Charming penguin contributions in $B\to K^*\pi$, $K(\rho,\omega,\phi)$ decays

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(Received 1 August 2003; published 3 December 2003)

We evaluate the decays $B\to K^*\pi$, $K(\rho,\omega,\phi)$ adding the long distance charming penguin contributions to the short distance: tree+penguin amplitudes. We estimate the imaginary part of the charming penguin contribution by an effective field theory inspired by heavy quark effective theory and parametrize its real part. The final results for branching ratios depend on only two real parameters and show a significant role of the charming penguin contributions. The overall agreement with the available experimental data is satisfactory.

DOI: 10.1103/PhysRevD.68.114001 PACS number(s): 13.25.Hw

I. INTRODUCTION

The CLEO II [1,2], the BaBar [3,4] and the Belle [5] Collaborations have recently reported data on the B meson nonleptonic decay channels into a light vector and a pseudoscalar meson:

$$B\to PV. \tag{1}$$

These data are of the utmost importance for the determination of the angles of the unitarity triangle. In particular, if one of the two mesons in the final state is a strange particle the decays (1) offer several channels for the extraction of the γ angle, thus providing alternatives to the $K\pi$ decay mode. These nonleptonic decays have been proposed long ago [6–18] as a method for the extraction of the γ angle. By this strategy one relates the data to a theoretical amplitude given as a sum of tree and penguin short distance contributions, whose relative weak phase is indeed γ. The usual computational scheme is the factorization hypothesis, either in the naive or in the QCD based version [19–26]. Here one neglects nonfactorizable contributions as one proves that they are of the order of Λ/m_b and therefore negligible in the $m_b\to\infty$ limit.

There is a class of contributions however that, although suppressed in the infinite heavy quark mass limit, cannot be neglected *a priori*. These are long distance contributions known in the literature as charming penguin diagrams [27–32]. Induced by the nonleptonic Hamiltonian $\propto V_{cb}^*V_{cs}\bar{b}\gamma_\mu(1-\gamma_5)c\,\bar{c}\gamma^\mu(1-\gamma_5)s+\text{Fierz}+\text{H.c.}$, they cannot contribute in the vacuum saturation approximation. However, the insertion of at least two charmed particles (charm+anticharm) between the currents can produce sizable contributions. As a matter of fact the $O(\Lambda\times m_b^{-1})$ suppression is compensated by the Cabibbo-Kobayashi-Maskawa enhancement and the actual role of these contributions can be established only as a result of some dynamical calculations. In [30] and [31] we estimated these contributions for the $B\to PP$ decay channels and found that for the $K\pi$ case they are indeed relevant. Their imaginary part can be evaluated with some confidence using the heavy meson chiral effective theory corrected for the light meson hard momenta. On the other hand, the real part is less predictable; for example in [30] and [31] the results strongly depend on the cutoff in the loop integrals. Sizable effects were also found in the $K^-\chi_{c0}$ (J/ψ) channel [33].

In the present paper we give an estimate of the charming penguin contributions in the charmless decays (1) for the cases $K^*\pi$ and $K(\rho,\omega,\phi)$. We do not consider the particles η,η' in the final state because the pseudoscalar SU(3) singlet is most probably contaminated by the glue and the evaluation of these contributions cannot be reliably done within our theoretical scheme. Because of the above mentioned difficulties we compute by the effective theory only the imaginary part of the amplitude, $\mathcal{I}_{\mathcal{LD}}$. For the real part of the amplitude, $\mathcal{R}_{\mathcal{LD}}$, we assume a simple parametrization $\mathcal{R}_{\mathcal{LD}}\propto\mathcal{I}_{\mathcal{LD}}$, fixing the two proportionality constants [one for each of the $\mathrm{SU}(3)_f$ multiplets comprising the light vector mesons] by a fit. A similar calculation has been recently attempted by [32]; these authors assume a more phenomenological approach and parametrize the amplitude in the more general way (two complex numbers), using SU(3) flavor symmetry to relate the different amplitudes. Therefore our calculation can provide a dynamical test of the model in [32].

The outline of the paper is as follows. In Sec. II we estimate the tree diagram contribution (or short-distance contribution), computed in the factorization approximation. In Sec.

III we estimate the absorptive part of the charming penguin contribution. It is evaluated by use of the effective Lagrangian for light and heavy mesons based on the heavy quark effective theory (for a review see [34]). The main uncertainty of this approach is the extrapolation to hard light meson momenta and in this context we use an estimate of form factor given by us in [30]. In Sec. IV we present numerical results for the branching ratios and the asymmetry. In our previous papers we estimated the real part of the charming penguin contributions by using a cutoff Cottingham formula; the results were strongly cutoff dependent and for this reason we give here a simple parametrization of the real part and we estimate the relevant two real parameters by comparing with the data. The overall result for both of the branching ratios and the *CP*-asymmetries is rather satisfying and points to a significant role of the charming penguins in the $B \to K^*\pi$, $K(\rho,\omega,\phi)$ decay modes.

