STUDIES IN
CLASSIFICATION, DATA ANALYSIS, AND KNOWLEDGE ORGANIZATION

M. Vichi · O. Opitz
Editors

Classification and Data Analysis

Theory and Application

Springer

Studies in Classification, Data Analysis, and Knowledge Organization

11

Springer
Berlin
Heidelberg
New York
Barcelona
Hong Kong
London
Milan
Paris
Singapore
Tokyo

Titles in the Series

H.-H. Bock and P. Ihm (Eds.)
Classification, Data Analysis, and Knowledge Organization. 1991
(out of print)

M. Schader (Ed.)
Analyzing and Modeling Data and Knowledge. 1992

O. Opitz, B. Lausen, and R. Klar (Eds.)
Information and Classification. 1993
(out of print)

H.-H. Bock, W. Lenski, and M.M. Richter (Eds.)
Information Systems and Data Analysis. 1994
(out of print)

E. Diday, Y. Lechevallier, M. Schader, P. Bertrand, and B. Burtschy (Eds.)
New Approaches in Classification and Data Analysis. 1994
(out of print)

W. Gaul and D. Pfeifer (Eds.)
From Data to Knowledge. 1995

H.-H. Bock and W. Polasek (Eds.)
Data Analysis and Information Systems. 1996

E. Diday, Y. Lechevallier and O. Opitz (Eds.)
Ordinal and Symbolic Data Analysis. 1996

R. Klar and O. Opitz (Eds.)
Classification and Knowledge Organization. 1997

C. Hayashi, N. Ohsumi, K. Yajima, Y. Tanaka, H.-H. Bock, and Y. Baba (Eds.)
Data Science, Classification, and Related Methods. 1998

I. Balderjahn, R. Mathar, and M. Schader (Eds.)
Classification, Data Analysis, and Data Highways. 1998

A. Rizzi, M. Vichi, and H.-H. Bock (Eds.)
Advances in Data Science and Classification. 1998

Maurizio Vichi · Otto Opitz
Editors

Classification and Data Analysis

Theory and Application

Proceedings of the Biannual Meeting of the
Classification Group of Società Italiana di Statistica (SIS)
Pescara, July 3–4, 1997

With 97 Figures
and 78 Tables

Springer

Prof. Maurizio Vichi
University "G. D'Annunzio" di Chieti
Dipartimento Metodi Quantitativi e
Teoria Economica
Viale Pindaro 42
I-65127 Pescara, Italy

Prof. Dr. Otto Opitz
University of Augsburg
Lehrstuhl für Mathematische Methoden
der Wirtschaftswissenschaften
D-86135 Augsburg
Germany

ISBN 3-540-65633-2 Springer-Verlag Berlin Heidelberg New York

Cataloging-in-Publication Data applied for
Die Deutsche Bibliothek – CIP-Einheitsaufnahme
Classification and data analysis: theory and application; proceedings of the biannual meeting of the Classification Group of Società Italiana di Statistica (SIS), Pescara, July 3–4, 1997; with 78 tables/Maurizio Vichi; Otto Opitz ed. – Berlin; Heidelberg; New York; Barcelona; Hong Kong; London; Milan; Paris; Singapore; Tokyo: Springer, 1999
(Studies in classification, data analysis, and knowledge organization)
ISBN 3-540-65633-2

Softcover-Design: Erich Kirchner, Heidelberg

SPIN 10700416 32/2202-5 4 3 2 1 0 – Printed on acid-free paper

Preface

International Federation of Classification Societies

The International Federation of Classification Societies (IFCS) is an agency for the dissemination of technical and scientific information concerning classification and data analysis in the broad sense and in as wide a range of applications as possible; founded in 1985 in Cambridge (UK) from the following Scientific Societies and Groups: British Classification Society - BCS; Classification Society of North America -CSNA; Gesellschaft für Klassifikation - GfKl; Japanese Classification Society - JCS; Classification Group of Italian Statistical Society - CGSIS; Société Francophone de Classification - SFC. Now the IFCS includes the following Societies: Dutch-Belgian Classification Society - VOC; Polish Classification Section - SKAD; Portuguese Classification Association - CLAD; Group-at-Large; Korean Classification Society - KCS.

