Available online at www.sciencedirect.com

ScienceDirect

Procedia Structural Integrity 6 (2017) 69–76

Structural Integrity Procedia

www.elsevier.com/locate/procedia

XXVII International Conference "Mathematical and Computer Simulations in Mechanics of Solids and Structures". Fundamentals of Static and Dynamic Fracture (MCM 2017)

A Bayesian approach for controlling structural displacements

Maria Grazia D'Urso[a,]*, Antonella Gargiulo[a], Salvatore Sessa[b]

[a]*DICeM – Department of Civil and Mechanical Engineering, Università degli Studi di Cassino e del Lazio Meridionale, Via G. Di Biasio 43, 03043, Cassino (RM), Italy*

[b]*Department of Structures for Engineering and Architecture, University of Naples Federico II, Via Claudio 21, 80125 Napoli, Italy*

Abstract

Bayesian Networks represent one of the most powerful and effective tools for knowledge acquisition in the observation of physical phenomena affected by randomness and uncertainties. The methodology is the result of several developments concerning the Bayesian statistical theory and permits, by inference, to update the statistics describing physical variables by the observation of experimental evidences. In general, Bayesian Networks have become a very popular and versatile approach in *problem solving* strategies because of their capability in enhancing the status of knowledge of a physical problem domain and to characterize expected outcomes. In particular, this work presents a strategy performing the Bayesian updating of the mechanical and geometrical properties of a steel structure. Based on high-precision topographical measurements, such a strategy has the purpose of accurately estimating the structural displacements expected during the structural life-cycle.

Peer-review under responsibility of the MCM 2017 organizers.

Keywords: Bayesian Network; displacements; conditional probability; probabilistic inference observation; error model.

1. Introduction

Monitoring of displacements, deflections and ground settlements of civil infrastructures can be performed by periodically performing topographical surveys detecting the coordinates and mutual locations of a set of control

* Corresponding author. Tel.: +39-0776-299-4309.
E-mail address: durso@unicas.it

Peer-review under responsibility of the MCM 2017 organizers.
10.1016/j.prostr.2017.11.011

points. Moreover, the interpretation of survey measurements is usually performed by adopting statistical strategies in order to account for instrumental errors and estimating confidence intervals of the results.

An appealing benefit of Bayesian Networks is their capability in accounting for *a priori* statistical characterization of the structural parameters which is updated by survey observations and, subsequently, can predict future structural responses. In this sense, differently from traditional statistical approaches, Bayesian updating can forecast anomalous behaviors before their occurrencies. In fact, it is well known that the accuracy of survey results obtained by least squares approaches is significantly influenced by the adopted stochastic model. On the contrary, use of the Bayesian approach requires far less observations to get a desired accuracy of the displacement measurement.

Bayesian Networks, implemented in conjunction with Markov Chains and Monte Carlo Simulations, permit to determine an efficient relationship between the prior knowledge of the structural model and the survey observations. In this respect, an effective and accurate characterization of the prior statistics of the structural domain represents an essential aspect of the identification process, especially in presence of limited observations or when their detection involves expensive activities, since it permits to forecast structural responses, although with limited confidence, even in absence of experimental evidences.

The application of Bayesian updating to structural models concerns mainly two different aspects: *parameter learning* is focused on the characterization of the marginal probabilities of the adopted structural parameters while *structure learning* concerns the relationships between different parameters and their conditional probabilities. Both these tasks are performed in *machine learning* methodologies in which optimization algorithms, analyzing experimental outcomes, determine the mutual dependency of parameters and responses. Moreover, observations permit to update the prior statistics of the structural parameters by an *inference* process. Finally, the updated parameters can be used to forecast future structural responses by performing reliability analysis algorithms.

The present research analyzes the outcomes of a structural survey of a steel truss vault in order to characterize its constitutive parameters and to detect possible anomalies. In particular, the vertical displacements of the structural nodes have been detected by a total station; subsequently, the recorded data have been interpreted by a Bayesian network characterizing the relationships between displacements and mechanical parameters. It is worth to be emphasized that the inference procedure accounts for the whole set of observed displacements and their correlation so that parameters' updating assumes the capabilities of a multi-objective identification process.