II. SHORT-DISTANCE NONLEPTONIC WEAK MATRIX ELEMENTS

The effective Hamiltonian for nonleptonic B decays is given by

$$\mathcal{H}_{\rm eff}=\frac{G_F}{\sqrt{2}}\left[V_{ub}^*V_{us}(c_1O_1^u+c_2O_2^u)+V_{cb}^*V_{cs}(c_1O_1^c+c_2O_2^c)-V_{tb}^*V_{ts}\left(\sum_{i=3}^{10}c_iO_i+c_gO_g\right)\right] \tag{2}$$

where c_i are the Wilson coefficients evaluated at the normalization scale $\mu=m_b$ [35–39] and next-to-leading order QCD radiative corrections are included. O_1 and O_2 are the usual tree-level operators, O_i ($i=3,\ldots,10$) are the penguin operators and O_g is the chromomagnetic gluon operator. The c_i in Eq. (2) are as follows [39]: $c_2=1.105$, $c_1=-0.228$, $c_3=0.013$, $c_4=-0.029$, $N\ c_5=0.009$, $c_6=-0.033$, $c_7/\alpha=0.005$, $c_8/\alpha=0.060$, $c_9/\alpha=-1.283$, $c_{10}/\alpha=0.266$. Moreover, for the current quark masses we use the values

$$m_b=4.6\ \text{GeV}\quad m_u=4\ \text{MeV}$$
$$m_d=8\ \text{MeV}\quad m_s=0.150\ \text{GeV}. \tag{3}$$

We define the T-matrix element

$$(2\pi)^4\delta^4(p_B-p_\pi-p_{K*})\times\mathcal{M}_{K*\pi}=\langle K^*\pi|T|B\rangle, \tag{4}$$

with a similar definition for the $K(\rho,\omega,\phi)$ final state. We separate short-distance and long-distance contributions to the weak $B\to K^*\pi$ decay:

$$\mathcal{M}=\mathcal{M}_{\mathcal{SD}}+\mathcal{M}_{\mathcal{LD}}, \tag{5}$$

and evaluate the short-distance part of the amplitude $\mathcal{M}_{\mathcal{SD}}$ using the operators in Eq. (2) in the factorization approximation [with $\mathcal{M}_{\mathcal{SD}}(B\to f)\equiv-\langle f|\mathcal{H}_{\rm eff}|B\rangle$]. The results are in Table I. Here $a_i=c_i+(c_{i+1}/3)$ ($i=$odd) and $a_i=c_i+(c_{i-1}/3)$ ($i=$even). Moreover, if V is a vector meson and $P^{(\prime)}$ is a pseudoscalar meson, we use the following definition for the matrix elements of weak currents:

$$\langle P(p)|A_\mu|0\rangle=-if_Pp_\mu,\quad \langle V(\varepsilon,p)|V_\mu|0\rangle=f_Vm_V\varepsilon_\mu^*, \tag{6}$$

and

$$\langle P'(p')|V_\mu|P(p)\rangle=F_1(q^2)\left[(p_\mu+p'_\mu)-\frac{m_P^2-m_{P'}^2}{q^2}q_\mu\right]+F_0(q^2)\frac{m_P^2-m_{P'}^2}{q^2}q_\mu \tag{7}$$

$$\langle V(\epsilon,p')|V^\mu-A^\mu|P(p)\rangle=\frac{2V(q^2)}{M_P+M_V}\epsilon^{\mu\nu\alpha\beta}\epsilon_\nu^*p_\alpha p'_\beta-i(M_P+M_V)\epsilon^{*\mu}A_1(q^2)+i\frac{(\epsilon^*\cdot q)}{M_P+M_V}(p+p')^\mu A_2(q^2)+i(\epsilon^*\cdot q)\frac{2M_V}{q^2}q^\mu[A_3(q^2)-A_0(q^2)], \tag{8}$$

where

$$A_3(q^2)=\frac{M_V-M_P}{2M_V}A_2(q^2)+\frac{M_V+M_P}{2M_V}A_1(q^2) \tag{9}$$

and $A_3(0)=A_0(0)$.

III. ABSORPTIVE PART

The computation of the discontinuity of the charming penguin contribution diagrams contributing to $B\to K^*\pi$ gives (cf. analogous diagrams in Fig. 2 of Ref. [30])

$$\begin{aligned}\text{Disc}\mathcal{M}_{\mathcal{LD}}&=2i\,\text{Im}\,\mathcal{M}_{\mathcal{LD}}\\&=i(2\pi)^2\int\frac{d^4q}{(2\pi)^4}\delta_+(q^2-m_{D_s}^2)\delta_+(p_{D(*)}^2-m_D^2)\\&\quad\times\mathcal{M}(B\to D_s^{(*)}D^{(*)})\mathcal{M}(D_s^{(*)}D^{(*)}\to K^*\pi)\\&=i\frac{m_D}{16\pi^2m_B}\sqrt{\omega^{*2}-1}\int d\mathbf{n}\mathcal{M}(B\to D_s^{(*)}D^{(*)})\\&\quad\times\mathcal{M}(D_s^{(*)}D^{(*)}\to K^*\pi),\end{aligned} \tag{10}$$

TABLE I. Factorized $\mathcal{M}_{\mathcal{SD}}$ amplitudes.

