Bari-TH/97-289
CPT-97/P.3560
hep-ph/9711343

QCD Sum Rule Analysis of $B \to (K, K^*)(\ell^+\ell^-, \nu\bar{\nu})$ Decays

PIETRO SANTORELLI

Dipartimento di Fisica, Universitá di Bari, Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy, and Centre de Physique Théorique-CNRS , Marseille, France.

Abstract

We use three-point QCD sum rules to calculate the form factors governing the rare exclusive decays $B \to (K, K^*)\ \ell^+\ell^-$, $B \to (K, K^*)\ \nu\bar{\nu}$. We predict the branching ratios, the invariant mass distributions of the lepton pair for $B \to (K, K^*)\ \ell^+\ell^-$, and the spectra of missing energy for $B \to (K, K^*)\ \nu\bar{\nu}$. The forward-backward asymmetry in $B \to K^*\ell^+\ell^-$ provides us with interesting tests of the Standard Model and its extensions.

Talk given at the
IVth International Workshop on Progress in Heavy Quark Physics,
Rostock, Germany, September 20-22, 1997.
To appear in the Proceedings.

QCD Sum Rule Analysis of $B \to (K, K^*)(\ell^+\ell^-, \nu\bar{\nu})$ Decays

Pietro Santorelli

Dipartimento di Fisica, Universitá di Bari, Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy, and Centre de Physique Théorique-CNRS, Marseille, France.

Abstract

We use three-point QCD sum rules to calculate the form factors governing the rare exclusive decays $B \to (K, K^*)\ \ell^+\ell^-$, $B \to (K, K^*)\ \nu\bar{\nu}$. We predict the branching ratios, the invariant mass distributions of the lepton pair for $B \to (K, K^*)\ \ell^+\ell^-$, and the spectra of missing energy for $B \to (K, K^*)\ \nu\bar{\nu}$. The forward-backward asymmetry in $B \to K^*\ell^+\ell^-$ provides us with interesting tests of the Standard Model and its extensions.

Introduction

The rare B-meson decays induced by the flavour changing neutral current $b \to s$ transition represent important channels for testing the Standard Model (SM) and for searching for the effects of possible new interactions [1]. In the SM these decays are forbidden at tree level, and occur only through loop diagrams. For this reason, physics beyond the SM can modify sensitively this kind of processes.

In order to test the SM predictions in the case of exclusive processes, one needs to take into account nonperturbative QCD contributions parameterized in terms of form factors. In this respect, the main problem, in the case of $B \to (K, K^*)(\ell^+\ell^-, \nu\bar{\nu})$ decays, is the large kinematical range for the squared momentum transfer (q^2) to the lepton pair, which prevents us from making assumptions on the q^2 behaviour of hadronic matrix elements. This difficulty can be overcome by using several approaches, for example the three-point function QCD sum rule technique [2], which is based on general features of QCD and allows us to compute hadronic matrix elements in a large part of the q^2 range. Three-point function QCD sum rules, first used to compute the pion form factor [3], have been applied to heavy meson semileptonic [4] and rare radiative decays [5]. In the following we discuss the results obtained applying this method to calculate the relevant hadronic matrix elements in the $B \to (K, K^*)(\ell^+\ell^-, \nu\bar{\nu})$ decays.

Effective Hamiltonian

The effective $\Delta B = -1$, $\Delta S = 1$ Hamiltonian governing, in the SM, the rare transition $b \to s\ell^+\ell^-$ can be written in terms of a set of local operators [6]:

$$\mathcal{H}_W = 4\,\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)\,, \tag{1}$$

where G_F is the Fermi constant and V_{ij} are elements of the Cabibbo-Kobayashi-Maskawa matrix. We neglect terms proportional to $V_{ub}V_{us}^*$ since the ratio $|V_{ub}V_{us}^*/V_{tb}V_{ts}^*|$ is of the order 10^{-2}. The operators O_i are written in terms of quark and gluon fields and their expressions can be found, for example, in Ref. [7]. For the numerical value of the Wilson coefficients $C_i(\mu)$ we follow the paper [7]: the next-to-leading logarithmic corrections are included only in the coefficient C_9, since at the leading approximation O_9 is the only operator responsible of the transition $b \to s\ \ell^+\ell^-$. Moreover, in our numerical calculations

