framework the CP violating out of equilibrium decay of N_i can produce a lepton asymmetry that is then converted into a baryon asymmetry by sphalerons. Such a mechanism is known as *baryogenesis via leptogenesis* [5] and can provide a viable origin for BAU. The possible correlations between the high energy scale CP violation and the low energy one, which comes from the diagonalization of Eq. (2) and it is still waiting for an experimental confirmation, has been intensively studied in literature [6–26].

The existence of such kind of relation can be naively understood by inverting the relation of Eq. (2), namely

$$M_R = -M_D^T M_\nu^{-1} M_D. \qquad (3)$$

The lepton asymmetry generated by leptogenesis depends on the right handed neutrino couplings and their masses, namely by the matrices M_R and M_D. By fixing the Dirac neutrino mass matrix M_D, one gets from Eq. (3) a connection between light neutrino mass matrix M_ν and lepton asymmetry that is related to the high energy scale CP violation. Unfortunately the experiments do not provide direct information about the Dirac mass matrix, and therefore in order to fix M_D we have to evoke some theoretical arguments as also given for example in [6,10,21]. Furthermore, one can observe that from Yukawa interactions of Eq. (1), in order to fit the charged lepton masses, the Yukawa matrix must be very hierarchical, namely $Y_{11}^\ell \ll Y_{22}^\ell \ll Y_{33}^\ell$. The origin of such hierarchy represents one of the biggest challenge of flavour physics, and several ideas has been developed to solve such a puzzle, like for instance the use of Froggatt-Nielsen (FN) flavour sym-

metries [27,28]. In this paper we do not focus our attention on such a problem, but rather we observe that, belonging the left-handed neutrinos to the same $SU(2)_L$ doublets of the charged leptons, if some symmetry enforces a hierarchy in the charged lepton Yukawas, it is reasonable that some hierarchy is also present in the Dirac neutrino coupling Y^ν. Another possibility to fix the structure of the Dirac mass matrix is to assume a Grand Unified gauge group like $SO(10)$ that implies $M_D \approx M_{\rm up}$, where $M_{\rm up}$ denotes the up-type quark mass matrix.[1] These two examples will be shortly described in appendix, while here we just remark that both scenarios imply a hierarchical Dirac Yukawa matrix, namely almost diagonal and with $Y^\nu_{11} \ll Y^\nu_{22} \ll Y^\nu_{33}$. It follows that also the right-handed neutrino mass matrix results to be hierarchical unless the entries of the matrix M_ν are strongly constrained. Thus, barring any particular assumption we expect a hierarchical heavy right-handed neutrino mass spectrum,

$$M_{R_1} \ll M_{R_2} \ll M_{R_3}, \tag{4}$$

with M_{R_1}, M_{R_2} and M_{R_3} denoting the eigenvalues of M_R. Note that if the hierarchy is too strong it could give problem in the origin of baryogenesis via leptogenesis. Indeed, in case the heaviest of the right-handed neutrinos has a mass of about $M_{R_3} \approx 10^{14}$ GeV (that is around the grand unified scale) the lightest right-handed mass could be below the Davidson-Ibarra limit $M_{R_1} < 10^9$ GeV [29], which is the lower limit for a lepton asymmetry generated by the decay of the lightest right-handed neutrino to be sufficiently large,[2] as shown also in [10].

A mechanism to obtain a reasonable value for the lepton asymmetry in this context has been proposed in [31]. The idea is quite simple: by imposing a compact spectrum for the right-handed masses on the l.h.s. of Eq. (3), it follows a condition for neutrino mass parameters on the r.h.s. of Eq. (3) (see also [10,33,34]) once the structure of the Dirac neutrino mass matrix M_D is fixed (by hand or by some model). In this paper we proceed on the path set by [31], extending that numerical study to an investigation on the dependence of the results on the neutrino mass mixing parameter. While that work focused on the analysis of leptogenesis for best fit values (of the time) of the mixing angles of the PMNS matrix, we provide an extension through Monte Carlo simulations which span the part of the parameter space allowed by experimental constraints, without restraining to best fit values, to explore the regions leading to an efficient leptogenesis.. In the next section we review the notation and we

deeper explain the basic idea of [31]. In Sect. 2 we review the basic ideas and relations of leptogenesis. In Sect. 4 we report our results with and without imposing the constraint coming from leptogenesis. Then in Sect. 5 we give our conclusions.

2 Compact right-handed spectrum: implication for neutrino phenomenology

By following the notation of [31] we put,

$$M_D = V_L^\dagger M_D^{\rm diag} V_R, \tag{5}$$

where $V_{L,R}$ are unitary matrices and $M_D^{\rm diag} = (M_{D_1}, M_{D_2}, M_{D_3})$. Once the above expression is used in Eq. (3) one has,

$$M_R = -V_R^T M_D^{\rm diag} A M_D^{\rm diag} V_R ;$$
$$A \equiv V_L^* M_\nu^{-1} V_L^\dagger. \tag{6}$$

As stated in the introduction, here we assume the matrix M_D to be known, namely the numerical values of the entries of the matrices $V_{L,R}$ and $M_D^{\rm diag}$ are given (see also "Appendix"). We take here the case of normal ordering of the neutrino masses; it has in fact been shown that the possibility of inverted ordering is disfavoured under our assumptions [32]. Then it is clear form Eq. (6) that in order to have a compact M_R spectrum some particular conditions have to be assumed on the matrix A. Such conditions have been analyzed in [31] (see section III.A of such a paper for a detailed discussion), and here we just report the main result. From [31] one gets that a compact spectrum can be obtained if the matrix A is such that,

$$\left| \frac{A_{33}}{A_{22}} \right| \le \frac{M_{D_2}^2}{M_{D_3}^2}, \quad \left| \frac{A_{23}}{A_{22}} \right| \le \frac{M_{D_2}}{M_{D_3}}. \tag{7}$$