Process	$\mathcal{M}_{\mathcal{SD}}$
$B^+\to K^{*0}\pi^+$	$+G_F\sqrt{2}F_1^{B\to\pi}(m_{K^*}^2)f_{K^*}m_{K^*}V_{tb}^*V_{ts}\left[a_4-\frac{a_{10}}{2}\right](\varepsilon^*\cdot p_B)$
$B^+\to K^{*+}\pi^0$	$-G_Fm_{K^*}\left\{F_1^{B\to\pi}(m_{K^*}^2)f_{K^*}[V_{ub}^*V_{us}a_2-V_{tb}^*V_{ts}(a_4+a_{10})]+A_0^{B\to K^*}(m_\pi^2)f_\pi\left[V_{ub}^*V_{us}a_1+V_{tb}^*V_{ts}\frac{3}{2}(a_7-a_9)\right]\right\}(\varepsilon^*\cdot p_B)$
$B^0\to K^{*0}\pi^0$	$-G_Fm_{K^*}\left\{A_0^{B\to K^*}(m_\pi^2)f_\pi\left[V_{ub}^*V_{us}a_1-V_{tb}^*V_{ts}\frac{3}{2}(a_9-a_7)\right]+F_1^{B\to\pi}(m_{K^*}^2)f_{K^*}V_{tb}^*V_{ts}\left[a_4-\frac{a_{10}}{2}\right]\right\}(\varepsilon^*\cdot p_B)$
$B^0\to K^{*+}\pi^-$	$-G_F\sqrt{2}F_1^{B\to\pi}(m_{K^*}^2)f_{K^*}m_{K^*}[V_{ub}^*V_{us}a_2-V_{tb}^*V_{ts}(a_4+a_{10})](\varepsilon^*\cdot p_B)$
$B^+\to K^0\rho^+$	$+G_F\sqrt{2}A_0^{B\to\rho}(m_K^2)f_Km_\rho V_{tb}^*V_{ts}\left[a_4-\frac{1}{2}a_{10}-\frac{(2a_6-a_8)m_K^2}{(m_b+m_d)(m_d+m_s)}\right](\varepsilon^*\cdot p_B)$
$B^+\to K^+\rho^0$	$-G_Fm_\rho\left\{A_0^{B\to\rho}(m_K^2)f_K\left[V_{ub}^*V_{us}a_2-V_{tb}^*V_{ts}\left(a_4+a_{10}-\frac{2(a_6+a_8)m_K^2}{(m_b+m_u)(m_u+m_s)}\right)\right]\right.$ $\left.+F_1^{B\to K}(m_\rho^2)f_\rho\left[V_{ub}^*V_{us}a_1-\frac{3}{2}V_{tb}^*V_{ts}(a_7+a_9)\right]\right\}(\varepsilon^*\cdot p_B)$
$B^0\to K^0\rho^0$	$-G_Fm_\rho\left\{F_1^{B\to K}(m_\rho^2)f_\rho\left[V_{ub}^*V_{us}a_1-V_{tb}^*V_{ts}\frac{3}{2}(a_7+a_9)\right]+A_0^{B\to\rho}(m_K^2)f_KV_{tb}^*V_{ts}\left[a_4-\frac{1}{2}a_{10}-\frac{(2a_6-a_8)m_K^2}{(m_b+m_d)(m_d+m_s)}\right]\right\}(\varepsilon^*\cdot p_B)$
$B^0\to K^+\rho^-$	$-G_F\sqrt{2}A_0^{B\to\rho}(m_K^2)f_Km_\rho\left[V_{ub}^*V_{us}a_2-V_{tb}^*V_{ts}\left(a_4+a_{10}-\frac{2(a_6+a_8)m_K^2}{(m_b+m_u)(m_u+m_s)}\right)\right](\varepsilon^*\cdot p_B)$
$B^+\to K^+\omega$	$-G_Fm_\omega\{F_1^{B\to K}(m_\omega^2)f_\omega[V_{ub}^*V_{us}a_1-V_{tb}^*V_{ts}(2(a_3+a_5)+\frac{1}{2}(a_7+a_9))]$ $+A_0^{B\to\omega}(m_K^2)f_K\left[V_{ub}^*V_{us}a_2-V_{tb}^*V_{ts}\left(a_4+a_{10}-\frac{2(a_6+a_8)m_K^2}{(m_b+m_u)(m_u+m_s)}\right)\right]\}(\varepsilon^*\cdot p_B)$
$B^0\to K^0\omega$	$-G_Fm_\omega\left\{F_1^{B\to K}(m_\omega^2)f_\omega\left[V_{ub}^*V_{us}a_1-V_{tb}^*V_{ts}\left(2(a_3+a_5)+\frac{1}{2}(a_7+a_9)\right)\right]\right.$ $\left.-A_0^{B\to\omega}(m_K^2)f_KV_{tb}^*V_{ts}\left(a_4-\frac{1}{2}a_{10}-\frac{(2a_6-a_8)m_K^2}{(m_b+m_d)(m_d+m_s)}\right)\right\}(\varepsilon^*\cdot p_B)$
$B^{+,0}\to K^{+,0}\phi$	$+G_F\sqrt{2}F_1^{B\to K}(m_\phi^2)f_\phi m_\phi V_{tb}^*V_{ts}\left[a_3+a_4+a_5-\frac{a_7+a_9+a_{10}}{2}\right](\varepsilon^*\cdot p_B)$

where the integration is over the solid angle and a sum over polarizations is understood. A similar equation holds for the $K(\rho,\omega,\phi)$ final state. The amplitudes for the decays $B\to D_s^{(*)}D^{(*)}$ are computed by factorization and using information on the Isgur-Wise function (see below). Diagrams for the calculation of the amplitudes $D_s^{(*)}D^{(*)}\to K^*\pi$ are in Fig. 1. A similar diagram can be drawn for the $D_s^{(*)}D^{(*)}\to K(\rho,\omega,\phi)$ amplitudes.

The effective Lagrangian to compute these diagrams can be written as follows (see e.g. [34]):

$$\mathcal{L}=i\langle H_bv^\mu D_{\mu ba}\overline{H}_a\rangle+ig\langle H_b\gamma_\mu\gamma_5\mathcal{A}^\mu_{ba}\overline{H}_a\rangle+i\beta\langle H_bv^\mu(\mathcal{V}_\mu-\rho_\mu)_{ba}\overline{H}_a\rangle+i\lambda\langle H_b\sigma^{\mu\nu}F_{\mu\nu}(\rho)_{ba}\overline{H}_a\rangle. \tag{11}$$