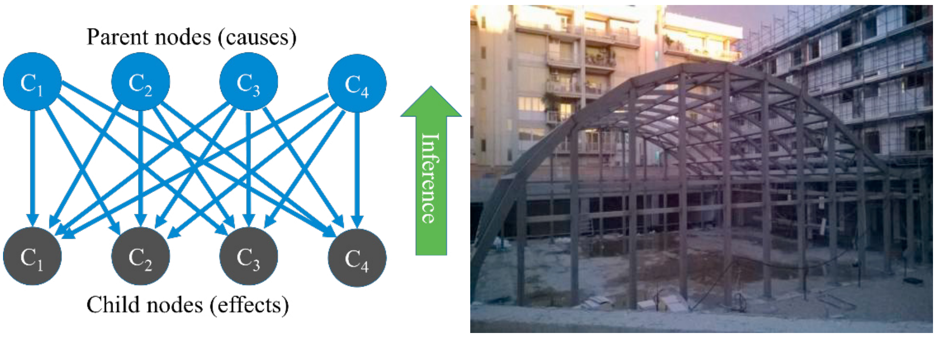

Fig. 1.(a) Example of Bayesian Network nodes and connections; (b) Case-study: a steel truss barrel vault.

2. Probabilistic Inference updating

Bayesian Networks are defined by means of random variables (nodes) mutually connected, see, e.g., Fig. 1(a). Variables not depending upon different nodes (namely *parent* variables) are characterized by marginal Probability Density Functions (PDFs) while variables influenced by different nodes of the network (defined as *child* variables) are characterized by PDFs conditioned by the value of their parents. Use of conditional probability to establish co*nnections* between parent and child nodes is particularly feasible to represent the modularity of random systems constituted by redundant components; moreover, implemented frameworks available in the literature that perform both parameters and structure learning and graphically represent variables and connections, make their use particularly intuitive in common practice.

Observation of experimental data permits to set the actual value of some variables, defined as *evidences*, for which a deterministic characterization is assumed. Purpose of the Bayesian Network consists in assessing, by statistical inference, the *updated*, or *posterior,* probabilistic distributions of the remaining nodes representing the global statistical characterization of an *event* or *scenario*. In brief, probabilistic inference, representing the pivotal phase of the machine learning process, determines the probability distributions $P[X_i|E]$ of the random variable X_i for an observed event E which characterize the probabilistic behavior of the analyzed system.

To fix ideas, starting from the nodes set as evidences, conditioned probability distributions are updated by inference, intuitively performed by the application of the Bayes' Theorem, so that the evidence *propagates* among the network , as shown in Casaca et al. (2008) and Straub (2010).

Conditional probabilities have been defined by assuming discrete descriptions of the random variables by Conditional Probability Tables (CPT). Bayesian inference updating consists in computing, by means of optimization algorithms, the posterior values of the CPT, and subsequently the discrete probability distribution of each node, relevant to one or more evidences introduced in the network. Computations have been performed by the freeware *Genie* which provides an exhaustive framework for Bayesian Network analysis.

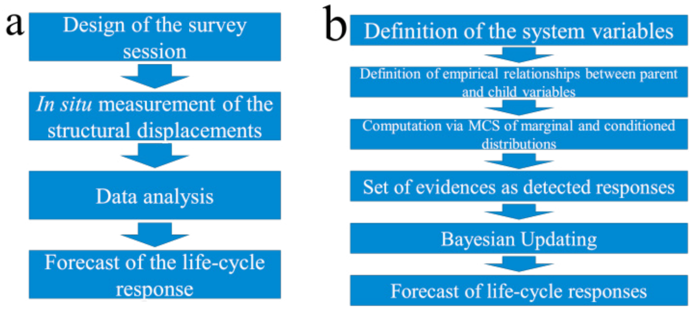

Fig. 2. (a) Workflow of a topographical survey session; (b) workflow of the Bayesian Updating procedure

3. The case-study Bayesian Network

The case-study analyzed in this research consists in a steel truss barrel vault, shown in Figure 1(b), which was monitored during its construction phases by two topographical surveys, detecting the structural displacements, performed on February 28th and March 22nd 2015, respectively.

The survey campaign aimed to detect possible anomalous behaviors. In particular, while classical procedures for the statistical assessment of survey data analyse the evolving values of displacements and can identify anomalously large values only after that they have occurred, the Bayesian updating workflow, reported in Figure 2(a), is capable of interpreting the values-in-time of the responses, update the statistics of the structural parameters and forecast the expected maximum values of the response which are compared with the values determined by the structural design.