Biannual Meeting of the Classification and Data Analysis Group of SIS

The biannual meeting of the Classification and Data Analysis Group of Società Italiana di Statistica (SIS) was held in Pescara, July 3 - 4, 1997.

The 69 papers presented were divided in 17 sessions. Each session was organized by a chairperson with two invited speakers and two contributed papers from a call for papers. All the works were referred. Furthermore, during the meeting a discussant was provided for each session. A short version of the papers (4 pages) was published before the conference.

The scientific program covered the following topics:

- *Classification Theory*

Fuzzy Methods - Hierarchical Classification - Non Hierarchical Classification - Optimisation approach in Classification. - Classification of Multiway Data - Probabilistic Methods for Clustering - Consensus and Comparison Theories in Classification - Spatial data and Clustering - Validity of Clustering - Neural Networks and Classification - Genetic Algorithms - Classification with Constraints

- *Multivariate Data Analysis*

Categorical Data Analysis - Factor Analysis and Related Methods - Discrimination and Classification - Visual Treatment in Data Analysis Symbolic Data Analysis - Non Linear Data Analysis

- *Multiway Data Analysis*

Analysis of Multiway Data - Panel Data Analysis

- *Proximity Structure Analysis*

Multidimensional Scaling - Similarities and Dissimilarities -

- *Software Developments for Classification and Data Analysis*

Algorithms for Hierarchical and Non Hierarchical Classification - Computer Data Visualization. Statistical Algorithms for Multivariate Analysis

- ***Applied Classification and Data Analysis in Social, Economic, Medical, and other Sciences***

Classification and Data Analysis of Textual Data - Data Analysis in Economics - Classification and Discrimination Approaches in Medical Science

The present volume contains 45 referred papers presented in four chapters as follows:

Classification

- Methodologies in Classification
- Fuzzy clustering and fuzzy methods

Other Approaches for Classification

- Discrimination and Classification
- Regression Tree and Neural Networks

Multivariate and Multidimensional Data Analysis

- Proximity Methodologies in Classification
- Factorial methods
- Spatial Analysis
- Multiway Data Analysis
- Multivariate analysis

Case Studies

Acknowledgements

The Editors would like to extend their sincere thanks to the authors whose enthusiastic participation made the present meeting possible. We are very grateful to the reviewers of the short papers and the time spent in their professional reviewing work. We are also grateful to the chairpersons and discussants of the sessions that also provided very useful suggestions to the authors.

Special thanks are due to the Local Organizing Committee of Pescara: Mauro Coli, Tonio Di Battista, Stella Iezzi, Eugenia Nissi, Antonio Pacinelli, Tonino Sclocco, Maurizio Vichi (*chairman*)

Finally, thanks are extended to Springer-Verlag, Heidelberg.

Maurizio Vichi

TABLE OF CONTENTS

PART I: Classification

Methodologies in Classification

Fuzzy Clustering and Fuzzy Methods

PART II : Other Approaches for Classification

Discrimination and Classification

Spatial Analysis

Multiway Data Analysis

Multivariate Data Analysis

PART IV: Case Studies

Applied Classification and Data Analysis

Shewhart's Control Chart: Some Observations

Massimiliano Giacalone
Department of Mathematics and Statistics, University of Naples "Federico II".
Complesso Monte S. Angelo, Via Cinthia, 80126 Naples.
e-mail: max@matstat.dms.unina.it

Abstract: Data Analysis in Shewhart's Control Chart, to use the original m samples n sized intensities, is the main subject of this paper. Given $m \times n$ intensities we examine three alternatives to sintetize the variability: a) arithmetic mean of m standard deviations $\left({}^{'}S\right)$; b) root mean square of m variances $\left({}^{''}S\right)$; c) global dispersion $\left({}^{'''}S\right)$. We prefer the global dispersion to estimate parent population σ^2.