Here $\langle\cdots\rangle$ means the trace,

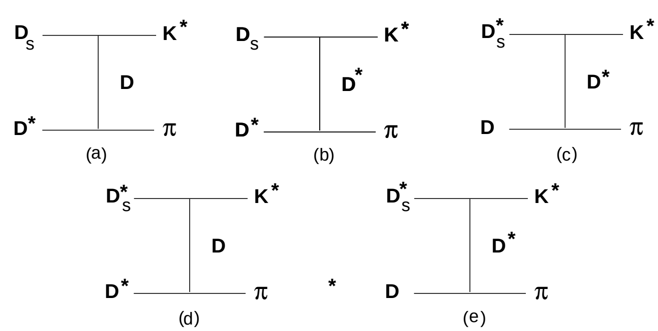

FIG. 1. Diagrams for the calculation of the $D_s^{(*)}D^{(*)}\to K^*\pi$ amplitudes.

$$H=\frac{1+\not{v}}{2}(-P_5\gamma_5-i\not{P}), \tag{12}$$

and P_5,P^μ are annihilation operators of the pseudoscalar P and vector V heavy mesons normalized as follows:

$$\langle 0|P_5|P\rangle=\sqrt{M_H}, \quad \langle 0|P_\mu|V(\epsilon)\rangle=\epsilon_\mu\sqrt{M_H}. \tag{13}$$

The first term in the Lagrangian (11) contains the kinetic term for the heavy mesons giving the P and P^* propagators, $i/(2v\cdot k)$ and $-i(g^{\mu\nu}-v^\mu v^\nu)/(2v\cdot k)$ respectively. However, since the residual momentum is not small we will use the complete form of the propagators instead of their approximate expressions [30,31]. The interactions among heavy and light mesons are obtained by the other three terms. In the second term there are interactions among the heavy mesons and an odd number of pions coming from the expansion $\mathcal{A}_\mu$. Here $\mathcal{A}_\mu=\frac{1}{2}(\xi^\dagger\partial_\mu\xi-\xi\partial_\mu\xi^\dagger)$. Expanding the field $\xi=\exp(i\pi/f)$ and taking the traces it provides the coupling in the lower vertices of Fig. 1. The last two terms give the upper vertices of Fig. 1. The octet of vector resonances (ρ, K^*, etc.) is introduced as the gauge multiplet according to the hidden gauge symmetry approach of Ref. [40]. We put

$$\rho_\mu=i\frac{g_V}{\sqrt{2}}\hat{\rho}_\mu \tag{14}$$

where $\hat{\rho}$ is the usual Hermitian 3×3 matrix of flavor SU(3) comprising the nonet of light vector mesons and g_V is determined by vector meson dominance as follows: $g_V\simeq5.8$ [40].

Also the parameter β can be fixed by vector meson dominance. This corresponds to assuming that, for the heavy pseudoscalar mesons H, the coupling of the electromagnetic current to the light quarks is dominated by the ρ,ω,ϕ vector mesons. This produces the numerical result

$$\beta=\frac{\sqrt{2}m_V}{g_V f_V}\approx0.9. \tag{15}$$

For g we take the recent experimental result from the CLEO Collaboration, obtained by the full width of D^{*+} [41]; this gives a value $g=0.59\pm0.07\pm0.01$.

It can be noted that while these determinations consider soft pions, we are interested in the coupling of a hard pion to D and D^*. This introduces a correction that can be parametrized by a form factor, see below for a discussion.

Let us now consider the parameter λ in Eq. (11). In [34] one can find an estimate of this parameter based on the effective chiral Lagrangian for heavy mesons. We present here an update of this analysis and new numerical results.

It is useful to begin by writing down the phenomenological heavy-to-heavy current in the leading $1/m_Q$ approximation:

$$J^{cb}_\mu=-\xi(v\cdot v')\langle\overline{H}^{(c)}_a\gamma_\mu(1-\gamma_5)H^{(b)}_a\rangle, \tag{16}$$

which gives rise to

$$\langle D(v')|\overline{c}\gamma_\mu b|B(v)\rangle=\sqrt{M_BM_D}\xi(v\cdot v')(v+v')_\mu,$$

$$\langle D^*(v',\epsilon)|\overline{c}\gamma_\mu b|B(v)\rangle =\sqrt{M_BM_D}\xi(v\cdot v')\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v^\alpha v'^\beta,$$

$$\langle D^*(v',\epsilon)|\overline{c}\gamma_\mu\gamma_5 b|B(v)\rangle =\sqrt{M_BM_D}\xi(v\cdot v')i[(1+v\cdot v')\epsilon^*_\mu-(\epsilon^*\cdot v)v'_\mu], \tag{17}$$

where $\xi(v\cdot v')$ is the Isgur-Wise form factor. Note that, by this definition, the choice of phases in Eq. (17) agrees with Eqs. (8) and (9). We also write the phenomenological heavy-to-light leading current for coupling to light pseudoscalar mesons of the heavy mesons in the multiplet H

$$L^\mu_a=\frac{i\hat{F}}{2}\langle\gamma^\mu(1-\gamma_5)H_b\xi^\dagger_{ba}\rangle. \tag{18}$$

Here $\hat{F}$ is related to leptonic decay constant of the pseudoscalar heavy meson B appearing in Eq. (6) by

$$f_B=\frac{\hat{F}}{\sqrt{M_B}}. \tag{19}$$

Several numerical analyses based on QCD sum rules have been performed, see for a review e.g. [34]. The result we shall use is

$$\hat{F}=0.30\pm0.05\ \ \mathrm{GeV}^{3/2} \tag{20}$$

which is obtained neglecting radiative corrections since they are neglected also in the evaluation of the Isgur-Wise form factor that the parametrization we use below is based on.