It is worth to be emphasized that the updated statistics of the network variables is highly influenced by the definition of the assumed prior distributions which, subsequently, must be properly computed by simulations consistent with the physical behaviour of the model. Main phases of the procedure for the Bayesian Network characterization and of the parameters updating are reported in Figure 2(b).

4. Geometrical scheme of the survey net

The monitored sail-shaped vault consists in a net of steel columns and beams and presents a 40x50 m rectangular plan. Main beams consist in 9 frames made of square-piped elements and vertical molded plate columns and are connected by pin joints to 13 secondary beams presenting square cross sections.

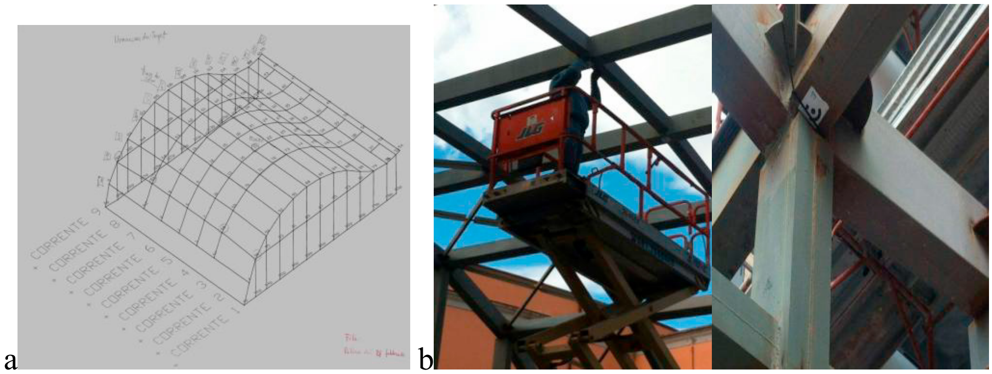

Fig. 3. (a) Axonometric representation of the net adopted in the topographical survey; (b) Joint targets monitored by the topographical survey.

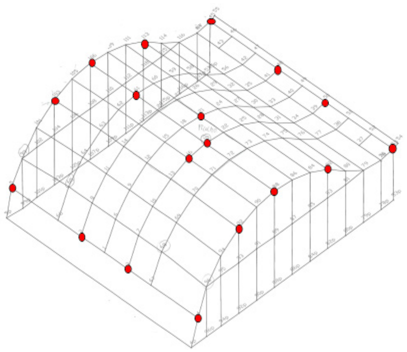

Fig. 4. Monitored points included in the Bayesian Network.

Surveys have been performed at the end of the steel frames realization in order to control the displacements generated by the dead loads and investigate the correct installation of the structural elements. To this end, the monitored joints, whose numbering is shown in Figure 3(a), have been equipped with 94 targets, shown in Figure 3(b), so that measurements concern the mutual distances between consecutive targets, azimuthal and zenithal angles.

In order to reduce the computational effort of the updating procedure, the responses included in the Bayesian network, shown by red bullets in Figure 4, are relevant to 14 joints located on the vault boundary and 4 nodes located at the top beams. Measurements have been performed by using a high precision total station type Leica TPR 30 equipped with innovative optical and digital technologies.

Table 1 shows the detected relative vertical altitudes $\Delta z_{mar.}$and Δz_{feb}, measured in the February 2015 and March 2015 survey sessions, respectively, and the relative vertical displacements Δu computed as $\Delta u = \Delta z_{mar.} - \Delta z_{feb}$.

5. Structure of the Bayesian Network

The Bayesian Network used in this research has been defined by prior PDFs of the structural parameters of interest and by conditional PDFs between those parameters and the esteemed responses. A schematized representation of the adopted network is reported in Figure 5(a).

It is worth to be emphasized that some variables of interest, which are described below, are directly involved in the updating procedure as evidences or target parameters. Different parameters, although included in the computational definition of the network, are not represented for brevity.

To illustrate the physical meaning of all the involved random variables, the network nodes are arranged by different typologies. In particular, the parent variables are:

1. **Structural parameters**: represented in black, consist in the parameters characterizing the structural model, such as the Young's modulus, see, e.g., Fig. 6(a), loads, joint performance coefficients etc.