In this way the entries of the matrix, which would be hierarchically large, are suppressed due to the largeness of M_{D3}. In the following for simplicity we assume an even more stringent condition by requiring,

$$A_{23} = A_{33} = 0. \tag{8}$$

Under these assumptions, up to the first order in the small quantity $M_{D_1}^2/M_{D_3}^2$, two of the three eigenvalues of the right-handed neutrino mass matrix are degenerate, in particular,

$$M_{R_1} \approx |A_{22}| M_{D_2}^2,$$
$$M_{R_2} \approx M_{R_3} \approx |A_{13}| M_{D_1} M_{D_3}. \tag{9}$$

From Eq. (6) it is clear that we can write the matrix A in terms of observable neutrino mixing parameters as,

$$A = V_L^* U_{\rm PMNS} \left(M_\nu^{\rm diag} \right)^{-1} U_{\rm PMNS}^T V_L^\dagger, \tag{10}$$

---

where,

$$
U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix}
$$
$$
\times \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}, \quad (11)
$$

and M_ν^{diag} is a function of the lightest active neutrino and of the two square mass differences Δm_{21}^2 and Δm_{31}^2. In (11) θ_{12}, θ_{23} and θ_{13} are the mixing angles, δ is the Dirac CP phase and α and β are the Majorana phases. The two positions $A_{23} = A_{33} = 0$ are complex equations that can be used to predict four neutrino mixing parameters from the other ones (remind that V_L must be considered as given). Since the parameters m_1, δ, α, and β are still experimentally undetermined[3] the better choice is to use the two complex equations $A_{23} = A_{33} = 0$ to obtain these unknown parameters as a function of the measured ones,

$$
\Delta m_{21}^2, \ \Delta m_{31}^2, \ \theta_{12}, \ \theta_{23}, \ \theta_{13}. \quad (12)
$$

The latter ones are the input parameters of our numerical study that is shown in the Sect. 3.

3 Neutrino mixing parameters and Leptogenesis

It is convenient in this section to use a basis where the right-handed neutrino mass matrix is diagonal. In this basis for simplicity we denote,

$$
M_R^{\text{diag}} = W^\dagger M_R W^* = \text{diag}(M_1, M_2, M_3). \quad (13)
$$

where, since M_R is symmetric, W is a unitary matrix. The matrix M_D then becomes,

$$
\hat{M}_D = M_D W^*. \quad (14)
$$

Let us define the CP asymmetry in the decay of the i-th right-handed neutrino (with $i = 1, 2, 3$) in the l_α lepton doublet (with $\alpha = 1, 2, 3$) as the quantity,

$$
\epsilon_{i\alpha} = \frac{\Gamma_{N_i \to l_\alpha\phi} - \Gamma_{N_i \to \bar{l}_\alpha\phi}}{\Gamma_{N_i \to l_\alpha\phi} + \Gamma_{N_i \to \bar{l}_\alpha\phi}}, \quad (15)
$$

whith Γ's denoting the rates of the the corresponding decay processes. As well known, the asymmetries do not appear in a tree level computation of the decay rates, but rather they originate, at the lowest order, from the interference of tree

level and one loop diagrams. The corresponding expressions, endowed with a regulating factor that gives contribution only in the case of quasi-degenerate neutrino mass spectrum,[4] are reported below,

$$
\epsilon_{i\alpha} = \frac{1}{8\pi v^2} \sum_{k \neq i} \left[A_{ik} f\left(\frac{M_k^2}{M_i^2}\right) + B_{ik} g\left(\frac{M_k^2}{M_i^2}\right) \right], \quad (16)
$$

where $v = 174$ GeV is the electroweak *v.e.v.*, M_i are the masses defined in Eq. (13) and,

$$
g(x) = \frac{1 - x}{(1-x)^2 + \left(\frac{\Gamma_i}{M_i} - x\frac{\Gamma_k}{M_k}\right)^2}, \quad (17)
$$

$$
f(x) = \sqrt{x}\left[g(x) + 1 - (1+x)\log\frac{1+x}{x}\right], \quad (18)
$$

$$
A_{ik} = \frac{\text{Im}\left[\hat{M}_{Di\alpha}^\dagger \hat{M}_{D\alpha k}\left(\hat{M}_D^\dagger \hat{M}_D\right)_{ik}\right]}{\left(\hat{M}_D^\dagger \hat{M}_D\right)_{ii}}, \quad (19)
$$

$$
B_{ik} = \frac{\text{Im}\left[\hat{M}_{Di\alpha}^\dagger \hat{M}_{D\alpha k}\left(\hat{M}_D^\dagger \hat{M}_D\right)_{ki}\right]}{\left(\hat{M}_D^\dagger \hat{M}_D\right)_{ii}}. \quad (20)
$$

The total decay rate of the i-th right-handed neutrino, Γ_i, can be easily calculated from tree level diagrams as:

$$
\Gamma_i = \frac{M_i}{8\pi v^2}\left(\hat{M}_D^\dagger \hat{M}_D\right)_{ii}. \quad (21)
$$

