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International Environmental Agreements with Social Externalities: A Global Emission Game with Asymmetric Players

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Abstract

In this work we present a Global Emission Game with N asymmetric players, in which the pay-off of cooperators is affected by a Social Externalities, that we assume to be a positive function. We refer to the notion of self-enforcing agreements to study the stability of the coalition and we assume that the N players are divided in two homogeneous groups, developed and developing countries. Moreover, the externalities doesn't depend on emissions, but only on the number of players in coalition. So, it has no effects on the choice of optimal emissions, but only on the stability of the coalition.

Keywords: Global Emission Games, self-enforcing agreements, Social Externalities, asymmetric players

1 Introduction

Environmental issues are a major theme for current politicians. Most of these issues, like climate change, acid rain or ozone depletion, are related to transboundary pollution and for that require global policies. There have been many occasions in which sovereign countries met to discuss the problems and find solutions. The Montreal Protocol on Substances that Deplete the Ozone Layer (1987) and the Kyoto Protocol for the reduction of Green House Gases (1997) are just the main examples of International Environmental Agreements (IEAs). Obviously, the problem deals with several economic issues. If we see to coalition formation process, we lie within the class of coordination problems. A natural approach to this class of problems is the Game Theory. So, in the last two decades extensive literature has been developed, since the works of Carraro and Siniscalco [7] and Barrett [2].
Cooperative games stream assumes that a coalition is formed and the main focus is on how to divide the surplus of cooperation between players who join the agreement. On the other side, non-cooperative games stream starts from the consideration that there is no supranational authority that can force players to cooperate, so the central point here is how the coalition formation process works, assuming that each player looks at his interest. Since in this case we have the well-known problem of *free-riding*, then we need some conditions that guarantee the stability of the coalition. We deal with the latter approach and we model a global emission game (see [11]). As usual, we consider a two-stage game: the first stage is the *membership game*, in which players make the choice about the entrance in the coalition, then we have the *emission game*, in which cooperators maximize the joint welfare, while defectors choice emissions by their own. For that, we look for a partial cooperative equilibrium ([19]; [20]; [8]; [3]). Naturally, the membership game is a *metagame*, because each player, choosing whether join or not in coalition, anticipates the choice of the other players and the relative outcomes in terms of emission levels. So, we first solve the maximization problems of the emission game and then we use the optimal emission to approach the membership game. To find a stable coalition, we need to refer to the self-enforcing stability conditions like in [9]. Briefly speaking, we say that a coalition of k players is stable if no one inside has an incentive to withdraw (*internal stability*) and no one outside has an incentive to join (*external stability*).
There are two issues on which we focus. First of all, we assume some degree of asymmetry between players, despite most of the literature assumes a world with symmetric countries. This choice is based on some practical considerations, the most important of which is the need to involve developing countries in emissions' reduction process. On the one hand, we have that developing countries have no responsibility for the actual level of pollution, moreover

other economic issues are central in their economic agenda, like to increase the wealth per capita, to build infrastructure, to increase the level of instruction, etc. So, environment is a "luxury" good. On the other side, in the list of developing countries we have some like China, India, Brazil or South Africa that give a significant contribution to the increment of pollution. In this sense, we need to design agreements that can involve this kind of players in environment protection policies. In our model, we first consider a world with N asymmetric countries and we solve the emission game. To approach the membership game, we assume that players are divided into two homogeneous groups: developed and developing countries.

The second issue is that if we want a stable and large coalition, if possible the grand coalition, we need to create some mechanisms that incentive players to cooperate, or that disincentive players to not cooperate. In literature, several proposals to design a self-enforcing agreement, have been presented. For example, we have models based on IEAs supported by *trigger strategies* (see e.g., [2]; [1]), that's to say that if a player in coalition defects, the other cooperators can punish him. Or, the gain from cooperation can be used, via *transfer scheme* (see e.g., [14]; [23]), to enlarge a stable coalition. Finally, we have the way known as *issue linkage* (see e.g., [18]; [4]; [16]), that tries to make the agreement economically advantageous linking the IEAs to another agreement, that could be R&D, or trade or another economic issue.

