



INSTABILITY OF DE SITTER SPACE AND CORPUSCULAR NATURE OF GRAVITY

Wolfgang Mück

UNIVERSITÀ DI NAPOLI "FEDERICO II"

INFN, SEZIONE DI NAPOLI

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40 YEARS OF RESEARCH (VERY SMALL SELECTION)

GIBBONS, HAWKING 1977

any geodesic observer in dS feels an isotropic heat
bath of particles, $T = \frac{H}{2\pi}$,
horizon is thermodynamically stable

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PERLMUTTER, RIESS, SCHMIDT 1998-99

accelerated expansion of the universe,
 $\sim 68\%$ dark energy,
 $\Lambda \Rightarrow$ late-time cosmology is asymptotically dS

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physical dS vacuum should be time-asymmetric,
cosmological constant evaporates

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IR divergences in dS lead to large loop corrections,
IR regime requires non-perturbative treatment
(similarity with information paradox in BHs)

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IR divergences in dS lead to catastrophic particle
production and breaking of dS symmetry

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DVALI GOMEZ 2014

constant $\Lambda > 0$ is incompatible with corpuscular
picture of dS (“quantum N -portrait”),
decay of coherent graviton state by condensate
depletion

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RAJAMARAN 2016

IR divergence in graviton propagator is removed by
spontaneous deformation of dS background

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cosmological expansion in dS produces soft gravitons, which change the vacuum,
dS Page time $t_{dS} \sim M_p^2 H^{-3}$ is the relevant time scale

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dS is a classical approximation (coherent state) of some quantum evolution,
quantum break time $t_{dS} \sim M_p^2 H^{-3}$

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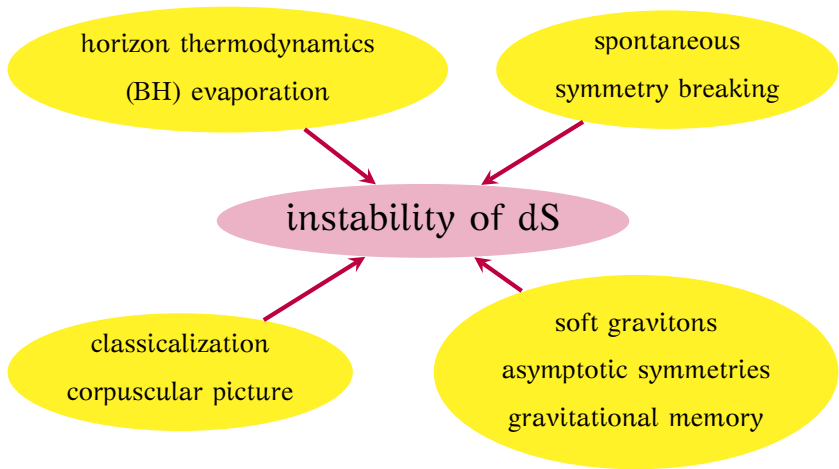
DVALI, GOMEZ, ZELL 2017

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MARKKANEN 2017

renormalized stress energy tensor for conformally coupled scalar in dS implies

$$\dot{H} \sim H^4 M_p^{-2} \quad \Rightarrow \quad t_{dS} \sim M_p^2 H^{-3}$$



- ① peculiar things we know
 - Unruh effect, BH evaporation
- ② de Sitter instability
 - review of two approaches
- ③ corpuscular picture of gravity
 - BHs, dS space, some speculations about Dark Matter