The evolution of right-handed neutrino species and the lepton asymmetries are described by a set of Boltzmann equations for the unknown abundances properly normalized, namely $Y_X = n_X/s$, where n_X is the number density of the X species and $s = \frac{2\pi^2}{45}g_S^* T^3$ is the total entropy density.[5] In terms of the abundance of left-handed α-leptons the corresponding asymmetry is defined as $Y_{\Delta l_\alpha} \equiv Y_{l_\alpha} - Y_{\bar{l}_\alpha}$. Since sphaleronic processes, which are at equilibrium when leptogenesis occurs, preserve the charge $B - L$, it is convenient to express the equations in terms of the $B - L$ asymmetry for the α-flavor $Y_{\Delta\alpha} \equiv Y_B/3 - Y_{\Delta L_\alpha}$, where Y_B is the total baryon asymmetry and $Y_{\Delta L_\alpha}$ is the total lepton asymmetry, involving both the left-handed and the right-handed leptons.

The corresponding Boltzmann equations for right-handed neutrinos, written in terms of the dimensionless variable $z = \frac{M}{T}$ (M being a convenient mass scale) involve only the term describing neutrino decays and inverse decays. The equations for the lepton asymmetries, instead, have to take into account

---

[3] Here for simplicity we consider the normal hierarchy only. Note that m_1 has to be replaced by m_3 in case of inverse hierarchy.

[4] Although we have proven above that our case of study leads to a compact spectrum with two nearly degenerate neutrinos, it turns out that the numerical differences between the two masses, although extremely small, make the regulating factors negligible in determining the final value of the yield.

[5] At the time of interest for leptogenesis $g_S^* = 106.75$; this result derives from all Standard Model species being in equilibrium.

neutrino decays and inverse decays, as well as the so-called *washout processes*. These are all the processes (lepton and Higgs decays, inverse decays and scatterings) which tend to wash out the initial baryon asymmetry. The set equations then reads,

$$sHz\frac{dY_i}{dz} = -\gamma_i \left(\frac{Y_i}{Y_i^{\text{eq}}} - 1 \right),$$
(22)

$$sHz\frac{dY_{\Delta\alpha}}{dz} = -\sum_i \epsilon_{i\alpha}\gamma_i \left(\frac{Y_i}{Y_i^{\text{eq}}} - 1 \right)$$
$$+ \sum_i \frac{\gamma_{i\alpha}}{2} \left(\frac{Y_{\Delta l_\alpha}}{Y_\alpha^{\text{eq}}} + \frac{Y_{\Delta H}}{Y_H^{\text{eq}}} \right),$$
(23)

where i is the index for right-handed neutrinos. In these equations,

$$\gamma_i = Y_i^{\text{eq}} s \, \Gamma_i \frac{K_1\left(\frac{M_i}{T}\right)}{K_2\left(\frac{M_i}{T}\right)},$$
(24)

is the thermally averaged total decay rate, with $K_{1,2}$ the first and second order modified Bessel functions, and $\gamma_{i\alpha} = \gamma_i P_{i\alpha}$ are the decay rates projected onto the α flavor.[6] The corresponding projectors are deduced, once again, by explicitly computing the tree level diagrams for the decay processes. The corresponding expressions are,

$$P_{i\alpha} = \frac{\hat{M}_{Di\alpha}^\dagger \hat{M}_{D\alpha i}}{\left(\hat{M}_D^\dagger \hat{M}_D\right)_{ii}}.$$
(25)

Finally, the equilibrium abundances in (22) and (23) are the following:

- For right-handed neutrinos we used the equilibrium distribution function of a non relativistic particle with mass M_i and 2 degrees of freedom corresponding to the two polarizations of the Majorana neutrino $Y_i^{\text{eq}} = \frac{45M_i^2 z^2}{2\pi^4 g_S^* M^2} K_2\left(\frac{M_i z}{M}\right)$. It is not completely correct to take the non relativistic distribution to describe the right-handed neutrino, whose mass becomes of importance only after it has come out of equilibrium; it would be more correct to take the complete Fermi–Dirac distribution and numerically integrate it. Because of numerical simpicity we decided however to keep the non relativistic distribution; this is an approximation used in most works on leptogenesis.
- For the Higgs particle we used the equilibrium distribution function of a massless boson with the 2 degrees of

freedom corresponding to the $SU(2)$ doublet structure of the Higgs $Y_H^{\text{eq}} = \frac{45\zeta(3)}{g_S^* \pi^4}$.
- For the lepton doublet we used the equilibrium distribution function of massless fermions with the 2 degrees of freedom corresponding to the $SU(2)$ doublet structure of the particles $Y_\alpha^{\text{eq}} = \frac{135\zeta(3)}{4\pi^4 g_S^*}$.

In order to have the Boltzmann equations only in terms of Y_i and $Y_{\Delta\alpha}$ it is necessary to express all asymmetries in the form $Y_{\Delta l_\alpha} = A_{\alpha\beta} Y_{\Delta\beta}$ and $Y_{\Delta H} = C_\beta Y_{\Delta\beta}$, where [35]

$$A = \frac{1}{2148} \begin{pmatrix} -906 & 120 & 120 \\ 75 & -688 & 28 \\ 75 & 28 & -688 \end{pmatrix}, \quad C = -\frac{1}{358} \begin{pmatrix} 37 \\ 52 \\ 52 \end{pmatrix}.$$
(26)

These relations are deduced from the equilibrium conditions for the reactions occurring at the time of leptogenesis. We stress here that, since reactions can go out of equilibrium at specific temperatures, they strongly depend on the value of temperature when leptogenesis occurs.