The effective theory approach gives predictions for the form factor $V(q^2)$ at high q^2 and relates it to the parameter λ. We can match this result with the theoretical calculations coming from light cone sum rules [42,43] (LCSR) and lattice QCD [44–46]. To do this we compute the form factor at $q^2\simeq q^2_{max}$ where it is dominated by the nearest low-lying vector meson pole. Computing the form factor at $q^2\simeq q^2_{max}$ $\equiv(M_B-M_V)^2$ and at leading order in $1/m_Q$, one gets [47]

$$V(q^2_{\max})=\frac{g_V}{\sqrt{2}}\lambda\hat{F}\frac{M_P+M_V}{\sqrt{M_P}}\frac{1}{M_V+\Delta} \tag{21}$$

where Δ is the appropriate mass splitting, i.e., for the transition $B\to K^\star$, $\Delta=m_{B^*_s}-m_B\simeq135$ MeV. From the LCSR and lattice QCD analyses for the transition $B\to K^*$ we infer the value $V=1.5$ at $q^2\equiv\overline{q^2}=17$ GeV^2. In order to compare with Eq. (21) we assume that in the range $q^2\in(\overline{q^2},q^2_{max})$ the form factors are dominated by the B^*_s simple pole. This gives $V(q^2_{max})=1.8$ and, as a result,

$$\lambda=+0.56\ \ \mathrm{GeV}^{-1}. \tag{22}$$

We note that to determine λ we used a positive sign of $V(q^2_{max})$. The same sign would be obtained using HQET and by assuming that the strange mass is large, see Eq. (17). We also note that a different result for the phase and the magnitude of λ was obtained in [34]. The difference in magnitude was due to the different methods used. Here we use LCSR and lattice QCD, while in [34] an extrapolation is used from the $D\to K^*$ form factor. As to the phase, it is due to a different choice we use here for the phase factor of the heavy vector meson, see Eq. (12).

To compute the matrix elements we use the following kinematics:

$$p^\mu = m_B v^\mu = (m_B,\vec{0}), \quad p^\mu_{D(*)} = m_D v'^\mu, \quad q = p - p_{D(*)}. \tag{23}$$

where $\omega^* = (m_B^2 + m_D^2 - m_{D_s}^2)/2m_D m_B$ and the angular integration is over the directions of the vector $\vec{v}\,' = \vec{n}\sqrt{\omega^{*2}-1}$. Our results are as follows:

$$\mathcal{M}[B(v)\to D_s(q)D^*(\epsilon,v')] = -K(m_B+m_D)\epsilon^*\cdot v, \tag{24}$$

$$\mathcal{M}[B(v)\to D_s^*(\eta,q)D(v')] = -Km_{D_s}\eta^*\cdot(v+v'), \tag{25}$$

$$\mathcal{M}[B(v)\to D_s^*(\eta,q)D^*(\epsilon,v')] = -iKm_{D_s}\eta^{*\mu}\epsilon^{*\alpha}[i\epsilon_{\alpha\lambda\mu\sigma}v'^\lambda v^\sigma - g_{\mu\alpha}(1+\omega^*) + v_\alpha v'_\mu], \tag{26}$$

where $K = (G_F/\sqrt{2})V^*_{cb}V_{cs}a_2\sqrt{m_B m_D}f_{D_s}\xi_{IW}(\omega^*)$. On the other hand, for the $K^*\pi$ final state we have

$$\mathcal{M}[D_s(q)D^*(\epsilon,v')\to K^*(p_K,\hat{\epsilon})\pi(p_\pi)] = -\frac{2gF^2(|\vec{p}_\pi|)}{f_\pi}\frac{g_V}{\sqrt{2}}\sqrt{\frac{m_{D^*}}{m_{D_s}}}\epsilon_\lambda\hat{\epsilon}^*_\sigma\left[\frac{2\beta m_D q^\sigma p^\lambda_\pi}{(m_D v'-p_\pi)^2-m_D^2} + \frac{4\lambda m_{D^*}G^{\sigma\lambda}(p_\pi,p_K,v')}{(m_{D^*}v'-p_\pi)^2-m^2_{D^*}}\right], \tag{27}$$

$$\begin{aligned}\mathcal{M}[D_s^*(\eta,q)D(v')\to K^*(p_K,\hat{\epsilon})\pi(p_\pi)] = {}& -\frac{2gm_{D^*}F^2(|\vec{p}_\pi|)}{f_\pi}\frac{g_V}{\sqrt{2}}\sqrt{\frac{m_D}{m_{D_s}}}\frac{\eta_\lambda\hat{\epsilon}^*_\sigma}{(m_{D^*}v'-p_\pi)^2-m^2_{D^*}}\\ &\times\left[2\beta q^\sigma\left(+p^\lambda_\pi - \frac{v'\cdot p_\pi}{m_{D^*}}p^\lambda_K\right) - 4\lambda m_{D_s}H^{\sigma\lambda}(p_\pi,p_K,v')\right],\end{aligned} \tag{28}$$

$$\begin{aligned}\mathcal{M}[D_s^*(\eta,q)D^*(\epsilon,v')\to K^*(p_K,\hat{\epsilon})\pi(p_\pi)] = {}& +\frac{2gm_{D^*}F^2(|\vec{p}_\pi|)}{f_\pi}\frac{g_V}{\sqrt{2}}\sqrt{\frac{m_{D^*}}{m_{D_s^*}}}\epsilon^{\alpha\mu\nu\lambda}\eta_\tau\hat{\epsilon}^*_\sigma\epsilon_\rho\left[\frac{4\lambda m_D q_\alpha(p_K)_\mu p^\rho_\pi}{(m_D v'-p_\pi)^2-m_D^2}\frac{\delta^\sigma_\nu\delta^\tau_\lambda}{m_{D^*}}\right.\\ &\left.+\frac{v'_\alpha(p_\pi)_\mu\delta^\rho_\nu}{(m_{D^*}v'-p_\pi)^2-m^2_{D^*}}\{2\beta q^\sigma\delta^\tau_\lambda + 4\lambda m_{D_s^*}[p^\tau_K\delta^\sigma_\lambda - (p_K)_\lambda g^{\sigma\tau}]\}\right],\end{aligned} \tag{29}$$