QFT IN CURVED SPACE-TIME

peculiarities

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graph TD; A([peculiarities]) --> B[notions of particles and the vacuum are observer dependent]; A --> C[presence of horizons]; B --- B1[Unruh effect]; B --- B2[cosmological particle production]; C --- C1[entanglement, mixed states, open quantum systems]; C --- C2[thermodynamics, Hawking radiation]; C --- C3[information paradox, firewall argument];
```

notions of particles and the vacuum are observer dependent

Unruh effect

cosmological particle production

presence of horizons

entanglement, mixed states, open quantum systems

thermodynamics, Hawking radiation

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It's not Quantum Gravity

QFT IN CURVED SPACE-TIME

“failure” of QFT at horizons



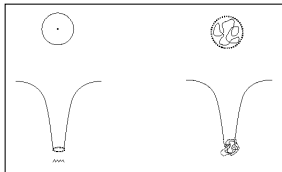
BHs: classical background = coarse grained picture
breaks down at the horizon

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fuzzballs
(Mathur)



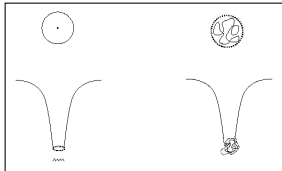
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quantum N -portrait
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BH = BEC of gravitons

$$N \sim (M/M_p)^2$$

$$\lambda \sim r_s$$

QFT IN CURVED SPACE-TIME

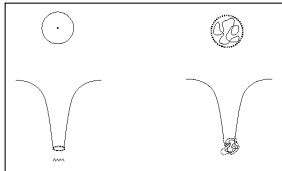
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fuzzballs
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cosmological
(apparent)
horizons
?



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$$N \sim (M/M_p)^2$$

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UNRUH EFFECT

[BIRRELL, DAVIES][WALD]

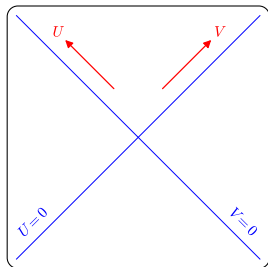
massless scalar in 2d Minkowski space

$$ds^2 = -dU dV$$

$$U = T - X \quad V = T + X$$

positive frequency modes ($\omega > 0$)

$$u_{in,\omega}^M \sim e^{-i\omega U} \quad u_{out,\omega}^M \sim e^{-i\omega V}$$



$$\phi^M = \sum_{\omega > 0} \left(u_{in,\omega}^M a_{in,\omega} + \overline{u_{in,\omega}^M} a_{in,\omega}^\dagger + u_{out,\omega}^M a_{out,\omega} + \overline{u_{out,\omega}^M} a_{out,\omega}^\dagger \right)$$

$$\text{Minkowski vacuum} \quad a_{in,\omega} |M\rangle = a_{out,\omega} |M\rangle = 0$$

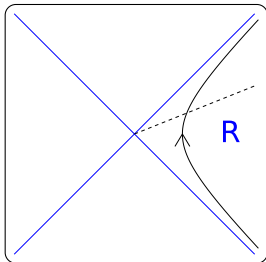
massless scalar in a Rindler wedge (R)

$$U = -\frac{1}{a} e^{-au} < 0 \quad V = \frac{1}{a} e^{av} > 0$$

$$ds^2 = -e^{a(v-u)} du dv$$

positive frequency modes ($\sigma > 0$)

$$u_{in,\sigma}^R \sim e^{-i\sigma u} \quad u_{out,\sigma}^R \sim e^{-i\sigma v}$$



$$\phi^R = \sum_{\sigma>0} (u_{in,\sigma}^R b_{in,\sigma}^R + u_{out,\sigma}^R b_{out,\sigma}^R + c.c.)$$

Rindler (R) vacuum $b_{in,\sigma}^R |R\rangle = b_{out,\sigma}^R |R\rangle = 0$

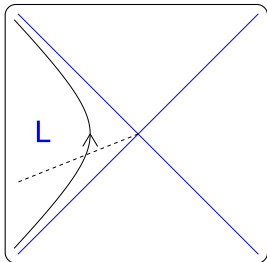
massless scalar in a Rindler wedge (L)