We explore another mechanism, that is the Social Externalities introduced in [6]. Our motivation is based on the consideration that an IEA should be seen within a network of relations between sovereign countries. For this reason we think that it is not credible that someone has the power to punish a country like China or has the will to transfer a certain amount to convince it to reduce its emissions.

What we say is that when players have to take the choice of joining the agreement, they consider all possible earnings due to the relationship that they build participating to the agreement. The objective of this work is to extend the framework of Cabon-Dhersin and Ramani [6], considering a world with asymmetric countries.

In Section 2 we present the model and its functional forms, introducing the externalities and the optimization problems. In Section 3 we characterize the optimal emissions and the consequent optimal welfares. In Section 4 we discuss about the stability conditions, we present some conditions for which a stable coalition exists and we present some numerical simulations. Section 5 concludes.

2 The Model

In this section we present the model. We consider a world with N asymmetric countries that correspond to our set of players I, where $I = \{1, ..., N\}$. We make some commonly assumptions about the functional forms of the model. We denote with $f_i(e_i)$ the production function of player i, where $e_i \geq 0$ describes the emissions of the player. The idea behind is that there is a direct correlation between production and pollution[1]. Moreover $D_i(E)$ is the damage-cost function for player i, where E is the global emission, $E = \sum_{p=1}^{N} e_p$. So, for each player i, we consider

$$f_i(e_i) = \delta_i \left(\alpha_i e_i - \frac{1}{2} e_i^2 \right),$$
$$D_i(E) = \beta_i \left(\sum_{i=1}^{N} e_i \right),$$

where e_i is the emission of player i, and the parameters α_i, β_i (vulnerability to environmental damage) and δ_i (shifting marginal benefits) are strictly positive. We consider a quadratic production function that is a classic choice in literature, and a linear damage-cost function, that is not uncommon in literature (see e.g., [15]; [5]; [23]) and supported by some empirical estimations (see [17]). Nevertheless, the differences between a linear damage-cost function and a more realistic quadratic function (see e.g., [22]; [13]; [21]) are almost all quantitative, but not qualitative.

We assume that the grand coalition is the social optimum, so we look for a mechanism that can involve in emission reduction process as many countries as possible. For this reason we introduce in the model the concept of Social Externalities (see [6]). We suppose that when countries decide to join in a coalition, their welfare is affected by an extra-profit that is uncorrelated with the optimization problem, but depends only on the number of cooperators. The idea behind is that countries could be motivated to sign an environmental agreement, although their primary interest is not the environment. Classical example is Russia, that ratified the Kyoto protocol with the hope to have more consideration when its entry in World Trade Organization (WTO) would have been voted. So, we assume that discuss about environment is an occasion to create a link between countries and this link can bring mutual benefit in many economic fields that do not deal with environment protection. It follows that the larger the network, the greater the externalities.

[1]Mathematically, we assume that emissions are by-product of industrial activities x_i, that's to say $e_i = g(x_i)$. If we take $g(\cdot)$ as a smooth function, we can invert it and write $x_i = g^{-1}(e_i) := f_i(e_i)$.

From a mathematical point of view, we add a strictly positive function to the cooperators' welfare, and we assume that this function in not related to the emissions. So, we have that the externalities has effects on the stability of the coalition, but not on the choice of optimal emissions. Its immateriality makes the concept of Social Externalities vague, because potentially within it we have all the possible relations that countries could establish. But we think that the loss of descriptive power (e.g., respect the idea of issue linkage) is acceptable, considering the great flexibility that we gain with this approach. We present in the following the functional forms of the model by choosing the production, the damage-cost and the externality functions.
Without loss of generality, we suppose that the first k players in the set I join the coalition, and maximize the joint welfare, while the remaining $(N-k)$ players act by their own, maximizing their single welfare. So, we have the optimization problem, for each player j not in coalition, as follows:

$$\max_{e_j} w_j^{NC} = \max_{e_j}\left\{\delta_j\left(\alpha_j e_j - \frac{1}{2}e_j^2\right) - \beta_j\left(\sum_{i=1}^{N} e_i\right)\right\}. \tag{1}$$