where

$$G^{\sigma\lambda}(p_\pi,p_K,v') = -(v'\cdot q)[g^{\sigma\lambda}(p_K\cdot p_\pi) - p^\sigma_\pi p^\lambda_K] - (q\cdot p_\pi)[v'^\sigma p^\lambda_K - g^{\sigma\lambda}(v'\cdot p_K)] - q^\lambda[p^\sigma_\pi(p_K\cdot v') - v'^\sigma(p_K\cdot p_\pi)], \tag{30}$$

$$H^{\sigma\lambda}(p_\pi,p_K,v') = g^{\sigma\lambda}\left(p_K\cdot p_\pi - \frac{v'\cdot p_\pi}{m_{D^*}}(m^2_{K^*} - p_K\cdot q)\right) - p^\lambda_K\left(p^\sigma_\pi + \frac{v'\cdot p_\pi}{m_{D^*}}q^\sigma\right). \tag{31}$$

Equation (27) corresponds to the sum of diagrams (a) and (b) of Fig. 1; Eq. (28) corresponds to diagram (c) and, finally, Eq. (29) corresponds to the sum of diagrams (d) and (e). Similar results hold for the $K(\rho,\omega,\phi)$ final states and we do not reproduce them here for the sake of brevity. $F(|\vec{p}_\pi|)$ is a form factor taking into account that in the vertex $DD^*\pi$ the pion is not soft and therefore the coupling constant should be corrected. Its determination by a quark potential model is discussed in [30] using a quark model. The central value we use is $F(|\vec{p}_\pi|) = 0.065$. In the absence of more detailed information, the same form factor is adopted for the upper vertices in Fig. 1 as well.

TABLE II. Theoretical values for $\mathcal{M}_{\mathcal{SD}},\mathcal{M}_{\mathcal{LD}}$. Units are GeV.

Process	$\mathcal{M}_{\mathcal{SD}}\times 10^8$	Im $\mathcal{M}_{\mathcal{LD}}\times 10^8$
$B^+\to K^{*0}\pi^+$	$+1.45$	-2.14
$B^+\to K^{*+}\pi^0$	$+1.02-0.79i$	-1.52
$B^0\to K^{*0}\pi^0$	$-0.60-0.08i$	$+1.52$
$B^0\to K^{*+}\pi^-$	$+0.85-1.01i$	-2.14
$B^+\to K^0\rho^+$	$+0.17$	-2.74
$B^+\to K^+\rho^0$	$+0.39-0.64i$	-1.94
$B^0\to K^0\rho^0$	$+0.48-0.11i$	$+1.94$
$B^0\to K^+\rho^-$	$-0.28-0.74i$	-2.74
$B^+\to K^+\omega$	$-0.27-0.63i$	-1.94
$B^0\to K^0\omega$	$+0.03-0.10i$	-1.94
$B^+\to K^+\phi$	1.30	-2.58
$B^0\to K^0\phi$	1.30	-2.58

IV. NUMERICAL RESULTS

Let us first consider the short distance contribution. Using, for the CKM matrix elements, $\rho=0.2296$, $\eta=0.3249$, $A=0.819$, and for the involved form factors the value collected in Table IV of Ref. [48] (a different determination based on QCD sum rules is in [49]), one gets the results reported in Table II. These values correspond to the following value of the angle γ of the unitarity triangle:

$$\gamma=\arctan\left(\frac{\eta}{\rho}\right)\simeq 54.8°. \tag{32}$$

Next we consider the long distance absorptive part. For its parameters we assume the values of the previous section. For the D_s decay constant we use $f_{D_s}=\hat{F}/\sqrt{m_{D_s}}$, with $\hat{F}$ given in Eq. (20); for the Isgur-Wise form factor we use the parametrization

$$\xi(v\cdot v')=\left(\frac{2}{1+v\cdot v'}\right)^{\hat{\rho}^2} \tag{33}$$

with $\hat{\rho}^2\simeq 1$. This parametrization agrees with the results for the $b\to c$ exclusive decays obtained by the CLEO and Belle Collaborations. It is a fit to the results obtained by the QCD sum rules method, see [34].

The numerical results for the imaginary part of $\mathcal{M}_{\mathcal{LD}}$ are given in Table II. Typical sizes of the different contributions to $B^-\to\overline{K}^{*0}\pi^-$ are as follows. From the two terms in Eq. (27): Im $\mathcal{M}^{a,b}_{\mathcal{LD}}=-1.32\times 10^{-8}$; from the terms in Eq. (28): Im $\mathcal{M}^{c}_{\mathcal{LD}}=-0.16\times 10^{-8}$; from the terms in Eq. (29): Im $\mathcal{M}^{d,e}_{\mathcal{LD}}=-0.66\times 10^{-8}$. For the $B^-\to K^0\rho^-$ channel we find: Im $\mathcal{M}^{a,b}_{\mathcal{LD}}=-1.52\times 10^{-8}$; Im $\mathcal{M}^{c}_{\mathcal{LD}}=-0.70\times 10^{-8}$; Im $\mathcal{M}^{d,e}_{\mathcal{LD}}=-0.52\times 10^{-8}$. It can be noted that the phase of $\mathcal{M}_{\mathcal{SD}}$ is purely weak while the phase in $\mathcal{M}_{\mathcal{LD}}$ is only due to strong interactions.