$$U = \frac{1}{a} e^{av} > 0 \quad V = -\frac{1}{a} e^{au} < 0$$

$$ds^2 = -e^{a(v-u)} du dv$$

positive frequency modes ($\sigma > 0$)

$$u_{in,\sigma}^L \sim e^{-i\sigma u} \quad u_{out,\sigma}^L \sim e^{-i\sigma v}$$



$$\phi^L = \sum_{\sigma>0} (u_{in,\sigma}^L b_{in,\sigma}^L + u_{out,\sigma}^L b_{out,\sigma}^L + c.c.)$$

Rindler (L) vacuum $b_{in,\sigma}^L |L\rangle = b_{out,\sigma}^L |L\rangle = 0$

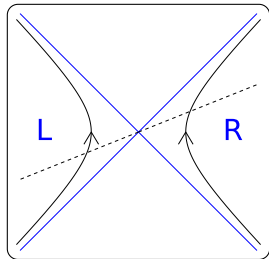
Minkowski Hilbert space in terms of Rindler modes

R-out, L-in sector

$$u_{out,\sigma}^R \sim \begin{cases} e^{-\frac{i\sigma}{a} \ln(aV)} & V > 0 \\ 0 & V < 0 \end{cases}$$

$$u_{in,\sigma}^L \sim \begin{cases} 0 & V > 0 \\ e^{\frac{i\sigma}{a} \ln(-aV)} & V < 0 \end{cases}$$

$$\overline{u_{out,\sigma}^R}, \quad \overline{u_{in,\sigma}^L}$$

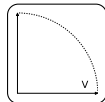


$$\phi^M = \sum_{\sigma>0} (u_{in,\sigma}^R b_{in,\sigma}^R + u_{out,\sigma}^L b_{out,\sigma}^L + c.c.)$$

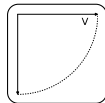
Fourier components with respect to V

$$u_{out,\sigma}^R(\pm\omega) \sim \int_0^\infty e^{\pm i\omega V} e^{-\frac{i\sigma}{a} \ln(aV)} dV$$

$$u_{out,\sigma}^R(\omega) \sim i e^{\frac{\pi\sigma}{2a}} \int_0^\infty dy e^{-\omega y} e^{-\frac{i\sigma}{a} \ln(ay)}$$



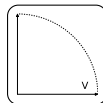
$$u_{out,\sigma}^R(-\omega) \sim -i e^{-\frac{\pi\sigma}{2a}} \int_0^\infty dy e^{-\omega y} e^{-\frac{i\sigma}{a} \ln(ay)}$$



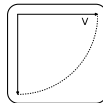
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repeat for

$$u_{in,\sigma}^L, \overline{u_{out,\sigma}^R}, \overline{u_{in,\sigma}^L}$$

positive frequency combinations

$$u_{out,\sigma}^R + e^{-\frac{\pi\sigma}{a}} \overline{u_{in,\sigma}^L} \quad u_{in,\sigma}^L + e^{-\frac{\pi\sigma}{a}} \overline{u_{out,\sigma}^R}$$

Bogoliubov transformation

$$d_{\sigma}^1 = \left(1 - e^{-\frac{2\pi\sigma}{a}}\right)^{-1/2} (b_{out,\sigma}^R - e^{-\frac{\pi\sigma}{a}} b_{in,\sigma}^L \dagger)$$

$$d_{\sigma}^2 = \left(1 - e^{-\frac{2\pi\sigma}{a}}\right)^{-1/2} (b_{in,\sigma}^L - e^{-\frac{\pi\sigma}{a}} b_{out,\sigma}^R \dagger)$$

$|M\rangle$ is entangled state in $|R\rangle \times |L\rangle$ Fock space

$$d_{\sigma}^1 |M\rangle = d_{\sigma}^2 |M\rangle = 0 \quad \Rightarrow \quad |M_{out,\sigma}\rangle = \sum_n p_n(\sigma) |n_{out,\sigma}^R\rangle \times |n_{in,\sigma}^L\rangle$$

$$p_n(\sigma) = \left(1 - e^{-\frac{\sigma}{T}}\right)^{-1/2} e^{-\frac{n\sigma}{2T}}$$

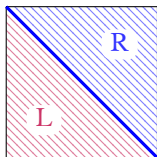
$$T = \frac{a}{2\pi}$$

Unruh
temperature

thermal density matrix

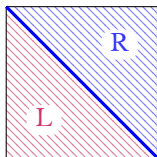
$$\rho_R = \text{tr}_L |M_{out,\sigma}\rangle \langle M_{out,\sigma}| = \sum_n |p_n(\sigma)|^2 |n_{out,\sigma}^R\rangle \langle n_{out,\sigma}^R|$$