For the coalition, we have a joint welfare maximization and moreover we hypothesize that a Social Externalities, related to the number of players in coalition only, affects the welfare. So the optimization problem is:

$$\max_{e_1\ldots e_k}\sum_{i=1}^{k} w_i^C = \max_{e_1\ldots e_k}\sum_{i=1}^{k}\left[\delta_i\left(\alpha_i e_i - \frac{1}{2}e_i^2\right) - \beta_i\left(\sum_{h=1}^{k} e_h + \sum_{h=k+1}^{N} e_h\right) + s_i\right], \tag{2}$$

where the parameter $s_i > 0$ represent the Social Externalities.

3 Emission Solutions and Welfares

In order to find the optimal emissions for the defectors, we solve the first order conditions of the problem (1). Deriving respect e_j and imposing the result equal to zero, we have

$$\frac{\partial w_j^{NC}}{\partial e_j} = \alpha_j\delta_j - \delta_j e_j - \beta_j = 0.$$

Rearranging, we can find the expression for non-cooperative emissions, as follows

$$e_j^{NC} = \alpha_j - \frac{\beta_j}{\delta_j}. \tag{3}$$

In a similar way, we calculate the partial derivative of the problem (2), with respect to e_i, where $i = 1, \ldots, k$, and find the emission of every player in coalition by solving the k-dimensional system obtained by imposing that all the derivatives to be equal to zero

$$\begin{cases} \alpha_1\delta_1 - \delta_1 e_1 - \sum_{h=1}^{k}\beta_h = 0, \\ \vdots \\ \alpha_i\delta_i - \delta_i e_i - \sum_{h=1}^{k}\beta_h = 0, \\ \vdots \\ \alpha_k\delta_k - \delta_k e_k - \sum_{h=1}^{k}\beta_h = 0. \end{cases}$$

Then the emission for each player i in coalition is:

$$e_i^C = \alpha_i - \frac{1}{\delta_i}\sum_{h=1}^{k}\beta_h. \tag{4}$$

To find the welfares of cooperative and non-cooperative players, we have to compute $w_i(e_1^C, \dots, e_k^C, e_{k+1}^{NC}, \dots, e_N^{NC})$ for any $i \in I$, that we denote by $w_i^C(k)$ for $i \in \{1, \dots, k\}$ and by $w_j^{NC}(k)$ for $j \in \{k+1, .., N\}$

$$\begin{cases} w_i^C(k) = & \delta_i\left[\alpha_i\left(\alpha_i - \frac{1}{\delta_i}\sum_{h=1}^{k}\beta_h\right) - \frac{1}{2}\left(\alpha_i - \frac{1}{\delta_i}\sum_{h=1}^{k}\beta_h\right)^2\right] + \\ & -\beta_i\left[\sum_{p=1}^{k}\left(\alpha_p - \frac{1}{\delta_p}\sum_{h=1}^{k}\beta_h\right) + \sum_{p=k+1}^{N}\left(\alpha_p - \frac{\beta_p}{\delta_p}\right)\right] + \\ & + \sum_{h=1}^{k} s_h, \\ & \\ w_j^{NC}(k) = & \delta_j\left[\alpha_j\left(\alpha_j - \frac{\beta_j}{\delta_j}\right) - \frac{1}{2}\left(\alpha_j - \frac{\beta_j}{\delta_j}\right)^2\right] + \\ & -\beta_j\left[\sum_{p=1}^{k}\left(\alpha_p - \frac{1}{\delta_p}\sum_{h=1}^{k}\beta_h\right) + \sum_{p=k+1}^{N}\left(\alpha_p - \frac{\beta_p}{\delta_p}\right)\right], \end{cases}$$

in which i is a cooperative player, j is a defector and the coalition consists of k players.