2. **Model error ε_{fem}**: represented in light blue, it models the inaccuracies related to the finite element model. The node has unitary-mean Gaussian distribution, shown in Fig. 6(b), with coefficient of variation (c.o.v.) of 30%.

Survey measurement error ε_l: represented in violet, it is related to the instrumental error of the survey sessions. It is characterized by a unitary-mean Gaussian distribution, shown in Fig. 6(c), with 5% c.o.v.

Child variables can be summarized as:

1. **Theoretical displacements $u_{fem,i}$**: represented in blue, consists in the absolute displacements of the monitored nodes computed by a finite element analysis. Progressive index *i* denotes the node of the survey net for which the displacement is computed.
2. **Real displacements u_i**: reported in red, denote the real, physical displacement of each node. Their outcomes are esteemed as $u_i = u_{fem,i}\, \varepsilon_{fem}$.
3. **Detected relative displacements Δ_{ij}:** represented in green will be adopted as evidences of the network. Such quantities represent the relative displacement detected between two consecutive nodes corrected by the instrumental error: $\Delta_{ij} = (u_i - u_j)\, \varepsilon_l$.

In conclusion, the model is made of 53 nodes, related by 59 dependencies. The probability distributions of all the random variables have been discretized in order to implement and analyze the network by Genie 2.0, a freeware framework for scientific research purposes. A part of the network implemented in Genie is shown in Figure 5(b) in which parent variables are represented in violet, child nodes are depicted in light blue and dependencies are represented as arrows.

Table 1. Target nodes vertical displacements and altitudes.

Target	$Z_{Febbraio}$ [m]	Z_{Marzo} [m]
5	2.104862	2.044688
103	6.699921	6.641457
106	7.547846	7.486530
113	6.186326	6.127835
45	3.984066	3.938086
48	3.961980	3.904306
50	3.964270	3.910218
53	4.016172	3.951600
82	6.191537	6.132470
88	7.537745	7.483738
92	6.678969	6.619585
4	2.101707	2.045437
3	2.139384	2.079672
2	2.139901	2.079416
61	7.550593	7.496081
21	7.428238	7.357290
19	7.541229	7.458407
16	7.396085	7.314307

Δ	$\Delta z_{febbraio}$ [m]	Δz_{marzo} [m]	Δu [m]
Δ 103-5	4.595059	4.596769	0.001710
Δ 106-103	0.847925	0.845073	0.002852
Δ 113-106	1.361521	1.358695	0.002826
Δ 45-113	2.202260	2.189749	0.012511
Δ 48-45	0.022086	0.033780	0.010694
Δ 50-48	0.002291	0.005912	0.004621
Δ 53-50	0.051902	0.041383	0.010519
Δ 82-53	2.175364	2.180870	0.005505
Δ 88-82	1.346209	1.351268	0.005060
Δ 92-88	0.858779	0.864153	0.005377
Δ 4-92	4.577262	4.574148	0.003114
Δ 3-4	0.037677	0.034234	0.003443
Δ 2-3	0.000517	0.000256	0.000773
Δ 5-2	0.035039	0.034748	0.000311
Δ 61-106	0.002747	0.009551	0.006804
Δ 21-61	0.122355	0.138791	0.016436
Δ 19-21	0.112991	0.101117	0.011874
Δ 16-19	0.145144	0.144100	0.000944
Δ 88-16	0.141660	0.169431	0.010199

Some parent nodes PDFs are represented in, 6(b) and 6(c). Probabilistic dependencies, numerically defined by conditional PDFs, characterize the likelihood that a child variable assumes a specific value as function of all possible states of its parent variables.

Such a dependency has been determined by theoretical considerations, simplified models available in the literature and a Finite Element-based Monte Carlo simulation. Since the software needs the definition of a finite number of variable states, PDFs have been suitably discretized.

The generation of the conditional PDFs of the child variables has been performed by the following steps:

1. Random generation of *m* occurrences of the parent variables;
2. Computation of *m* corresponding finite element responses;
3. Computation of the absolute displacement occurrences by applying the model error;
4. Determination of the relative displacements occurrences by combining absolute displacements and survey error;

5. Definition of *n* intervals (states) for each variable;
6. Determination of the conditional PDFs by statistical analysis of the generated occurrences performed by an *ad-hoc* algorithm implemented in Matlab.