UNRUH EFFECT



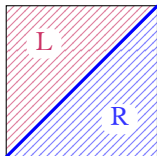
$$\text{R: } ds^2 = -e^{av} dv dU$$

$|M\rangle$ $\begin{cases} \rightarrow \text{thermal for R-out modes} \\ \rightarrow \text{vacuum for R-in modes} \end{cases}$



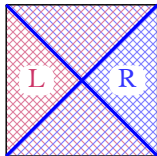
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EFFECTIVE CFT ON THE HORIZON

[PADMANABHAN]

generic metric with horizon

$$ds^2 = -f(r) dt^2 + \frac{f'(r)^2}{4a^2 f(r)} dr^2 + g_{ab}(r, x) dx^a dx^b$$

horizon

$$f(r_h) = 0$$

interacting scalar field

$$\nabla^2 \phi - V'(\phi) = 0$$

$$\xi = \frac{1}{2a} \ln f(r)$$

$$f(r) > 0$$

$$r \rightarrow r_h$$

$$(-\partial_t^2 + \partial_\xi^2) \phi = 0$$

2d massless scalar

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CFT know-how:

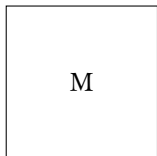
$$ds^2 = C(x^+, x^-) dx^+ dx^-$$

EM tensor

$$\langle T_{\pm\pm} \rangle = -\frac{1}{12\pi} C^{1/2} \partial_\pm^2 C^{-1/2}$$

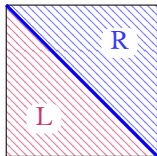
$$\langle T_{+-} \rangle = \frac{1}{24\pi} \partial_+ \partial_- \ln C$$

BHs: THREE VACUA



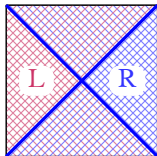
$$\langle T_{\mu\nu} \rangle = 0$$

Hartle-Hawking vacuum, time-symmetric



$$\langle T_{Uv} \rangle = \langle T_{UU} \rangle = 0, \quad \langle T_{vv} \rangle = -\frac{a^2}{48\pi}$$

Unruh vacuum, time-asymmetric, **evaporation**



$$\langle T_{uv} \rangle = 0, \quad \langle T_{vv} \rangle = \langle T_{uu} \rangle = -\frac{a^2}{48\pi}$$

Boulware vacuum, time-symmetric, **singular**

BH EVAPORATION: PAGE TIME AND EMITTED QUANTA

Schwarzschild BH emits Hawking quanta with thermal spectrum

$$T = (8\pi M)^{-1} \quad A = 16\pi M^2 \sim S \quad (\hbar = G = 1)$$

Planck's formula

$$\frac{dL}{d\omega} = \frac{\#A}{8\pi^2} \frac{\omega^3}{e^{\omega/T} - 1}$$

$$L = \frac{\#\pi^2}{120} AT^4$$

$$L = -\frac{dM}{dt} = \frac{\#}{15 \cdot 2^{11}\pi} M^{-2}$$

Page time

$$t_{\text{Page}} = 5 \cdot 2^{11} \pi \#^{-1} M^3$$

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number flux $\frac{d\Gamma}{d\omega} = \frac{1}{\omega} \frac{dL}{d\omega}$

$$\Gamma = \frac{dN}{dt} = \frac{\#\zeta(3)}{4\pi^2} AT^3 = \frac{\#\zeta(3)}{128\pi^4} M^{-1}$$

number of emitted quanta

$$N = \int_0^{t_{\text{Page}}} \Gamma dt = \frac{120\zeta(3)}{\pi^3} M^2 \sim S$$

dS in comoving coordinates

$$ds^2 = -(1 - H^2 r^2) dt^2 + (1 - H^2 r^2)^{-1} dr^2 + r^2 d\Omega^2$$

$$\text{ideal fluid: } -p = \varepsilon = \frac{3H^2}{8\pi G} \quad \text{apparent horizon } r_h = H^{-1}$$

horizon thermodynamics

$$M_h = V_h \varepsilon \quad V_h = \frac{4}{3} \pi r_h^3$$

$$S_h = \frac{A_h}{4\hbar G} \quad A_h = 4\pi r_h^2$$

$$dM_h = T_h dS_h - p dV_h$$

$$T_h = -\frac{\hbar}{2\pi} H$$

DE SITTER PAGE TIME

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$$T_h = -\frac{\hbar}{2\pi} H$$