Remark 3.1 *We want to highlight that this game is a positive externalities and superadditive game. Positive externalities says that $\forall i \in C$ and $\forall j \in NC$, then $w_j(C) \geq w_j(C\backslash i)$. Moreover, there exists at least a player $z \in NC$, such that $w_z(C) > w_z(C\backslash i)$. This is clearly true, because if a player withdraws from the coalition the emissions of non-cooperator remain the same, but the damage-cost increase. A game is superadditive if, the aggregate payoff of the coalition, $w^C(k)$, is at least equal to the aggregate payoff, when a player i defeats, plus the payoff of the defector: $w^C(k) \geq w^C(k\backslash i) + w_i(k\backslash i)$. It's possible to show that our game is always superadditive. A game that is both superadditive and with positive externalities is a fully cohesive game, that means that the aggregate payoff of the coalition is an increasing function of the size.*

4 Stability

To establish the number of players of a stable IEA we refer to notions of internal and external stability (see [9]). We want to highlight that these conditions are more stringent and there are different papers that try to propose different ways to face the problem (see [12]; [10]). The basic idea is that a coalition is stable if no one inside has an incentive to defect and no one outside has an incentive to join in. So, called w the pay-off of a player, a coalition of k players is stable if the following inequalities:

$$w_i^C(k) \geq w_i^{NC}(k-1), \qquad w_j^{NC}(k) \geq w_j^C(k+1),$$

are satisfied, where i is a cooperator and j is a defector. First condition is called *internal* stability, while the second one is called *external* stability.
Substituting optimal emissions into the welfare functions, we can make explicit the stability conditions. So, consider first the internal stability condition, that is given by the inequality $w_i^C(k) - w_i^{NC}(k-1) \geq 0$. So, simplifying and rearranging, we have

$$w_i^C(k) - w_i^{NC}(k-1) = -\frac{1}{2\delta_i}\left[\left(\sum_{h=1}^{k}\beta_h\right)^2 - \beta_i^2\right] + \beta_i\left[\frac{1}{\delta_i}\sum_{\substack{h=1\\h\neq i}}^{k}\beta_j + \beta_i\sum_{\substack{h=1\\h\neq i}}^{k}\frac{1}{\delta_h}\right] + + \sum_{h=1}^{k}s_h \geq 0,$$

which becomes

$$w_i^C(k) - w_i^{NC}(k-1) = -\frac{1}{2\delta_i}\left(\sum_{\substack{h=1\\h\neq i}}^{k}\beta_h\right)^2 + \beta_i^2\left(\sum_{\substack{h=1\\h\neq i}}^{k}\frac{1}{\delta_h}\right) + \sum_{h=1}^{k}s_h \geq 0. \qquad (5)$$

We now consider the external stability condition, that gives a number k such that the inequality $w_j^{NC}(k) - w_j^C(k+1) \geq 0$ holds for each player j outside coalition.
After some algebra, we obtain

$$w_j^{NC}(k) - w_j^C(k+1) = \frac{1}{2\delta_j}\left[\left(\sum_{h=1}^{k}\beta_h\right)^2 + 2\beta_j\sum_{h=1}^{k}\beta_h\right] + - \beta_j\left[\beta_j\sum_{h=1}^{k}\frac{1}{\delta_h} + \frac{1}{\delta_j}\sum_{h=1}^{k}\beta_h\right] - \sum_{h=1}^{k}s_h - s_j \geq 0,$$

which becomes

$$w_j^{NC}(k) - w_j^C(k+1) = \frac{1}{2\delta_j}\left(\sum_{h=1}^{k}\beta_h\right)^2 - \beta_j^2\left(\sum_{h=1}^{k}\frac{1}{\delta_h}\right) - \sum_{h=1}^{k}s_h - s_j \geq 0. \quad (6)$$