A. Branching ratios

From the results of Table II we can compute the branching ratios ($\mathcal{B}$) and the *CP* asymmetries. As explained in the Introduction, we have not presented an evaluation of the real part of $\mathcal{A}_{\mathcal{LD}}$. Attempts for the $K\pi$ channel can be found in [30,31]. Typical results are that the real part is of the same order of the imaginary part, but the uncertainties are large. To estimate the *CP* averaged branching ratios we add therefore to the imaginary part a real part as follows:

TABLE III. Branching ratios $\mathcal{B}$ (units 10^{-6}). In the second and third columns, theoretical values computed using only the short distance amplitude and the full amplitude (i.e. short distance and long distance); in the latter case the three values correspond respectively to $\mathcal{R}_{\mathcal{LD}}=(+1,0,-1)\times\mathcal{I}_{\mathcal{LD}}$, see text.

Process	$\mathcal{B}$ ($\mathcal{SD}$ only)	$\mathcal{B}$ ($\mathcal{SD}$+$\mathcal{LD}$)
$B^+\to K^{*0}\pi^+$	1.96	(4.71,6.22,16.3)
$B^+\to K^{*+}\pi^0$	1.56	(5.20,5.95,11.0)
$B^0\to K^{*0}\pi^0$	0.32	(2.51,2.10,5.66)
$B^0\to K^{*+}\pi^-$	1.50	(9.94,9.13,16.2)
$B^+\to K^0\rho^+$	0.03	(13.1,6.99,14.8)
$B^+\to K^+\rho^0$	0.52	(8.42,6.32,11.2)
$B^0\to K^0\rho^0$	0.21	(7.88,3.07,4.71)
$B^0\to K^+\rho^-$	0.54	(18.1,10.4,15.6)
$B^+\to K^+\omega$	0.44	(10.7,6.21,8.73)
$B^0\to K^0\omega$	0.01	(6.70,3.58,6.91)
$B^+\to K^+\phi$	1.55	(7.58,7.63,19.8)
$B^0\to K^0\phi$	1.42	(6.98,7.02,18.3)

$$\mathcal{M}_{\mathcal{LD}}=\mathcal{R}_{\mathcal{LD}}+i\mathcal{I}_{\mathcal{LD}} \tag{34}$$

and consider the values $\mathcal{R}_{\mathcal{LD}}=0,\pm\mathcal{I}_{\mathcal{LD}}$. The results are reported in Table III for the three values of $\mathcal{R}_{\mathcal{LD}}$, together with the results obtained considering only the short distance amplitude. They show the relevance of the long-distance contribution. The "best value" of $\mathcal{R}_{\mathcal{LD}}$ might be obtained by a fit to the available experimental data; these data are reported in Table IV and a comparison with the results of the previous table shows that the preferred values, except for the channels with ϕ in the final state, satisfy $\mathcal{R}_{\mathcal{LD}}\approx-\mathcal{I}_{\mathcal{LD}}$. Instead, the decay in $K\phi$ prefers $\mathcal{R}_{\mathcal{LD}}\approx+\mathcal{I}_{\mathcal{LD}}$. To find a "best value" of $\mathcal{R}_{\mathcal{LD}}$ we defined a χ^2 as

$$\chi^2\equiv\sum_i\left(\frac{Br_i^{th}-Br_i^{exp}}{\sigma(Br_i)}\right)^2 \tag{35}$$

where the Br_i^{exp} are reported in Table IV and the $\sigma(Br_i)$ are obtained summing quadratically the statistic and systematic errors in the same table. When the experiment provides an asymmetric error ${}^{\sigma_1}_{\sigma_2}$ a conservative error was assumed: $\sigma=\max(\sigma_1,\sigma_2)$. For the decay $\overline{B}^0\to\overline{K}^{*0}\pi^0$ we put $Br^{exp}=0$ and the corresponding error was fixed to be equal to the experimental upper limit $\sigma=3.6\times 10^{-6}$. We have first attempted a fit with only one real parameter $r=\mathcal{R}_{\mathcal{LD}}/\mathcal{I}_{\mathcal{LD}}$, which corresponds to the use of the SU(3) nonet symmetry for the light vector mesons. The minimum value of $\chi^2(=12.1)$ is obtained for $r=1.207$. Since however we do not expect the validity of the nonet hypothesis we have also

TABLE IV. *CP* averaged branching ratios $\mathcal{B}$ (units 10^{-6}). In the second column theoretical values computed using the present model. In the third column and fourth column theoretical computations based on Ref. [30]: Scenario (Sc.) 1 refers to QCD with factorization and free γ; scenario 2 refers to QCD+charming penguin contributions with constrained γ (see text). Experimental data are from CLEO [1,2], BaBar [3,4], and Belle [5] or averages from these data.

Process	$\mathcal{B}$ (this paper)	$\mathcal{B}$([32], Sc. 1)	$\mathcal{B}$([32], Sc. 2)	$\mathcal{B}$ (Expt.)
$B^+\to K^{*0}\pi^+$	15.6	7.889	11.080	12.1 ± 3.1 (av.)
$B^-\to K^{*-}\pi^0$	8.44	7.303	8.292	$7.1^{+11.4}_{-7.1}\pm1.0(<31)$ (CLEO)
$\overline{B}^0\to\overline{K}^{*0}\pi^0$	5.61			<3.6 (CLEO)
$B^0\to K^{*+}\pi^-$	12.0	9.760	10.787	19.3 ± 5.2 (av.)
$B^+\to K^0\rho^+$	14.1	7.140	14.006	
$B^+\to K^+\rho^0$	8.56	1.882	5.665	8.9 ± 3.6 (av.)
$B\to K^0\rho^0$	4.86	5.865	8.893	
$B^0\to K^+\rho^-$	11.6	6.531	14.304	15.9 ± 4.7 (av.)
$B^+\to K^+\omega$	6.19	2.398	6.320	$9.2^{+2.6}_{-2.3}\pm1.0$ (CLEO); <4 (BaBar); <7.9 (Belle)
$B^0\to K^0\omega$	6.27	2.318	5.606	6.3 ± 1.8 (av.)
$B^+\to K^+\phi$	9.11	8.941	9.479	8.9 ± 1.2 (av.)
$B^0\to K^0\phi$	8.39	8.360	8.898	8.7 ± 1.4 (av.)