horizon evaporation

$$-T_h \frac{dS_h}{dt} = \frac{\# \pi^2}{120 \hbar^3} A_h T_h^4$$

$$\Rightarrow r_h^3(t) = r_h^3(0) + \frac{\# L_P^2}{160\pi} t$$

dS Page time

$$t_{dS} \sim L_P^{-2} H^{-3}$$

SPONTANEOUS DEFORMATION OF DE SITTER

[RAJAMARAN]

physical graviton modes propagate like a minimally coupled massless scalar field

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massive scalar in dS

$$(\square - m^2) \phi = 0 \quad ds^2 = \frac{1}{H^2 \tau^2} (-d\tau^2 + d\mathbf{x}^2) \quad \tau \in (-\infty, 0)$$

$$\text{mode expansion} \quad \phi(x) = \sum_k \left[u_k(x) a_k + \overline{u_k(x)} a_k^\dagger \right]$$

$$\text{positive frequency modes} \quad u_k(\tau, \mathbf{x}) \sim (-\tau)^{3/2} e^{-i\mathbf{k} \cdot \mathbf{x}} H_\nu^{(1)}(-k\tau)$$

$$\text{with } \nu^2 = \frac{9}{4} - m^2 H^{-2}$$

SPONTANEOUS DEFORMATION OF DE SITTER

scalar propagator in dS is IR divergent for $m = 0$

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scalar propagator in dS is IR divergent for $m = 0$

Schwinger-Keldysh formalism (CTP)

$$G_{SK}(x,y) = \begin{pmatrix} iF(x,y) & G^R(x,y) \\ G^A(x,y) & 0 \end{pmatrix} \quad \begin{aligned} (\square - m^2) F(x,y) &= 0 \\ (\square - m^2) G^{R,A}(x,y) &= \delta(x,y) \end{aligned}$$

$$F(x,y) = \frac{1}{2} [\langle \phi(x)\phi(y) \rangle + \langle \phi(y)\phi(x) \rangle]$$

SPONTANEOUS DEFORMATION OF DE SITTER

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$$F(x,y) = \frac{1}{2} [\langle \phi(x)\phi(y) \rangle + \langle \phi(y)\phi(x) \rangle]$$

$$k \rightarrow 0 : \quad F(\mathbf{k}; \tau_1, \tau_2) \sim \begin{cases} \frac{H^2}{2k^3} & m = 0 \\ \frac{H^2}{2k^3} (k^2 \tau_1 \tau_2)^{\frac{m^2}{3H^2}} & m \neq 0 \end{cases}$$

$F(x,y)$ is ill-defined for $m = 0$!

SPONTANEOUS DEFORMATION OF DE SITTER

deformed dS

$$ds^2 = \frac{1}{H^2 \tau^2} [-d\tau^2 + f(\tau) d\mathbf{x}^2]$$

for example,

$$f(\tau) = \left(\frac{\tau}{\tau_0} \right)^\varepsilon \quad \varepsilon \ll 1$$

spontaneous deformation

$$f(\tau) \approx 1$$

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$$\square^{(ddS)} \phi = 0 \quad \Rightarrow \quad (\square^{(dS)} - m^2) \phi = 0 \quad m^2 \sim \varepsilon H^2$$

background deformation acts as IR regulator in graviton propagator

SPONTANEOUS DEFORMATION OF DE SITTER

Consider pure trace components

$$8h - 8\tau h' - 4\tau^2 h'' = \frac{8f' - 4\tau f''}{H^2 \sqrt{\kappa\tau}}$$

classical source

SPONTANEOUS DEFORMATION OF DE SITTER

Consider pure trace components

$$8h - 8\tau h' - 4\tau^2 h'' = \frac{8f' - 4\tau f''}{H^2 \sqrt{\kappa\tau}} + 2\sqrt{\kappa} H^2 h_{ij}^2 \tau^2 \stackrel{!}{=} 0$$

classical source tadpole

$$\langle h_{ij}(x) h_{ij}(x) \rangle \sim F(x, x) \sim \frac{1}{H^2 \tau^4 \varepsilon}$$