We can observe immediately that the Social Externalities facilitates the stability of a grand coalition. If we consider the effects of the externalities on the stability conditions we observe that it helps to obtain internal stability, but, on the other side, it has a negative effect on external stability. In others words, if the externality is large enough, we can have that every coalition is internal stable, while there is no size k for which we have external stability. So, players have an incentive to join in the coalition. Suppose now that there exist two kinds of player, developed countries, identified by subscript 1, and developing countries, denoted by 2. We assume that within each subgroup players are homogeneous, and that the coalition is arranged by n_1 players of kind 1 and n_2 of kind 2, with n_1 and n_2 positive integers such that $n_1 + n_2 \leq N$. So, we are assuming that within each group, all players have the same parameters: for countries of kind 1 we have $(\alpha_1, \delta_1, \beta_1, s_1)$, while for countries of kind 2 we have $(\alpha_2, \delta_2, \beta_2, s_2)$.
We must specify the stability conditions for developed countries and for developing ones. So, every condition generates two different inequalities.
For internal stability, we have the two inequalities

$$w_1^C(k) - w_1^{NC}(k-1) = -\frac{1}{2\delta_1}\Big((n_1-1)\beta_1 + n_2\beta_2\Big)^2 + \\ + \beta_1^2\left(\frac{n_1-1}{\delta_1} + \frac{n_2}{\delta_2}\right) + s_1n_1 + s_2n_2 \geq 0; \quad (7a)$$

$$w_2^C(k) - w_2^{NC}(k-1) = -\frac{1}{2\delta_2}\Big(n_1\beta_1 + (n_2-1)\beta_2\Big)^2 + \\ + \beta_2^2\left(\frac{n_1}{\delta_1} + \frac{n_2-1}{\delta_2}\right) + s_1n_1 + s_2n_2 \geq 0, \quad (7b)$$

while for external stability the conditions are

$$w_1^{NC}(k) - w_1^C(k+1) = \frac{1}{2\delta_1}\left(n_1\beta_1 + n_2\beta_2\right)^2 - \beta_1^2\left(\frac{n_1}{\delta_1} + \frac{n_2}{\delta_2}\right) + \\ - s_1(n_1+1) - s_2 n_2 \geq 0; \tag{8a}$$

$$w_2^{NC}(k) - w_2^C(k+1) = \frac{1}{2\delta_2}\left(n_1\beta_1 + n_2\beta_2\right)^2 - \beta_2^2\left(\frac{n_1}{\delta_1} + \frac{n_2}{\delta_2}\right) + \\ - s_1 n_1 - s_2(n_2+1) \geq 0. \tag{8b}$$

4.1 Existence of a Stable Coalition

At this point, we face two questions: whether the system of inequalities (7a), (7b), (8a) and (8b) admits solutions and, if yes, what these solutions are. Unfortunately, it is not analytically tractable in general.
So, in the following we proceed by two steps. First, under suitable assumptions on the parameters we are able to proof that a solution exists. After that, removing the above assumptions on the parameters, we make some numerical simulations to estimate the size and the composition of a stable coalition.
Then, we assume that the vector of coefficients (β_N, δ_S, s_N) is proportional to (β_S, δ_N, s_S) according to a natural number $\sigma \in \mathcal{N}$ assumed greater than 1. Then, there exists $\sigma \in \mathcal{N}, \sigma > 1$ such that

$$\frac{\beta_N}{\beta_S} = \frac{\delta_S}{\delta_N} = \frac{s_N}{s_S} = \sigma.$$