Obviously, the occurrences of the finite element displacements depend on the realizations of the structural parameters generated by the Monte Carlo procedure. It is worth to be emphasized that, because of the Markov hypothesis, conditional probabilities of each child variable depend exclusively on the state of their directly-connected parent variables. Two examples of the PDF entries relevant to the $U_{fem,i}$ and U_i displacements, discretized as matrices, are reported in Figures 7 and 8. Note that each variable depends on the states of the corresponding parent nodes so that the FEM displacements depends on the outcome of the Young's modulus while displacement U_i, in Fig. 8, depends on the states of $U_{fem,i}$ and ε_{fem}.

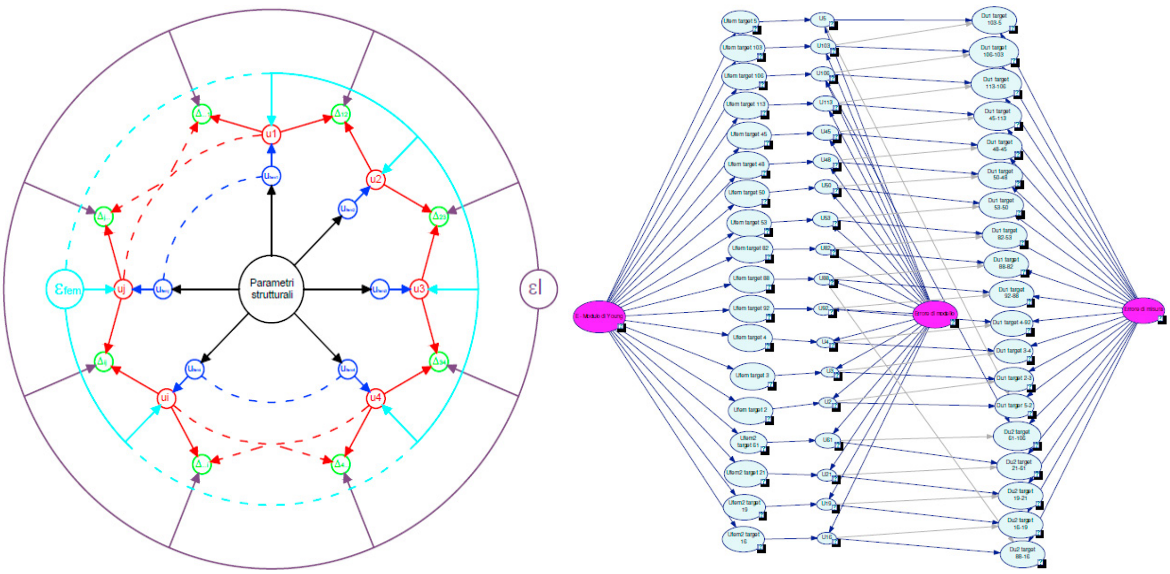

Fig. 5. (a) Scheme of the adopted Bayesian Network; (b) Outline of the Bayesian Network modeled in Genie 2.0.

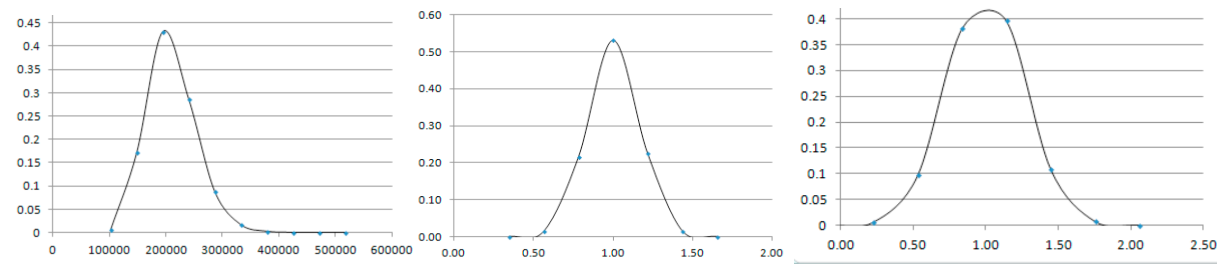

Fig. 6. (a) Young's modulus PDF; (b) FEM error PDF; (c) Survey error PDF.

6. Discussion of the Bayesian Updating results

Bayesian updating of the node PDFs can be performed as the values of variables D_{ij} are set, as evidence, equal to the survey measurements. Subsequently, the software computes by inference the posterior PDFs of each node.