As an alternative we suggest to analyze all the items of an unique random sample dimensioned in such a manner to have an efficient σ^2 estimate. A second introducted proposal is to use the Factory's needs: $\left(P_0, P_1, \alpha, \beta, L \text{ and } U\right)$. Some examples are given in the last session of the paper.

Keywords: Shewart's Control Chart, Sigma's Estimate, Data Analysis.

1. Introduction

Using S.C.C. (Shewhart's Control Chart) it is customary to operate 2 stages; a first stage devoted to data collection and limits $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s, UCL_s (Lower Control Limit and Upper Control Limit for mean and dispersion) computation.The second stage is devoted to chart's use.

In the first stage it is customary to produce $K = m \cdot N$ items, (in other words we have m lots N sized), to draw m single random samples n size from each lot N sized. The population is given by all items *produced and to be produced,* its mean is μ and its variance is σ^2; μ and σ^2 supposed stable in the first stage (*items produced*).

Let us call x_{ij} the ith intensity of the jth sample, so the jth sample mean is:

$$\overline{x_j} = \sum_i x_{ij}/n \qquad (i = 1,2,\ldots n)$$

$$s_j^2 = \sum_i \left(x_{ij} - \bar{x}_j\right)^2 / (n-1) \qquad (1)$$

is the jth sample variance estimate;

$$\overline{\overline{X}} = \sum_j \overline{x}_j/m,$$

is the mean of the sample means.
Sample mean synthesis create no problem, not the same happens for s^2 or s. Indeed some authors (W.A. Shewhart, 1931); (A. J. Duncan, 1965); (P.L. Piccari, 1974); (D.C. Montgomery, 1991) propose to compute:

$$'S = \sum s_j/m \tag{2}$$

Some other authors (Mittag-Rinne, 1993) propose to compute:

$$''S = \left\{\sum s_j^2/m\right\}^{1/2} \tag{3}$$

finally one may also compute:

$$'''S = \left\{\sum_i \sum_j \left(X_{ij} - \overline{\overline{X}}\right)^2 \Big/ (m \cdot n - 1)\right\}^{1/2}; \tag{4}$$

In this paper we study the rationale of each solution and we suggest an alternative proposal.

2. Synthesis analysis

Since root mean square is greater than or equal to arithmetic mean, we may write:

$$'S \leq ''S,$$

and declare that one of the introduced formulae can't be correct. Relation (2) is the main suspect because since:

$$E(s) \neq \sigma$$

The same may be said for (3), and this means $'S$ to be a biased σ estimate. Someone notes that, if the underlying population is normal, $'S$ actually estimates $\sigma \cdot c_2$; this is statistically correct but a little cumbersome. We remember that c_2 is a constant depending on the sample size n:

$$c_2 = \{2/(n-1)\}^{1/2} \cdot \Gamma\{(n-1)/2\};$$

tabulated values are presented in Duncan (1965).
Let us now consider the synthesis of sample variances (3). Kenney and Keeping

(1956), showed that:

$$E\left(s^2\right)=\sigma^2,$$

not only for simple samples, but also in presence of m simple samples. In case h independent samples are available from the universe, they suggest to use:

$$\hat{\sigma}^2 = Q/(U-h);$$

where

$$Q=n_1 s_1^2 + n_2 s_2^2 + \ldots\ldots + n_h s_h^2;$$

$$U=n_1 + n_2 + \ldots\ldots\ldots + n_k;$$

and s_i^2 is the variance in the ith sample consisting of n_i variates.
If $n_i=n$ is the same for every sample, we have:

$$\hat{\sigma}^2 = n\left(s_1^2 + s_2^2 + \ldots\ldots + s_h^2\right)/(U-h);$$

where $U=n\cdot h$. Clearly the last relation may be written in the form:

$$(n-1)/n\cdot\hat{\sigma}^2 = \left(s_1^2 + s_2^2 + \ldots\ldots + s_h^2\right)/h$$