tried a two parameter fit, corresponding to the two multiplets (octet+singlet) of the light vector mesons. This gives as a result $\mathcal{R}_{\mathcal{LD}}=-0.954\mathcal{I}_{\mathcal{LD}}$ for the (ρ, K^*, ω) set of particles and two solutions, $\mathcal{R}_{\mathcal{LD}}=-0.201\mathcal{I}_{\mathcal{LD}}$ and $\mathcal{R}_{\mathcal{LD}}=+1.21\mathcal{I}_{\mathcal{LD}}$, for the ϕ with the same $\chi^2=7.8$. The former solution would be preferred because it represents a less important deviation of the SU(3) nonet symmetry. Even if the two solutions produce the same values of the branching ratios they would produce different *CP* asymmetries. The results we find are in Table IV. Besides our data, we also present two model calculations presented in [32], where an analysis of all the PV (not only strange) final states is performed. The first model (called in that paper scenario 1) contains short distance terms (QCD factorization) and unconstrained γ angle. Other theoretical determinations based on QCD factorization are in Ref. [50], e.g. $\mathcal{B}(B^0\to K^+\rho^-)=12.1$. The authors of [32] do not agree with the conclusions of Du *et al.*, Ref. [50], because, differently from them, they include the $K^*\pi$ channels. The second model (scenario 2) uses QCD factorization and charming penguin contributions as in the present paper, but differently from this paper, where only two real free parameters are used, they employ two universal complex amplitudes multiplied by a computed Clebsh-Gordan coefficient. It is worthwhile to note that a significant agreement with the data is obtained with our simple hypothesis. We also note that the value γ of Eq. (32) obtained by a global fit to the CKM matrix [51] is compatible with the $B\to K^*\pi,\ K(\rho,\omega,\phi)$ branching ratios only if the charming penguin diagrams are included.

TABLE V. Theoretical values for the asymmetries; for the definitions see Eq. (39).

Asymmetry	$\mathcal{A}_\pi^{+0}$	$\mathcal{A}_\pi^{0+}$	$\mathcal{A}_\pi^{+-}$	$\mathcal{A}_\pi^{00}$	$\mathcal{A}_\rho^{+0}$	$\mathcal{A}_\rho^{0+}$	$\mathcal{A}_\rho^{+-}$	$\mathcal{A}_\rho^{00}$
A_{CP}	0.27	0	0.31	-0.04	0.27	0	0.30	-0.08

B. *CP* asymmetries

From previous results we can also compute the *CP* asymmetries for the various channels. The Belle Collaboration [5] reported the result

$$\mathcal{A}_{CP}^{K^-\omega}=\frac{\mathcal{B}(B^-\to K^-\omega)-\mathcal{B}(B^+\to K^+\omega)}{\mathcal{B}(B^-\to K^-\omega)+\mathcal{B}(B^+\to K^+\omega)}=-0.21\pm0.28\pm0.03. \tag{36}$$

The result we find agrees with the data:

$$\mathcal{A}_{CP}^{K^-\omega}=-0.37 \quad \text{(theory)}. \tag{37}$$

We also get

$$\mathcal{A}_{CP}^{\overline{K}^0\omega}=\frac{\mathcal{B}(\overline{B}^0\to\omega\overline{K}^0)-\mathcal{B}(B^0\to\omega K^0)}{\mathcal{B}(\overline{B}^0\to\omega\overline{K}^0)+\mathcal{B}(B^0\to\omega K^0)}=-0.05. \tag{38}$$

We can similarly compute the asymmetries for the $K^*\pi,K\rho$ channels, defined by

$$\mathcal{A}_\pi^{+0}=\frac{\mathcal{B}(B^+\to K^{*+}\pi^0)-\mathcal{B}(B^-\to K^{*-}\pi^0)}{\mathcal{B}(B^+\to K^{*+}\pi^0)+\mathcal{B}(B^-\to K^{*-}\pi^0)},$$

$$\mathcal{A}_\pi^{0+}=\frac{\mathcal{B}(B^+\to K^{*0}\pi^+)-\mathcal{B}(B^-\to \overline{K}^{*0}\pi^-)}{\mathcal{B}(B^+\to K^{*0}\pi^+)+\mathcal{B}(B^-\to \overline{K}^{*0}\pi^-)},$$

$$\mathcal{A}_\pi^{+-}=\frac{\mathcal{B}(B^0\to K^{*+}\pi^-)-\mathcal{B}(\overline{B}^0\to K^{*-}\pi^+)}{\mathcal{B}(B^0\to K^{*+}\pi^-)+\mathcal{B}(\overline{B}^0\to K^{*-}\pi^+)},$$

$$\mathcal{A}_\pi^{00}=\frac{\mathcal{B}(B^0\to K^{*0}\pi^0)-\mathcal{B}(\overline{B}^0\to \overline{K}^{*0}\pi^0)}{\mathcal{B}(B^0\to K^{*0}\pi^0)+\mathcal{B}(\overline{B}^0\to \overline{K}^{*0}\pi^0)}, \tag{39}$$

with analogous definitions for the $K\rho$ channel, with $\mathcal{A}_\pi \to \mathcal{A}_\rho$ and the changes $K^* \to K$ and $\pi \to \rho$.

The results are presented in Table V. They have a peculiar pattern and will therefore provide a crucial test of the present model when future experimental data for these asymmetries are available.

ACKNOWLEDGMENTS

We thank T.N. Pham for valuable collaboration at an early stage of this work. C.I. is partially supported by MIUR under COFIN PRIN-2001.

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