The quantities of interest for this survey are the real displacements U_i whose updated PDFs represent the probability distributions of the real displacements attained by the structure. Expected values of the displacement of each monitored node are reported in Figure 9(a) where anomalously high values are boxed in red and represented as red bullets in Figure 9(b). In particular, nodes belonging to the top beam (n. 21) and to the edge beam (n. 48 and 50) are expected to attain at displacement greater than one centimetre with probability values of, respectively, 46.3%, 64% and 65.6%. The latter two nodes are located on a doubled beam in proximity of some joints with columns and the high associated probability indicates a high risk that the region nearby such nodes can present significant damage. It is worth to be emphasized that a structural inspection, subsequent to the survey, has proved that some connections located nearby the two anomalous nodes were affected by manufacturing defects.

E	State0	State1	State2	State3	State4	State5	State6	State7	State8	State9
State0	0.00222	6.29E-06	2.43E-06	3.38E-06	9.88E-06	4.48E-05	0.000261	0.00188	0.010989	0.038462
State1	0.043616	0.000195	2.43E-06	3.38E-06	9.88E-06	4.48E-05	0.000261	0.00188	0.010989	0.038462
State2	0.470579	0.044788	0.000168	3.38E-06	9.88E-06	4.48E-05	0.000261	0.00188	0.010989	0.038462
State3	0.452815	0.629332	0.183048	0.008047	8.89E-05	4.48E-05	0.000261	0.00188	0.010989	0.038462
State4	0.030611	0.324049	0.797648	0.864466	0.570895	0.213537	0.037317	0.005639	0.010989	0.038462
State5	0.000159	0.00163	0.019131	0.127477	0.428986	0.786284	0.961639	0.986842	0.945055	0.807692

Fig. 7. Conditional PDF values of a displacement $U_{fem,i}$.

Err.Mod	State0						State1					
Ufem5	St.0	St.1	St.2	St3	St.4	St.5	St.0	St.1	St.2	St3	St.4	St.5
St.0	0.17	0.17	0.17	0.01	0.00	0.02	0.17	0.10	0.00	0.00	0.00	0.00
St.1	0.17	0.17	0.17	0.01	0.00	0.02	0.17	0.10	0.00	0.00	0.00	0.00
St.2	0.17	0.17	0.17	0.01	0.00	0.02	0.17	0.10	0.01	0.00	0.00	0.00
St.3	0.17	0.17	0.17	0.01	0.00	0.02	0.17	0.40	0.35	0.03	0.00	0.00
St.4	0.17	0.17	0.17	0.56	0.09	0.02	0.17	0.20	0.62	0.90	0.51	0.02
St.5	0.17	0.17	0.17	0.40	0.90	0.92	0.17	0.10	0.02	0.08	0.49	0.98

Err.Mod	State2						State3					
Ufem5	St.0	St.1	St.2	St3	St.4	St.5	St.0	St.1	St.2	St3	St.4	St.5
St.0	0.13	0.01	0.00	0.00	0.00	0.00	0.15	0.03	0.00	0.00	0.00	0.00
St.1	0.25	0.03	0.00	0.00	0.00	0.00	0.31	0.20	0.02	0.00	0.00	0.00
St.2	0.25	0.36	0.08	0.00	0.00	0.00	0.23	0.50	0.32	0.03	0.00	0.00
St.3	0.13	0.48	0.67	0.24	0.01	0.00	0.15	0.26	0.59	0.53	0.08	0.00
St.4	0.13	0.10	0.25	0.74	0.81	0.25	0.08	0.01	0.08	0.43	0.86	0.55
St.5	0.13	0.01	0.00	0.01	0.18	0.75	0.08	0.01	0.00	0.00	0.06	0.45