The constant $(n-1)/n$ is present because the authors started with $s_j^2=\Sigma_i\left(X_{ij}-\overline{X}\right)^2/n$ instead of s_j^2, but if the degrees of freedom are used, the result is correct and consistent with: $E\left(s^2\right)=\sigma^2$. This solution records time variations. In other words we have a trace of variability changes during data collection period.
Finally relation (4) is based on the whole group. It may be seen as the *total variance*, while $\bar{S}^2$ may be seen as *within variance*. Deviances are the same if *between variance* is equal to zero.
There is someone discouraging its use. For instance D.C. Montgomery (1991), affirms that the estimate of the process standard deviation σ used in constructing the control limits is calculated from the variability within each sample. Consequently, the estimate of σ reflects within-sample variability only. It is not correct the estimate of σ based on the usual quadratic estimator, say $\bar{\bar{S}}$, because if the sample means differ, then this will cause $\bar{\bar{S}}$ to be too large. Consequently, in this way ,σ could be overestimated.
A. J. Duncan (1965) shares the same opinion, and retaines that is not correct to estimate the process standard deviation from all the data (e. g. $\bar{\bar{S}}$) and use this in setting up limits for the $\overline{X}$-chart. The estimate of the process standard deviation to be used in setting up limits for the $\overline{X}$-chart must be computed from the within-sample variation to the exclusion of the between-sample variation.

Let us remember that if a production process presents stable between-sample variation it could be a good rule to look for the trouble and to remove it if possible.If the problem persist we do not see why to ignore it, computing the so called within variation. Another important remark is the difference between "first stage" and "second stage". In the second stage production must be monitored so that it is very useful to divide output into lots, let us say N sized, and investigate every single lot produced. If no trouble appears production can continue; on the contrary, if a trouble comes out it is much better to stop production and to look for happenings. In the second stage, points are regarded as independent events and O.C.C. (Operating Characteristic Curve) is computed under this assumption (G. Rouzet, 1957). In short, the division of production into lots N sized is a suitable procedure for the second stage as we said before.
The first stage problem is a different one. to estimate μ and σ^2 related to the character of interest. The subject involved is the *parent population* and its parameters. The division of items into lots N sized is not an essential operation. Perhaps the sample repetition is a mechanical consequence of the second stage technique, to some extent necessary if $n=5$, because μ and σ^2 estimates based on so a little sample should be extremely poor ones, so to have both ways saved some authors suggested to repeat the sample (and the lot) m times (Mittag-Rinne,1993). It seemed therefore a natural consequence to compute $\overline{X}_j$, s_j and $'S$, $''S$ and $'''S$.

3. Simulation

In order to emphasize our opinion we consider a simulation. We shall use Wold's Random Normal Deviates divided into lots N=50 sized, one numbers column for lot. From each column we draw one sample n sized and this operation will be repeated m (=20) times as in the first stage practice. We compute m $\bar{x}$ and m σ^2,and the synthesis $''S^2$ is compared with $'''S^2$. We define: DifTot = $\sigma^2 - ''S^2$ and DifUni = $\sigma^2 - '''S^2$. We also noted that here σ^2 is the population variance computed on $N \cdot m$ data = 1000 considering series of 100 samples. If DifTot<DifUni one point is given to $''S^2$, but if DifUni<DifTot then one point is given to $'''S^2$.
For series of samples $n=5$ sized we found more than 75% points for DifUni, then for $'''S^2$.

4. Alternative proposals

The first stage procedure is a very expensive one. Infact after m samples we must revise the production process, therefore to save time and money we

suggest to analyze all the items produced within the first stage and dimension this sample according to wanted protection.
Our suggestion seems particularly useful for destructive control analysis because with customary procedure if not analyzed items are out of tolerance, production-control costs increases.
Calling N the first stage lot size, we shall have: $\overline{X}=\sum_i X_i/N$, and $\overline{S}^2=\sum_i (X_i-X)^2/(N-1)$, as an unbiased σ^2 estimate.
A different suggestion is based on the introduction of Factory's needs $(P_0, P_1, \alpha, \beta, L \text{ and } U)$. Many authors, use symbol L for *Lower specification limit* and symbol U for *Upper specification limit.*
Now let us call P_0 the well known *Acceptable Quality Level* and we underline that it seems suitable subdivide P_0 into to parts, the one on the left ${}_LP_0$ (fraction of too small items) and the other on the right ${}_UP_0$ (fraction of too large items), of course ${}_LP_0+{}_UP_0=P_0$. This is enought for the computation of:

$$\overline{X}_0=(Lz_U-Uz_L)/(Z_U-Z_L); \qquad (Z_L<0)$$

$$\sigma_0=(U-L)/(Z_U-Z_L);$$

where Z_L is the normal standardized fractile given ${}_LP_0$, and Z_U the one given ${}_UP_0$. $\overline{X}_0$ and σ_0 are the parameters to be used for Shewhart's variables Control Chart computation.
The SCC so obtained is a very different tool because it privileges Factory's needs, whereas customary procedure privileges process capability. Therefore, once obtained the new SCC $(UCL_{\bar{x}}, LCL_{\bar{x}}, UCL_s)$ we must look if production process is able to output material just as designer wants (L and U).
For this test we must collect N data related to the character of interest and compute $S^2=\sum(X_i-\overline{X})^2/(N-1)$, the variance of the last N items produced and compare S^2 with σ_0^2. If $S^2<\sigma_0^2$ the process is capable.
According to capability studies experience it is better to accept the process if $S^2/\sigma_0^2<1.23$. We did not use here the so called natural tolerance concept because it is enought to compare directly variances in order to have the test accomplished.
If production process is not able we suggest an innovative maintenance keeping into account cost embroiled with this operation.
To complete Factory needs list we remember P_1 known as *Lot Tolerance Percent Defective;* α, the producer's risk or first type error probability; β, the consumer's risk or second type error probability. All these values must be contractually chosen.

5. An example

Let us take some data based on the: "Inside diameter for automobile engine piston rings" (Montgomery, 1991, pag. 234).
We have: $\overline{\overline{X}} = 74.001$; $'S = 0.0090$; $''S = 0.0099$; $'''S = 0.0101$. The limits for the $\bar{x}$ chart are:

$$UCL_{\bar{x}} = \overline{X} + A_3 \, 'S = 74.001 + (1.427)\,(0.009) = 74.014;$$

$$LCL_{\bar{x}} = \overline{X} - A_3 \, 'S = 74.001 - (1.427)\,(0.009) = 73.988;$$

and for the S chart:

$$UCL_S = B_4 \, 'S = (2.089)(0.009) = 0.019.$$

If we introduce : $L = 73.981$; $U = 74.021$; ${}_L P_o = 1\%$ ${}_U P_o = 1\%$, $(P = 2\%)$ we can compute the new parameters according to our proposal. We consider:

$$z_L = -2.32635; \quad z_L = 2.32635; \quad \chi^2_{0.002} = 16.92386;$$

$$\overline{X}_0 = (L \cdot z_U - U \cdot z_L)/(z_U - z_L) = 74.001;$$

$$\sigma_o = (U - L)/(z_U - z_L) = 0.0086.$$

Therefore:

$$UCL_{\bar{x}} = 74.001 + 3\,(0.0086)/\sqrt{5} = 74.01254;$$

$$LCL_{\bar{x}} = 74.001 - 3\,(0.0086)/\sqrt{5} = 73.98946;$$

$$UCL_s = \sigma_0 \sqrt{\chi^2/(n-1)} = 0.0086\sqrt{16.92386} = 0.0177.$$

We note that our UCL_s is based on the χ^2-distribution as suggested by Duncan (1965). Our limits are slightly narrower than Montgomery's ones, but if factory needs are the declared ones (L and U) we have a production process not capable. Here is very important the designer responsibility because a little larger tolerances would change the situation.
We repeat the observation outlined above. Customary control charts privileges process capability. In presence of a chart (*UCL* and *LCL* for $\bar{x}$ and s) we

must verify if designer's needs (L and U) are satisfied. On the contrary with our control chart designer needs are privileged but we do not know if production process is capable. In order to get this last peace of information we must compare $''S$ with σ_0. Of course if $''S < \sigma_0$ the process can satisfy designer's needs; on the contrary we must solve the trouble. For the process capability analysis many references are given by Montgomery (1991).