We remark that the case $\sigma = 1$ is not realistic because of the difference in technological advancement of the developed countries.
Thus, if σ is greater than one, we have that $\beta_1 > \beta_2$, $\delta_2 > \delta_1$ and $s_1 > s_2$. The idea behind is that developed countries are more sensitive to environmental damages, and then suffer higher marginal costs. Moreover, this sensitiveness is reflected also on the marginal shift parameters in the production function, that is smaller compared to the one of developing countries. It is natural to assume that developed countries are more prone to join in coalition. We want to highlight that the implication $\delta_1 < \delta_2$ doesn't mean that countries of kind 2 have an higher production function, because of the presence of parameters α_1 and α_2.
Given these assumptions on parameters, after some rearrangements we can rewrite the system as

$$\begin{cases} -y^2 + 2(2\sigma + t)y - 3\sigma^2 \geq 0, \\ -y^2 + 2(2 + t\sigma)y - 3 \geq 0, \\ y^2 - 2(t + \sigma)y - 2t\sigma \geq 0, \\ y^2 - 2(1 + t\sigma)y - 2t\sigma \geq 0, \end{cases} \tag{9}$$

where $y = \sigma n_1 + n_2$ and by the proportionality assumption $t = \sigma \frac{\delta_1 s_1}{\beta_1^2}$. The above system has not always solutions as in the following example.

Example 4.1 *Suppose in a global emission game that $\beta_1 = 2, \delta_1 = 4, s_1 = 2$ and $\sigma = 2$; then $t = 4$ and system (9) consists in the following three conditions*

$$-y^2 + 16y - 12 \geq 0, \qquad -y^2 + 20y - 3 \geq 0, \qquad y^2 - 18y - 16 \geq 0,$$

which are not compatible and in this case there exists no a stable coalition.

We suggest to find a suitable value of s_1 (then of s_2) in order to ensure a stable coalition. More precisely we prove the following result.

Proposition 4.2 (Existence of a stable coalition) *Let us assume that $s_1 = \frac{\beta_1^2}{\sigma\delta_1}$. Then there exists a natural number $\bar{y} \in [1+\sigma+r, 2+\sigma+r]$ satisfying system (9), where $r = \sqrt{1 + 4\sigma + \sigma^2}$.*

Proof 4.3 *By considering the assumed value $s_1 = \frac{\beta_1^2}{\sigma\delta_1}$, we have $t = 1$ and system (9) becomes:*

$$\begin{cases} -y^2 + 2(2\sigma + 1)y - 3\sigma^2 \geq 0, \\ -y^2 + 2(2 + \sigma)y - 3 \geq 0, \\ y^2 - 2(1 + \sigma)y - 2\sigma \geq 0. \end{cases}$$

The first equation is satisfied in the interval

$$[2\sigma + 1 - \sqrt{(2\sigma + 1)^2 - 3\sigma^2}, 2\sigma + 1 + \sqrt{(2\sigma + 1)^2 - 3\sigma^2}] =$$

$$[2\sigma + 1 - r, 2\sigma + 1 + r].$$

The second equation is satisfied in the interval

$$[2 + \sigma - \sqrt{(2 + \sigma)^2 - 3}, 2 + \sigma + \sqrt{(2 + \sigma)^2 - 3}] =$$

$$[2 + \sigma - r, 2 + \sigma + r]$$

and the last one in

$$]-\infty, 1 + \sigma - \sqrt{(1 + \sigma)^2 + 2\sigma}] \cup [1 + \sigma + \sqrt{(1 + \sigma)^2 + 2\sigma}, +\infty[=$$

$$]-\infty, 1 + \sigma - r] \cup [1 + \sigma + r, +\infty[$$

where $1 + \sigma - r < 0$*, and for* $\sigma \geq 1$ *we have*

$$2 + \sigma - r \leq 2\sigma + 1 - r \leq 1 + \sigma + r \leq 2 + \sigma + r \leq 2\sigma + 1 + r.$$

In fact, $2\sigma + 1 - r \leq 1 + \sigma + r$ *implies* $\sigma \leq 2r = 2\sqrt{1 + 4\sigma + \sigma^2}$*, i.e.* $3\sigma^2 + 16\sigma + 4 \geq 0$ *that is true for* $\sigma \geq 1$*, the rest of inequalities being immediate.*