Err.Mod	State4						State5						State6					
Ufem5	St.0	St.1	St.2	St3	St.4	St.5	St.0	St.1	St.2	St3	St.4	St.5	St.0	St.1	St.2	St3	St.4	St.5
St.0	0.25	0.11	0.01	0.00	0.00	0.00	0.17	0.25	0.05	0.00	0.00	0.00	0.17	0.17	0.17	0.02	0.00	0.02
St.1	0.25	0.42	0.11	0.00	0.00	0.00	0.17	0.25	0.29	0.02	0.00	0.00	0.17	0.17	0.17	0.10	0.00	0.02
St.2	0.13	0.32	0.50	0.13	0.00	0.00	0.17	0.17	0.49	0.29	0.02	0.00	0.17	0.17	0.17	0.44	0.09	0.02
St.3	0.13	0.11	0.36	0.66	0.24	0.00	0.17	0.17	0.15	0.61	0.44	0.01	0.17	0.17	0.17	0.35	0.55	0.14
St.4	0.13	0.01	0.03	0.20	0.73	0.77	0.17	0.08	0.00	0.08	0.53	0.89	0.17	0.17	0.17	0.08	0.34	0.74
St.5	0.13	0.01	0.00	0.00	0.02	0.23	0.17	0.08	0.00	0.00	0.01	0.09	0.17	0.17	0.17	0.02	0.01	0.05

Fig. 8. Conditional PDF values of a displacement U_i.

a

TARGET	UATTESI [m]	P(UATTESI)	Δu		
5	-0.00055839	0.616	Δu 103-5	0.001710	m
103	-5.4318E-05	0.385	Δu 106-103	0.002852	m
106	-0.00212491	0.615	Δu 113-106	0.002826	m
113	-0.00132408	0.604	Δu 45-113	0.012511	m
45	-0.00127318	0.753	Δu 48-45	0.010694	m
48	-0.01228179	0.64	Δu 50-48	0.004621	m
50	-0.01228146	0.656	Δu 53-50	0.010519	m
53	-0.00127327	0.465	Δu 82-53	0.005505	m
82	-0.00035566	0.603	Δu 88-82	0.005060	m
88	-0.00212337	0.622	Δu 92-88	0.005377	m
92	-0.00054246	0.575	Δu 4-92	0.003114	m
4	-0.00566351	0.856	Δu 3-4	0.003443	m
3	0.00123643	0.918	Δu 2-3	0.000773	m
2	0.0016422	0.409	Δu 5-2	0.000311	m
61	-0.00201967	0.756	Δu 61-106	0.006804	m
21	-0.01219386	0.463	Δu 21-61	0.016436	m
19	-0.00227373	0.491	Δu 19-21	0.011874	m
16	-0.00312786	0.73	Δu 16-19	0.000944	m
			Δu 88-16	0.010199	m

b

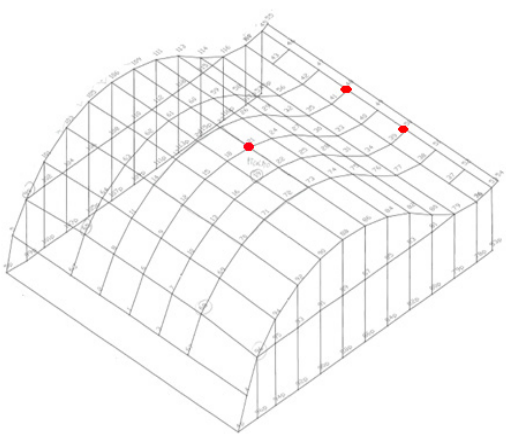

Fig. 9. (a) Expected values of the structural displacements; (b) Nodes with anomalous displacements.

7. Conclusions

A Bayesian Network conceived for the survey of a steel vault, aiming to update the structural parameters and forecast expected displacements and anomalous behaviours, has been presented. This was motivated by the fact that even the rough analysis of a preliminary survey, when the structure was subject to its self-weight only, indicated unexpectedly large displacements at the top of the vault and nearby the edge beam. Eventually, the Bayesian updating analysis described in this research confirmed such an anomalous behaviour which was caused by handcrafting defects of some steel joints between beam and column elements.

The main advantage of the proposed network consists in the fact that the updating analysis forecasts expected values of the displacements (namely displacements of nodes 21, 48 and 50, all resulting about 1.22 cm), which can be compared with the corresponding design values, before that they actually occur inducing significant damage. Moreover, the computed expected values are characterized by a probability distribution which determines the likelihood of the outcomes and the confidence of the results.

Despite of their efficiency, Bayesian Networks often present a strong mutual dependency of the considered variables resulting in significant computational effort. Moreover, such a demand increment tends to sensibly increase as the set of variables of the model is enriched in order to characterize more accurate responses.

For this reason, future work will investigate alternative strategies such as algorithms based on the likelihood principle or, especially, strategies focused on different formulation of the network dependencies in order to reduce the mutual dependencies and to obtain almost-Markovian structures.