tadpole cancellation fixes

$$\varepsilon^2 + \frac{2\kappa H^2}{15\pi^2} = 0$$

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Nice idea, but needs more study

QUANTUM BREAK TIME OF DE SITTER

[DVALI, GOMEZ, ZELL]

linearize gravity (with Λ)
around Minkowski

$$\epsilon_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} = -2\Lambda\eta_{\mu\nu}$$

metric =
approximation of dS
for $t_{cl} \ll \Lambda^{-1/2}$

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gravitons with Fierz–Pauli mass

$$\epsilon_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} + m^2 (h_{\mu\nu} - \eta_{\mu\nu} h) = 0$$

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They agree up to an additive constant! $m = \sqrt{\Lambda}$

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QUANTUM BREAK TIME OF DE SITTER

Quantum resolution of dS

trace component of
Fierz–Pauli graviton

$$\Phi_{cl} = \frac{\Lambda}{\sqrt{4\pi m^2}} \cos(mt)$$

($\hbar = G = 1$)

$$\langle N|\Phi|N\rangle = \Phi_{cl}$$
$$\Phi = \sum_{\mathbf{k}} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}}} \left(a_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^\dagger e^{i\mathbf{k}\cdot\mathbf{x}} \right)$$

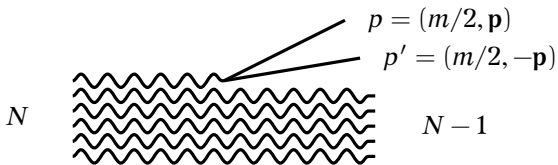
coherent state $|N\rangle = e^{\sqrt{N}(a_0 - a_0^\dagger)} |0\rangle$

$$N = \frac{V\Lambda^2}{8\pi m^3} \quad \text{with} \quad m = \sqrt{\Lambda}, \quad V \sim \Lambda^{-3/2}$$

$$N \sim \frac{1}{\hbar G \Lambda}$$

QUANTUM BREAK TIME OF DE SITTER

decay of the coherent state



decay rate

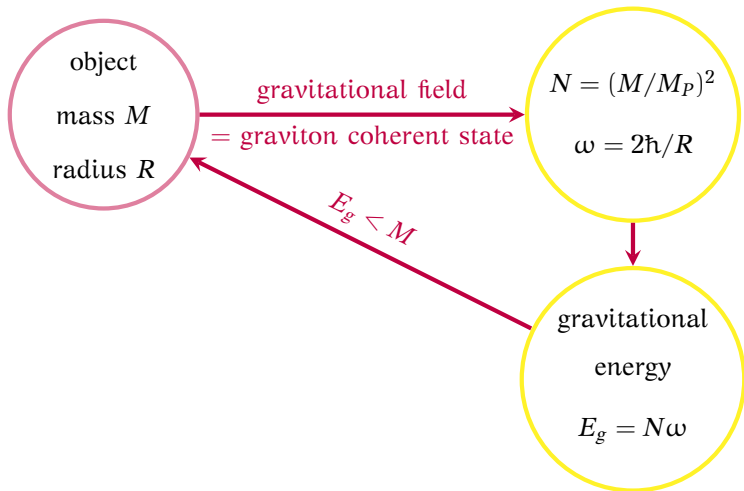
$$\Gamma \sim \Lambda^{1/2}$$

quantum break time

$$t_q \sim \Gamma^{-1} N \sim N \Lambda^{-1/2} \sim L_P^{-2} H^{-3} \sim t_{DS}$$

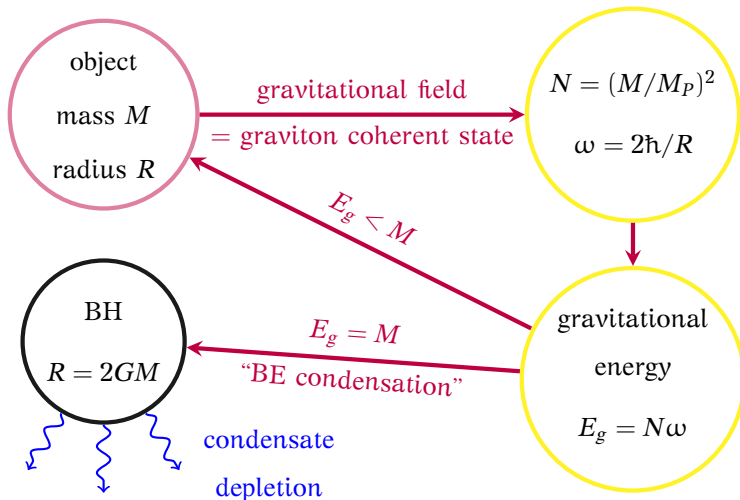
CORPUSCULAR PICTURE OF GRAVITY

[DVALI, GOMEZ]



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CORPUSCULAR PICTURE OF GRAVITY

static, spherically symmetric metric

$$ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2 d\Omega^2$$

Misner-Sharp mass function

$$f(r) = 1 - \frac{2G m(r)}{r}$$

horizon $f(r_h) = 0 \Rightarrow r_h = 2Gm(r_h)$

Quantum N -portrait

$$N = [m(r_h)/M_P]^2 \text{ gravitons with mean energy } \omega = 2\hbar/r_h$$

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For dS, turn the argument around:

Dark Energy density

$$m = \frac{4\pi}{3} R^3 \varepsilon, \quad N\omega = \frac{m^2}{M_P^2} \frac{2\hbar}{R} = m \Rightarrow$$

$$\varepsilon = \frac{3}{8\pi G R^2}$$

CORPUSCULAR PICTURE OF GRAVITY

2 masses, m_1, m_2 , of radii R_1, R_2 , distance $r_{12} \gg R_1, R_2$

of gravitons

$$N = \frac{(m_1 + m_2)^2}{M_p^2} = \frac{m_1^2}{M_p^2} + \frac{m_2^2}{M_p^2} + \frac{2m_1m_2}{M_p^2}$$

gravitational energy