Remark 4.4 *In order to have* $\bar{y} \in [1, N]$ *we have the following bound on* σ*: clearly* $1 + \sigma + r > 1$*, and solving* $2 + \sigma + r \leq N$ *we have* $\sigma \leq \frac{N^2 - 4N + 3}{2N}$*. Since we assume* $\sigma > 1$*, Proposition 4.2 is significant for* $N \geq 8$*, namely our analysis works for at least* 8 *countries world.*

Example 4.5 *Suppose in a global emission game that* $\beta_1 = 2, \delta_1 = 4$ *and* $\sigma = 2$*; then we find* $s_1 = 0.5$ *that ensures a stable coalition with size* $\bar{y} \in [6.6, 8.6]$*, namely* $\bar{y} = 7$*. Since* $2n_1 + n_2 = 7$*, it is possible to have a stable coalition if we have one of the following four possible combinations:* $n_1 = 0$ *and* $n_2 = 7$*,* $n_1 = 1$ *and* $n_2 = 5$*,* $n_1 = 2$ *and* $n_2 = 3$ *or* $n_1 = 3$ *and* $n_2 = 1$*.*

4.2 Simulations

We have shown that, under some assumptions on the parameters, a solution exists, but we did no discuss about the size and the composition of a stable coalition. In order to make more considerations, we present in the following some numerical simulations, in which the assumption of proportionality is removed. First, let us recall a result from [23], where in the same framework, the authors found that without externalities the maximal size of a stable coalition can be achieved with all developing countries, but only with two developed countries, if $\frac{\beta_1}{\beta_2}$ is large enough and $\frac{\delta_1}{\delta_2}$ is small enough.
So, we need first to calibrate the parameters and then we have to choose values which respect the constraint $e_i > 0$ for each $i \in I$. So, for $i = \{1, 2\}$, we take $\alpha_i \in [5, 10]$, $\delta_i \in [0.05, 0.3]$, $\beta_i \in [0.02, 0.2]$. Moreover, we assume that $s_i \in [0.02, 0.2]$ and that we have 10 countries, with $n_1 \in [0, 6]$ and $n_2 \in [0, 4]$, where clearly n_1 and n_2 are both natural numbers. This partition is based on World Bank classification of high income countries and upper-middle income countries, which are the two sets within that we have all developed countries and all developing countries that contribute significantly to global pollution.

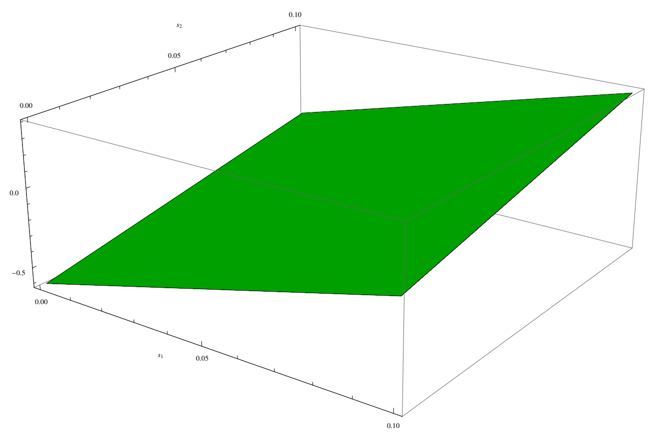

Figure 1: Internal stability for developed countries as function of s_1 and s_2, with $n_1 = 6$ and $n_2 = 4$

All simulations are made with software Wolfram Mathematica. The next step is to set parameters for a benchmark model. So, we take

$$\delta_1 = 0.15, \quad \delta_2 = 0.2, \quad \beta_1 = 0.07, \quad \beta_2 = 0.04.$$