The set of Eqs. (22) and (23) can now be numerically solved to obtain the asymptotic values of $Y_{\Delta\alpha}$. Once this has been done, the asymptotic value of the baryon asymmetry yield can be found through the sphaleron relation, derived from imposing the equilibrium condition on the chemical potential:[7]

$$Y_{\Delta B} = \frac{28}{79} \sum_\alpha Y_{\Delta\alpha}.$$
(27)

The experimental value of the baryon asymmetry yield is given by [36],

$$Y_{\Delta B} = (8.65 \pm 0.06) \times 10^{-11} \quad (68\% \text{ C.L.}).$$
(28)

4 Results

In order to proceed numerically we have to fix the Dirac neutrino mass matrix, m_D. Even if the procedure deligned in the previous section is completely general here we consider a $SO(10)$ inspired model that is described in more detail in appendix. In the $SO(10)$ framework we expect

$$M_D \approx M_{\text{up}}; \quad M_\ell \approx M_{\text{down}}.$$
(29)

---

[6] By this we mean the total decay rate in the α channel, comprised of both lepton and antilepton.

[7] A subtlety lies in the fact that these equilibrium conditions must be imposed not at the time at which leptogenesis happens; in fact, lepton number is produced at this time, but continues to be converted to baryon number until sphalerons run out of equilibrium. It is at this temperature that the equilibrium conditions must be imposed; therefore, the numerical coefficient in (27) does not depend on the leptogenesis temperature.

Assuming the down quark M_{down} and charged lepton M_ℓ mass matrices approximatively diagonal, it follows that the up quark mass matrix must be diagonalized by the CKM mixing matrix. Moreover if the scalar sector of the $SO(10)$ model does not contain the **120** irreducible representation, then both m_D and M^{up} are symmetric and,

$$M_D \approx V_{\text{CKM}}^\dagger M_{\text{up}}^{\text{diag}} V_{\text{CKM}}^*. \tag{30}$$

where $M_{\text{up}}^{\text{diag}} \equiv \{m_u, m_c, m_t\}$ that are the physical up, charm and top quark masses and,

$$V_{\text{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_{23} & \sin\omega_{23} \\ 0 & -\sin\omega_{23} & \cos\omega_{23} \end{pmatrix} \begin{pmatrix} \cos\omega_{13} & 0 & \sin\omega_{13}e^{i\omega} \\ 0 & 1 & 0 \\ -\sin\omega_{13}e^{i\omega} & 0 & \cos\omega_{13} \end{pmatrix}$$
$$\times \begin{pmatrix} \cos\omega_{12} & \sin\omega_{12} & 0 \\ -\sin\omega_{12} & \cos\omega_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{31}$$

where the values used for angles, masses and the phase are reported in Table (1). From Eq. (30) one gets that V_L is fixed, namely $V_L \equiv V_{\text{CKM}}$, and it can be replaced in Eq. (10). As already stated before, by using the conditions $A_{23} = A_{33} = 0$ in Eq. (10) one gets m_1, δ, α, β as a function of the input parameters (12).

The current bounds on neutrino mixing parameters can be found for example in Refs. [37–39]. For the present analysis we refer to the values given in [38].[8] In order to simplify the analysis we fix Δm_{21}^2 and Δm_{31}^2 to their best fit values, and we take randomly the three mixing angles within the corresponding 3σ ranges. In particular we randomly extract 15,000 points in this three dimensional space. For each input random point (12) we get a set of output values for the lightest neutrino mass and the three neutrino phases.

Before giving the results on the baryonic yield, which impose the needed constraints coming from leptogenesis, we point out that the order of magnitude for all the three heavy neutrino states is $\sim 10^9 - 10^{10}$ GeV. It is also of interest to notice that, even if our spectrum is compact, it never reaches the regime of full degeneracy, which allows us to gliss over the subtleties of resonant leptogenesis: for details see [31].

The results are given in Fig. 1 where we show the mass m_1 as a function of the Dirac phase δ (left panel) and the correlation between the Majorana phases α and β (right panel). In the left plot we report the 1σ (dot-dashed lines) and the 3σ (dashed lines) experimental range for the Dirac phase, while the dashed red line is the best fit value of the Dirac phase. We note that under the above hypotheses (see Eq. 8) a link between the lightest neutrino mass and the Dirac phase
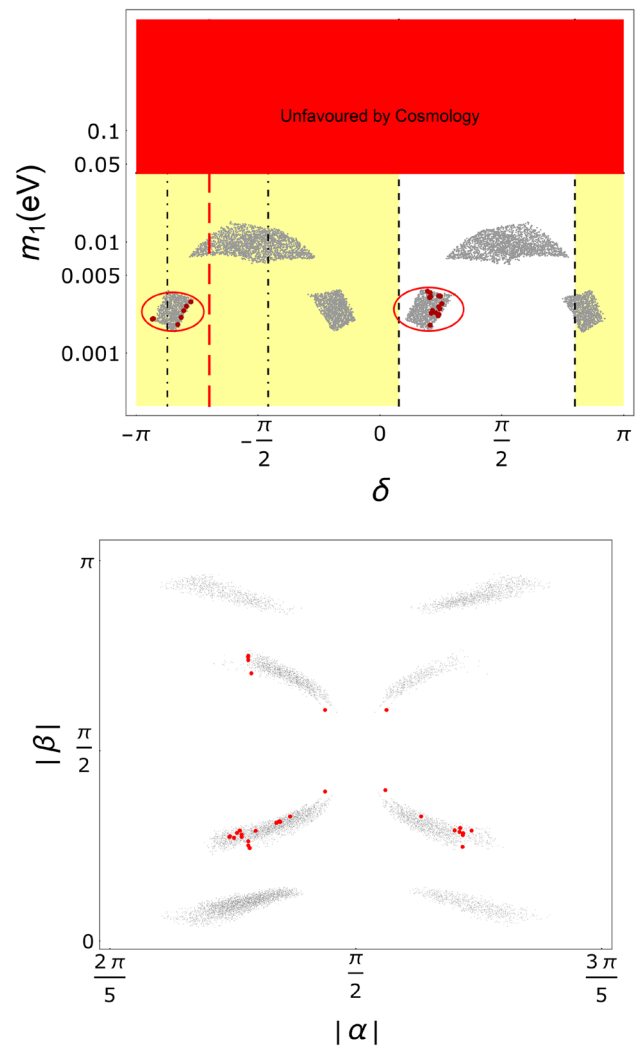