6. Use of the chart

In order to use the chart we draw a sample with $n = 5$ and compute $\bar{x}$ and s. With the following data ; 74.002; 73.990; 73.997; 74.003, 74.001, we obtain: $\bar{x} = 74.002$ $s = 0.002588$. The points are within limits so that production is good. Let us now try to use the s^2 chart as proposed by Duncan. We have: $s^2 = 0.0000067$ and $(\mathrm{UCL}_s)^2 = 0.000079 \cdot 4.230965 = 0.000313$; the point is within limits as before. If we suppose to have a point very near the limit e.g. s =0.0176, squaring it we get: $\sigma^2 = 0.00030976$ and it is also within limits.
If the point is just out of control e.g. s =0.0178, we get $s^2 = 0.00031684$ and the point is just out of control also in the new chart.

7. Conclusions

SCC (Shewart's Control Chart) is based on process ability to produce wanted items. Indeed, if control limits $(\mathrm{UCL}_{\bar{X}}\ \mathrm{LCL}_{\bar{X}}\ \mathrm{UCL}_S)$ are computed on either $''S^2$ or $'''S^2$, it is not worthy to insist on process ability. Clearly there is also the designer and their needs (L, U) to be considered so we must consider a capability study to test if they are in accordance.
We have seen how it is possible to get new control limits based on $\bar{\bar{X}}_0$ and σ_0 keeping into account designer's needs (L and U). Items must be output by production process so that now must look if it is able to do its work.
A different subject is the dispersion estimate related to SCC taking into account the presence of m lots. We have seen that 's is a biased statistic very cumbersome to be adjusted. A simulation leads to prefer $'''S^2$ but this means to use a σ^2chart instead of a σ chart. Nevertheless in our example we used $''S$ and $''S^2$ in order to simplify the discussion.
It seems worthy to remember that customary control chart construction privileges production process capability whereas our suggestion privileges designer needs. In every case we must verify the second coin's face to compare $''S$ with σ_0 or better $'''S$ with σ_0. Better else to compare $'''S^2$ with σ_0^2 because $'''S^2$ is a σ^2 unbiased estimate.

References

Duncan, A.J. (1965). *Quality Control and Industrial Statistics.* Richard D. Irwing, Inc. Homewood, Ill.

Kenney, A.F.& Keeping, E.S. (1956). *Mathematics of Statistics.* D. Van Nostrand Co. Inc., Princeton.

Mittag, H.G. & Rinne, H. (1993) *Statistical Methods of Quality Assurance.* Chapman & Hall, London.

Montgomery, D.C. (1991). *Introduction to Statistical Quality Control.* John Wiley & Sons. New York.

Piccari, P.L. (1974). *Manuale di Controllo di Qualità e Affidabilità.* ISEDI, Milano.

Rouzet, G. (1957). *Courbes d'Efficacité de la Carte de Contrôle pae Mesures en Fonction du Pourcentage de Pièces Défectueuses.* Revue de Statistique Appliquée, vol. V, 2, 19-32.

Shewart, W.A. (1931). *Economic Control of Quality of Manufactured Product.* D. Van Nostrand Co., Inc. New York.

Wold H., (1955) *Random Normal Deviates.* Department of Statistics University of London. Tracts for Computers Edited by E. S. Pearson D. Sc. n.XXV.

AUTHOR INDEX

S

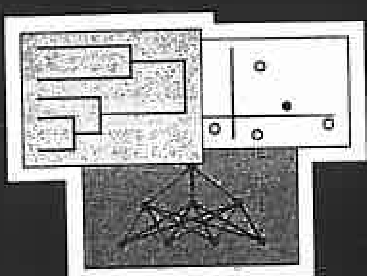
T

V

KEY WORDS

Classification and Data Analysis

The book provides new developments in classification, data analysis and multidimensional methods, topics which are of central interest to modern statistics. A wide range of topics is considered including methodologies in classification, fuzzy clustering, discrimination, regression tree, neural networks, proximity methodologies, factorial methods, spatial analysis, multiway and multivariate analysis.



ISBN 3-540-65633-2

9 783540 656333

http://www.springer.de