From now on, we evaluate the 32 possible coalitions given by the combinations of n_1 and n_2, in term of the internal and external stability conditions. The first simulations are made for a model without Social Externalites, so we take $s_1 = s_2 = 0$.
In this case we have that the only couples (n_1, n_2) that solve the stability conditions are $(3, 0), (0, 2), (0, 3)$. So we can have only small and homogeous coalitions.
We now introduce the Social Externalities, taking $s_1 = 0.1$ and $s_2 = 0.06$.
For this choice of parameters we verify that while the internal stability is always verified, the external one it never. That means that each player in coalition has no incentive to defeat and that each player outside has an incentive to join in. The final result is that the only stable coalition is the Grand Coalition.
We also want to see how the Social Externalities can be decisive to have a stable coalition of all players. In figure 1 and 2 we show the internal stability conditions, for developed and developing countries respectively, as functions of parameters s_1 and s_2. What emerges is that the Grand Coalition is unstable when we have no Social Externalities or when at least one of the two parameters converges to zero. The same kind of simulations on smaller coalition brings to similar results, at least from a qualitative point of view[2].
The next step is to make some sensitivity analysis on parameters β_i and δ_i, $i = 1, 2$.

[2]Clearly in this cases we have to consider the external stability also, but the results are specular to the internal stability conditions.

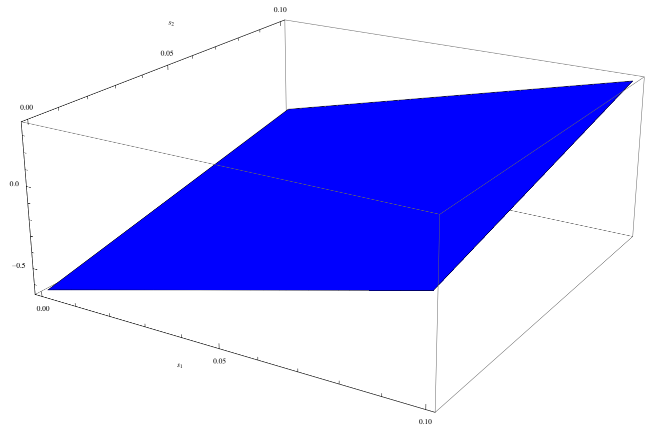

Figure 2: Internal stability for developing countries as function of s_1 and s_2, with $n_1 = 6$ and $n_2 = 4$

To study the effect of an increment of marginal damage, we first set $\beta_1 = 0.14$. In this case we have that internal stability is verified only when $n_1 \leq 3$, but for these coalitions we don't have external stability. So, it's impossible to sign an agreement. Similar, when we double the value of β_2, taking it equal to 0.08, we have that for developed countries the only stable coalition is $(2, 4)$, but we don't have the external stability for developing countries. So, once again, there is not a stable coalition.
Regarding the parameters δ_1 and δ_2, from our simulations it results that an increment of this value does not change the fact that the only stable coalition is the Grand Coalition. Instead, if we set $\delta_1 = 0.075$, we found that we have a unique solution given by $(n_1, n_2) = (6, 0)$. On the other side, if we set $\delta_2 = 0.1$, we have again no solution for stability conditions.

5 Conclusions

In this paper we investigated an N-player static game, in which we assumed asymmetric countries. The aim of this work was to verify whether a Social Externalities could bring to a Grand Coalition in a more realistic framework than the one of Cabon-Dhersin and Ramani [6].
So, we first solved the emission game assuming that a positive function, depending on the number of cooperators only, affects the welfares of the players in coalition. In this framework, we characterized the optimal solutions assuming that we have N asymmetric players.
After that, we considered the stability conditions for a self-enforcing agreement, developed in [9]. In this case, we assumed that players are divided into two homogenous groups, developed and developing countries. We were able to proof that a stable coalition exists, making some assumption about parame-

ters' relations. We showed that the effect of the externalities on the stability is to bring to a grand coalition. In fact, we can see from the stability conditions that the Social Externalities helps to make the internal condition positive, but makes the external condition unstable. In order to discuss about the size and the composition of a stable coalition, we made some numerical simulations that we show in subsection 4.2. We saw that the introduction of the Social Externalities in our benchmark model brings to an agreement signed by all players. Finally we make some sensitivity analysis in which we showed that a change of conditions could have a negative effect, that is greater than the positive one given by Social Externalities, bringing the Grand Coalition to be unstable.

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