Fig. 1 (First panel) Lightest neutrino mass vs the Dirac phase. The different vertical bands correspond to the experimental values for the Dirac phase (see text for details) and the horizontal band is the upper limit coming from Cosmology. In yellow the 3σ confidence band is evidenced. (Second panel) Majorana phases α and β for the numerically generated points. In both plots red points are obtained by imposing $Y_{\Delta B}$ within the 3-σ experimental range while grey points are not constrained from baryon asymmetry

is expected. The scatter plot of Fig. 1 (grey points in left panel) shows such correlation when the uncertainty on the input parameters is taken into account. It is interesting to observe that such a correlation is minimally smeared out by these observational uncertainties. Despite the high number of 15,000 generated points, only a small part of these (about 30) led to a value for the baryon asymmetry consistent with the experimental value given in (28) within a 3σ range. This extremely low rate of points leading to an efficient leptogenesis is probably due in the first place to the interplay between the many parameters entering our analysis (the Dirac phase, the two Majorana phases, the lightest neutrino mass, the mix-
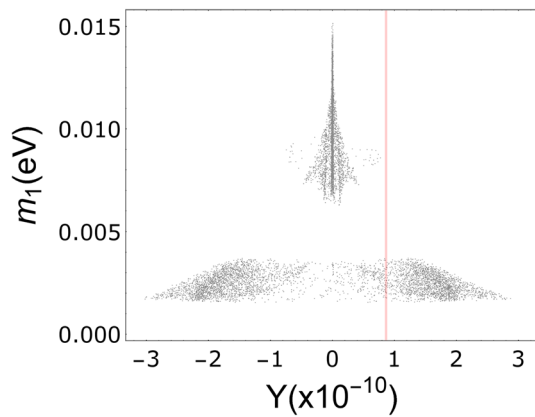
Fig. 2 Mass of the lightest neutrino as a function of the baryonic yield. The vertical band correspond to the $3 - \sigma$ experimental value
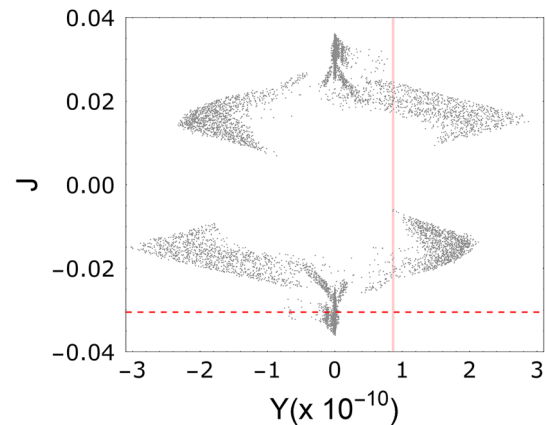

Fig. 3 Jarlskog invariant parameter as a function of the baryon asymmetry yield obtained. The vertical band correspond to the $3 - \sigma$ experimental value. The horizontal dashed line represents the value obtained by fixing all the oscillating parameters to their best fit values

ing angles), which requires very strict conditions in order to be succesful: the natural implication is a fine tuning in the set of parameters. Another point is our brute force approach to the Monte Carlo simulation: since the generation of the set of parameters is independent on the subsequent analysis, there is no feedback on the generation phase; one could imagine an improvement over our approach by concentrating the generation in the regions where the analysis shows an efficient leptogenesis to happen, but this is outside the aim of this paper. For the benefit of the reader we highlight the regions containing these points by circling them in red. We notice that one of these regions falls outside the 3σ confidence range for δ. Moreover, the lightest neutrino mass has an upper and lower limit that is not affected by present cosmological observations; the regions efficient for leptogenesis, in fact, have $m_1 \sim (0.002, 0.004)$ eV and $\delta \sim (-0.90\pi, -0.75\pi)$. Also the Majorana phases are constrained, in particular $|\alpha| \approx \pi/2$ and β is distributed in the range $(-\pi, +\pi)$ in two isolated regions centered around the values of 0.3π and -0.7π.

The neutrinoless double beta parameter $m_{\beta\beta} = \left| \sum_i m_i U^2_{PMNS_{ei}} \right|$ is found to lie between 0.001 eV and 0.02 eV, which is below the experimental bounds, set, for example, in [40].[9]

In Fig. 2 we show the evolution of the baryonic yield with the mass of the lightest neutrino: it is clear from the picture that only a tiny band of points leads to an efficient leptogenesis.