Nevertheless, Bayesian Networks represent a very effective approach defining a reliable computational tool, although its efficiency depends on suitable updating procedures of the statistical characterization, which results particularly suitable for multidisciplinary activities and for data exchange with different technologies such as spatio-temporal GIS systems.

Acknowledgments

Financial support from the Italian Ministry of Education, University and Research (MIUR) in the framework of the Project PRIN code 2015HJLS7E – is gratefully acknowledged.

References

Bensi M., Der Kiureghian A., Straub D. - Bayesian network modeling of correlated random variables drawn from a Gaussian random field - Structural Safety, 2011.

Cárdenas I.C., Al-jibouri S.S., Halman J.I. – A Bayesian Belief Networks Approach to Risk Control in Construction Projects – University of Twente, The Netherlands, 2012.

Casaca J., Mateus P., Coelho J. – Bayesian Estimation in Dam Monitoring Networks – 4th IAG Symposium on Geodesy for Geotechnical and Structural Engineering, Lisbon 2008.

Chizhova M., Korovin D., Gurianov A., Brodovskii M., Brunn A., Stilla U., Luhmann T. – Probabilistic reconstruction of the orthodox churches from precision point clouds using Bayesian network and cellular automata – The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, Volume XLII-2/W3, 2017 - 3D Virtual Reconstruction and Visualization of Complex Architectures, Nafplio, Greece, 1 – 3 March 2017.

D'Urso M.G., Russo P., Sessa S. – Sull'impiego del monitoraggio geomatico per la stima dei parametri nei modelli di calcolo degli spostamenti strutturali – Atti della 16a Conferenza Nazionale ASITA, Vicenza, vol. I, 523-530, 2012.

Growe S., Schroder T., Liedtke C.-E. – Use of Bayesian networks as judgement calculus in a knowledge based image interpretation system – International Archives of Photogrammetry and Remote Sensing, Vol. XXXIII, Part B3 – Amsterdam 2000.

Horney M. – Bayesian Networks – Technical Report n. 5 – Boston University – 2014

Jensen F.V., Nielsen T.D. – Bayesian Networks and Decision Graphs second edition – Springer Science + Business Media LLC, 2007 - ISBN-10: 0-387-68281-3

Niculita O., Skaf Z., Jennions I.K. - The Application of Bayesian Change Point Detection in UAV Fuel Systems – ScienceDirect, Procedia CIRP 22, 115-121, 2014.

Russell S.J., Norvig P. - Artificial intelligence: a modern approach. 2nd edition, Prentice-Hall, Englewood Cliffs, N.J., 2003.

Sansò F., De Lacy M.C. – The Bayesian Approach Applied to Significant Deformation Identification - Geodetic Deformation Monitoring From Geophysical to Engineering Roles, Springer – International Association of Geodesy Symposia Vol. 131,19-29 – March 17-19, 2005.

Sessa S., D'Urso M.G. - Employment of Bayesian networks for risk assessment of excavation processes in dense urban areas - Safety, Reliability, Risk and Life-Cycle Performance of Structures & Infrastructures - Eds: Deodatis, Ellingwood & Frangopol, 2013 Taylor & Francis Group, London, ISBN 978-1-138-00086-5; 3163-3169

Straub D. – Part I Bayesian Networks – Lecture Notes on Engineering Risk Analysis - Technische Universität München, Summer schools, 2010.

Straub D. – Part J Modeling with Bayesian Networks – Lecture Notes on Engineering Risk Analysis - Technische Universität München, Summer schools, 2010.

Tanir E., Felsenstein K., Yalcinkaya M. – Using Bayesian methods for the parameter estimation of deformation monitoring networks – Natural Hazards and Earth System Sciences, 8, 335-347, 2008.

Yu Xin,Wen Zongyong, Zhu Zhaorong, Xia Qiang, Shun Lan – Remote sensing image classification applied to the first national geographical information census of China – The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, Volume XLI-B7, 2016 - XXIII ISPRS Congress, Prague, Czech Republic 12 – 19 July 2016.

Zhu J., Santerre R., Chang X.W. – A Bayesian method for linear, inequality – constrained adjustment and its application to GPS positioning – Journal of Geodesy 78, 528-534, Springer-Verlag, 2005 DOI 10.1007/s00190-004-0425-y.