$$E_g = \frac{m_1^2}{M_p^2} \frac{2\hbar}{R_1} + \frac{m_2^2}{M_p^2} \frac{2\hbar}{R_2} + \frac{2m_1m_2}{M_p^2} \frac{\hbar}{r_{12}}$$

total mass

Newtonian gravitational potential

$$m_1 + m_2 = \underbrace{\left(m_1 - \frac{m_1^2}{M_p^2} \frac{2\hbar}{R_1} - G \frac{m_1 m_2}{r_{12}} \right)}_{E_1} + \underbrace{\left(m_2 - \frac{m_2^2}{M_p^2} \frac{2\hbar}{R_2} - G \frac{m_1 m_2}{r_{12}} \right)}_{E_2} + E_g$$

CORPUSCULAR SPECULATIONS ON DARK MATTER

[CADONI, CASADIO, GIUSTI, W.M., TUVERI]

“baryonic” mass μ inside a universe filled with dark energy

$$r_h = 2G(m_d + \mu)$$

$$N = \frac{m_d^2}{M_P^2} + \frac{2\mu m_d}{M_P^2} + \frac{\mu^2}{M_P^2}$$

dark energy

$$m_d = \frac{m_d^2}{M_P^2} \frac{2\hbar}{r_h + \delta} + \frac{2\mu m_d}{M_P^2} \frac{\hbar}{\lambda}$$

horizon shift

new length scale

$$2\mu G = r_\mu \ll \lambda \ll r_h$$

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$$\frac{\delta}{r_h} \left(1 - \frac{r_\mu}{\lambda}\right) = \frac{r_\mu}{\lambda} - \frac{r_\mu}{r_h}$$

1 equation, 2 unknowns

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generic solution must satisfy $r_\mu \ll \lambda \ll r_h$

$$\frac{\delta}{r_h} = \frac{r_\mu}{\lambda} + (1-\gamma)\frac{r_\mu^2}{\lambda^2} + \gamma\frac{r_\mu^2}{\lambda^2} - \frac{r_\mu}{r_h} + \mathcal{O}(r_\mu^3/\lambda^3)$$

$$\delta \approx \sqrt{\gamma^{-1}r_\mu r_h}$$

$$\lambda \approx \sqrt{\gamma r_\mu r_h}$$

γ : $\mathcal{O}(1)$ constant

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dS horizon shift \Rightarrow total apparent matter in dS universe

$$r_h \approx H^{-1} - GM = H^{-1} - \delta \quad \Rightarrow \quad M \approx \sqrt{\frac{2\mu}{\gamma GH}}$$

CORPUSCULAR SPECULATIONS ON DARK MATTER

(one) evidence of DM:
galaxy rotation curves

baryonic Tully–Fisher relation

$$v_f^4(r) \approx a_0 G \mu(r)$$

$$a_0 \approx \frac{1}{6} H$$

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MOND

$$g = \nu(g_N/a_0) g_N$$

$$\begin{cases} \nu(x \ll 1) \sim x^{-1/2} \\ \nu(x \gg 1) \sim 1 \end{cases}$$

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Verlinde 2016

elastic reaction of space–time
to presence of matter

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corpuseular origin ?

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CORPUSCULAR SPECULATIONS ON DARK MATTER

static, spherically symmetric
anisotropic fluid space-time

match BTF
kinematics

$$p_{\parallel} \sim \varepsilon \sim \frac{\sqrt{HG\mu}}{4\pi G} \frac{1}{r^2}$$

μ : baryonic mass

dS asymptotics

horizon shift

$$Hr_h \approx 1 - \frac{\sqrt{HG\mu/6}}{1 + \alpha}$$

$$\varepsilon = \frac{3H^2}{8\pi G} + \frac{\sqrt{HG\mu/6}}{4\pi G} \frac{\alpha}{r^2}$$
$$p_{\parallel} = -\frac{3H^2}{8\pi G} + \frac{\sqrt{HG\mu/6}}{4\pi G} \frac{1 - \alpha}{r^2}$$