In Fig. 3 we give the Jarlskog parameter [41] as a function of the baryon asymmetry yield, together with the red vertical band representing the 3σ confidence range for the baryon abundance experimentally measured. We see that requiring a baryon asymmetry within about a $3 - \sigma$ range around the experimental value of the baryon abundance, we get J in the

range $(-0.022, -0.018)$ (approximately independent of the sign). It is to be noted, however, that different signs predict different yields, due to the opposite value of the CP asymmetry.

We mention here that our method leads to the further possibility of determining a rather precise range of values for the right-handed neutrino masses: in particular, we find that the lightest neutrinos has masses in the range 2.46×10^9 GeV $\leq M_1 \leq 4.24 \times 10^9$ GeV, while the heaviest couple of neutrinos (which we recall to be quasi-degenerate in mass) have masses in the range 7.47×10^9 GeV $\leq M_2 \leq 11.67 \times 10^9$ GeV.

5 Conclusions

In this paper we analize the baryogenesis *via* leptogenesis scenario for a type-I seesaw mechanism at the basis of neutrino mass generation. In this framework, we assume a reasonable structure for Dirac neutrino mass matrix, namely symmetric and similar to up-type quark mass matrix, like occurring in SO(10) inspired models. These assumptions imply a relevant correlation between the parameters of CP violation at high and low energy, and this restricts low energy neutrino parameter space (already compatible with neutrino phenomenology) once we require consistency with the observed baryon asymmetry. Unfortunately, with the hierarchical structure induced on right-handed neutrino mass matrix by previous similarity hypothesis, it is not possible to obtain a viable leptogenesis realization, because the lightest right-handed neutrino mass results to be below the Davidson–Ibarra limit. One can circumvent this problem by imposing a fine tuning in the neutrino parameters, which providing a compact right-handed neutrino spectrum, allows to obtain an

---

[9] For a comparison with major neutrinoless double beta experiments see Table 2 of [40].

efficient leptogenesis. This fine tuning, through the numerical resolution of the Boltzmann equations ruling the yields evolution, provides the following allowed intervals for the lightest neutrino mass and the Dirac CP phase (in a 3σ range from the experimental values): $-0.90\pi < \delta < -0.75\pi$ and $m_1 \sim (0.002 - 0.004)$ eV. Further experimental constraints on these quantities might be used to eventually falsify the model. In case the predictions of our model should be fulfilled, the question might arise as to whether other leptogenesis models might be distinguishable from our own. A different model giving the same predictions for the experimentally observable quantities (the lightest neutrino mass and the Dirac phase) could only be differentiated by ours through the Majorana phases, the right-handed neutrino masses and more generally dynamical quantities (cross sections, decay rates...) for processes involving the newly added species. These quantities are outside the possibilities of present experimental capabilities as far as we know.

Appendix A: Examples of hierarchical lepton Yukawas

First example: FN scenarios

Assuming FN families symmetries we expect quite hierarchical Dirac Neutrino Yukawa interactions. Here we provide an example from Ref. [28] where the three left-handed $SU(2)_L$ doublets have $U(1)_{FN}$ charges $(3, 0, 0)$, the charged lepton right-handed $(3, 2, 0)$ and the right-handed singlets $(1, -1, 0)$. Moreover two FN scalars electroweak singlets has been introduced, θ and θ', with charges 1 and -1 respectively. Under such assumptions the resulting mass matrices are proportional to

$$M^\ell \propto \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \tag{32}$$

$$m_D \propto \begin{pmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{pmatrix}, \tag{33}$$

$$M_R \propto \begin{pmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{pmatrix}, \tag{34}$$

where $\lambda \propto \langle\theta\rangle$ and $\lambda' \propto \langle\theta'\rangle$. The proportional symbol is just to remember that each entry is multiplied by an arbitrary order one parameter. From such matrices we obtain

$$m_\nu \propto \begin{pmatrix} \lambda^6 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \\ \lambda^3 & 1 & 1 \end{pmatrix}, \tag{35}$$

predicting a large atmospheric angle. However, here we are not interested to give a realistic model that can explain all fermion observables but we want just to show an example of a Dirac Yukawa coupling providing hierarchical eigenvalues. Indeed, fixing $\lambda = 0.05$ and $\lambda' = 0.1$ we obtain eigenvalues of order $\mathcal{O}(10^{-9}, 10^{-3}, 1)$.

Second example: $SO(10)$ scenarios

In minimal $SO(10)$ models it is well known that the up-type quark mass matrix and the Dirac neutrino one are approximatly equal $M_{up} \approx m_D$. We note that if only one **10** irreducible representation appears in the scalar sector, then $M_{up} = m_D$ exactly. On the other hand, in this case the CKM would be exactly the identity matrix, leading to an unrealistic model. A possible minimal solution is to add a $\overline{\mathbf{126}}$ scalar representation [42]. The fit of all Standard Model observables in this minimal scenario has been studied in detail for instance in Ref. [43]. We also observe that if the scalar sector of the theory does not contain the **120** irreducible representation of $SO(10)$, the two matrices M_{up} and m_D are symmetric. These facts imply that m_D is diagonalized by a single unitary matrix (and not two) with quite small angles of the order of the CKM mixing matrix [44], and that its eigenvalues are hierarchical like the up quark masses.

We moreover consider an extension of the minimal $SO(10)$ scalar spectrum (with only **10** and $\overline{\mathbf{126}}$ scalar representations), with a **45** multiplet that takes v.e.v. in the T_{3R} direction of $SO(10)$ [45]. In this case the non-renormalizable operator $\mathbf{16}\,\mathbf{16}\,\mathbf{45}\,\overline{\mathbf{126}}$ gives a contribution only to right handed neutrino. Therefore, differently from the minimal $SO(10)$ model, the right handed neutrino mass is a free matrix.

Appendix B: Benchmark point

In this appendix we explicitly report a point obtained in our numerical analysis which can be used as benchmark or to test our results; in Table 1 we report the values of the parameters from the Standard Model (where ω has been used to denote the mixing angles and the CP phase of the CKM matrix, in order to avoid confusion with the angles of the PMNS matrix), the values of the parameters generated for the specific point given and the equilibrium yield obtained. We also report the CP asymmetry matrix for the above mentioned point:

Table 1 Input parameters used in our numerical analysis and corresponding output for a benchmark point

Constants	G_F (GeV^{-2})	1.166×10^{-5}
	M_W (GeV)	80.39
	v (GeV)	174
	m_u (eV)	6.7×10^5
	m_c (eV)	0.327×10^9
	m_t (eV)	99.1×10^9
	Δm^2 (eV^2)	3.84×10^{-3}
	δm^2 (eV^2)	11.8×10^{-5}
	ω_{12} (°)	13.02
	ω_{13} (°)	0.20
	ω_{23} (°)	2.35
	ω	1.20
Input parameters	θ_{12}	-0.580
	θ_{13}	0.142
	θ_{23}	0.861
Output results	m_1 (eV)	0.0026
	δ	3.505
	α	1.512
	β	-2.052
	Y	1.28×10^{-10}

$$\epsilon = \begin{pmatrix} 4.43 \times 10^{-7} & 2.54 \times 10^{-6} & -2.96 \times 10^{-6} \\ 5.03 \times 10^{-8} & 1.09 \times 10^{-6} & 6.72 \times 10^{-4} \\ 4.98 \times 10^{-8} & 1.09 \times 10^{-6} & 6.72 \times 10^{-4} \end{pmatrix} \quad (36)$$

References

1. P. Minkowski, Phys. Lett. **67B**, 421 (1977). https://doi.org/10.1016/0370-2693(77)90435-X
2. T. Yanagida, Conf. Proc. C **7902131**, 95 (1979)
3. M. Gell-Mann, P. Ramond, R. Slansky, Conf. Proc. C **790927**, 315 (1979). arXiv:1306.4669 [hep-th]
4. R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980). https://doi.org/10.1103/PhysRevLett.44.912
5. M. Fukugita, T. Yanagida, Phys. Lett. B **174**, 45 (1986). https://doi.org/10.1016/0370-2693(86)91126-3
6. M.S. Berger, B. Brahmachari, Phys. Rev. D **60**, 073009 (1999). https://doi.org/10.1103/PhysRevD.60.073009. arXiv:hep-ph/9903406
7. G. Bambhaniya, P.S. Bhupal Dev, S. Goswami, S. Khan, W. Rodejohann, Phys. Rev. D **95**(9), 095016 (2017). https://doi.org/10.1103/PhysRevD.95.095016. arXiv:1611.03827 [hep-ph]
8. W. Buchmuller, M. Plumacher, Int. J. Mod. Phys. A **15**, 5047 (2000). https://doi.org/10.1016/S0217-751X(00)00293-5. arXiv:hep-ph/0007176
9. L. Merlo, S. Rosauro-Alcaraz. arXiv:1801.03937 [hep-ph]
10. E.K. Akhmedov, M. Frigerio, A.Y. Smirnov, JHEP **0309**, 021 (2003). https://doi.org/10.1088/1126-6708/2003/09/021. arXiv:hep-ph/0305322
11. S. Pascoli, S.T. Petcov, A. Riotto, Phys. Rev. D **75**, 083511 (2007). https://doi.org/10.1103/PhysRevD.75.083511. arXiv:hep-ph/0609125
12. S. Pascoli, S.T. Petcov, W. Rodejohann, Phys. Rev. D **68**, 093007 (2003). https://doi.org/10.1103/PhysRevD.68.093007. arXiv:hep-ph/0302054
13. A.S. Joshipura, E.A. Paschos, W. Rodejohann, JHEP **0108**, 029 (2001). https://doi.org/10.1088/1126-6708/2001/08/029. arXiv:hep-ph/0105175
14. D. Falcone, F. Tramontano, Phys. Rev. D **63**, 073007 (2001). https://doi.org/10.1103/PhysRevD.63.073007. arXiv:hep-ph/0011053
15. M. Hirsch, S.F. King, Phys. Rev. D **64**, 113005 (2001). https://doi.org/10.1103/PhysRevD.64.113005. arXiv:hep-ph/0107014
16. F. Buccella, D. Falcone, F. Tramontano, Phys. Lett. B **524**, 241 (2002). https://doi.org/10.1016/S0370-2693(01)01409-5. arXiv:hep-ph/0108172
17. H. Goldberg, Phys. Lett. B **474**, 389 (2000). https://doi.org/10.1016/S0370-2693(00)00032-0. arXiv:hep-ph/9909477
18. R.N. Mohapatra, H.B. Yu, Phys. Lett. B **644**, 346 (2007). https://doi.org/10.1016/j.physletb.2006.12.005. arXiv:hep-ph/0610023
19. S. Antusch, S.F. King, JHEP **0601**, 117 (2006). https://doi.org/10.1088/1126-6708/2006/01/117. arXiv:hep-ph/0507333
20. J.R. Ellis, M. Raidal, T. Yanagida, Phys. Lett. B **546**, 228 (2002). https://doi.org/10.1016/S0370-2693(02)02691-6. arXiv:hep-ph/0206300
21. P.H. Frampton, S.L. Glashow, T. Yanagida, Phys. Lett. B **548**, 119 (2002). https://doi.org/10.1016/S0370-2693(02)02853-8. arXiv:hep-ph/0208157
22. A. Das, N. Okada, Phys. Lett. B **774**, 32 (2017). https://doi.org/10.1016/j.physletb.2017.09.042. arXiv:1702.04668 [hep-ph]
23. A. Das, N. Okada, Phys. Rev. D **88**, 113001 (2013). https://doi.org/10.1103/PhysRevD.88.113001. arXiv:1207.3734 [hep-ph]
24. G.C. Branco, T. Morozumi, B.M. Nobre, M.N. Rebelo, Nucl. Phys. B **617**, 475 (2001). https://doi.org/10.1016/S0550-3213(01)00425-4. arXiv:hep-ph/0107164
25. G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim, Phys. Lett. B **645**, 432 (2007). https://doi.org/10.1016/j.physletb.2006.12.060. arXiv:hep-ph/0609297
26. G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim, I. Masina, M.N. Rebelo, C.A. Savoy, Phys. Rev. D **67**, 073025 (2003). https://doi.org/10.1103/PhysRevD.67.073025. arXiv:hep-ph/0211001
27. C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B **147**, 277 (1979). https://doi.org/10.1016/0550-3213(79)90316-X
28. G. Altarelli, F. Feruglio, In *Venice 1999, Neutrino telescopes*, vol. 2, pp. 353–381. arXiv:hep-ph/9905536
29. S. Davidson, A. Ibarra, Phys. Lett. B **535**, 25 (2002). https://doi.org/10.1016/S0370-2693(02)01735-5. arXiv:hep-ph/0202239
30. F. Björkeroth, F.J. de Anda, I. de Medeiros Varzielas, S.F. King, JHEP **1701**, 077 (2017). https://doi.org/10.1007/JHEP01(2017)077. arXiv:1609.05837 [hep-ph]
31. F. Buccella, D. Falcone, C.S. Fong, E. Nardi, G. Ricciardi, Phys. Rev. D **86**, 035012 (2012). https://doi.org/10.1103/PhysRevD.86.035012. arXiv:1203.0829 [hep-ph]
32. F. Buccella, M. Chianese, G. Mangano, G. Miele, S. Morisi, P. Santorelli, JHEP **1704**, 004 (2017). https://doi.org/10.1007/JHEP04(2017)004. arXiv:1701.00491 [hep-ph]
33. G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim, M.N. Rebelo, Nucl. Phys. B **640**, 202 (2002). https://doi.org/10.1016/S0550-3213(02)00478-9. arXiv:hep-ph/0202030
34. E.J. Chun et al., Int. J. Mod. Phys. A **33**(05n06), 1842005 (2018). https://doi.org/10.1142/S0217751X18420058. arXiv:1711.02865 [hep-ph]
35. A. Abada, S. Davidson, A. Ibarra, F.-X. Josse-Michaux, M. Losada, A. Riotto, JHEP **0609**, 010 (2006). https://doi.org/10.1088/1126-6708/2006/09/010. arXiv:hep-ph/0605281
36. P.A.R. Ade et al. (Planck Collaboration), Astron. Astrophys. **594**, A13 (2016). https://doi.org/10.1051/0004-6361/201525830

37. I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, T. Schwetz, JHEP **1701**, 087 (2017). https://doi.org/10.1007/JHEP01(2017)087. arXiv:1611.01514 [hep-ph]

38. F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri, A. Palazzo, Phys. Rev. D **95**(9), 096014 (2017). https://doi.org/10.1103/PhysRevD.95.096014. arXiv:1703.04471 [hep-ph]

39. P.F. de Salas, D.V. Forero, C.A. Ternes, M. Tortola, J.W.F. Valle. arXiv:1708.01186 [hep-ph]

40. M. Agostini et al. [GERDA Collaboration], Phys. Rev. Lett. **120**(13), 132503 (2018). https://doi.org/10.1103/PhysRevLett.120.132503. arXiv:1803.11100 [nucl-ex]

41. C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985). https://doi.org/10.1103/PhysRevLett.55.1039

42. K.S. Babu, R.N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993). https://doi.org/10.1103/PhysRevLett.70.2845. arXiv:hep-ph/9209215

43. S. Bertolini, T. Schwetz, M. Malinsky, Phys. Rev. D **73**, 115012 (2006). https://doi.org/10.1103/PhysRevD.73.115012. arXiv:hep-ph/0605006

44. C. Patrignani et al. [Particle Data Group], Chin. Phys. C **40**(10), 100001 (2016). https://doi.org/10.1088/1674-1137/40/10/100001

45. G. Anderson, S. Raby, S. Dimopoulos, L.J. Hall, G.D. Starkman, Phys. Rev. D **49**, 3660 (1994). https://doi.org/10.1103/PhysRevD.49.3660. arXiv:hep-ph/9308333