	$F(0)$	M_P (GeV)		$F(0)$	β (GeV^{-2})
F_1	0.25 ± 0.03	5	A_1	0.37 ± 0.03	-0.023
F_0	0.25 ± 0.03	7	A_2	0.40 ± 0.03	0.034
V	0.47 ± 0.03	5	T_2	0.19 ± 0.03	-0.02
A_0	0.30 ± 0.03	4.8	F_T	-0.14 ± 0.03	
T_1	0.19 ± 0.03	5.3			

Table 1: Parameters of the form factors. The functional q^2 dependence is either polar: $F(q^2) = F(0)/(1-q^2/M_P^2)$ or linear: $F(q^2) = F(0)(1+\beta q^2)$.

we neglect the contribution coming from the penguin operators, $O_3 \div O_6$.
The four-quark operators O_1 and O_2 generate both short- and long-distance (resonant) contributions to the processes. These contributions can be taken into account by replacing C_9 with an effective Wilson coefficient given by [8, 9]:

$$C_9^{eff} = C_9 + (3C_1 + C_2)\left[h\left(\frac{m_c}{m_b}, \frac{q^2}{m_b^2}\right) + k\sum_{i=1}^{2}\frac{\pi\Gamma(\psi_i \to \ell^+\ell^-)M_{\psi_i}}{q^2 - M_{\psi_i}^2 + iM_{\psi_i}\Gamma_{\psi_i}}\right] , \qquad (2)$$

with the parameter k fixed according to the discussion in Ref. [8]. Finally, the effective $b \to s\ell^+\ell^-$ Hamiltonian can be recast in the following form:

$$\begin{aligned}\mathcal{H}_{eff}(b \to s\ell^+\ell^-) = \frac{G_F}{\sqrt{2}}\frac{\alpha_{em}}{2\pi}V_{ts}^*V_{tb}\Big\{ & - \frac{2im_b}{q^2}C_7(m_b)\left[\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\right]\left[\bar{\ell}\gamma^\mu\ell\right] \\ & + C_9^{eff}(m_b)\left[\bar{s}\gamma_\mu(1-\gamma_5)b\right]\left[\bar{\ell}\gamma^\mu\ell\right] \\ & + C_{10}(m_b)\left[\bar{s}\gamma_\mu(1-\gamma_5)b\right]\left[\bar{\ell}\gamma^\mu\gamma_5\ell\right]\Big\} . \qquad (3)\end{aligned}$$

On the other hand, the $b \to s\nu\bar{\nu}$ process is governed by the effective Hamiltonian

$$\mathcal{H}_{eff}(b \to s\nu\bar{\nu}) = \frac{G_F}{\sqrt{2}}\frac{\alpha_{em}}{2\pi\sin^2(\theta_W)}\, V_{ts}V_{tb}^*\, X\left(\frac{m_t^2}{M_W^2}\right)\, \left[\bar{b}\gamma^\mu(1-\gamma_5)s\right]\, \left[\bar{\nu}\gamma_\mu(1-\gamma_5)\nu\right] \qquad (4)$$

obtained from Z^0 penguin and box diagrams where the dominant contribution corresponds to a top quark intermediate state (θ_W is the Weinberg angle). The leading and the $\mathcal{O}(\alpha_s)$ corrections, deriving from two-loop diagrams, are taken into account in the X_0 and the X_1 term, respectively, of the function X:

$$X(x) = X_0(x) + \frac{\alpha_s}{4\pi}X_1(x) \;\; , \qquad (5)$$

which can be found in [10].

$B \to (K, K^*)$ form factors

We have evaluated in Refs. [11, 12] the matrix elements of the hadronic operators appearing in Eqs.(3),(4) between B and K, K^* states, using three-point function sum rules. The definition of the various form factors can be found in [11]. The parameters of the form factors are collected in Table 1. In Figure 1 we have plotted the q^2 behaviour of the

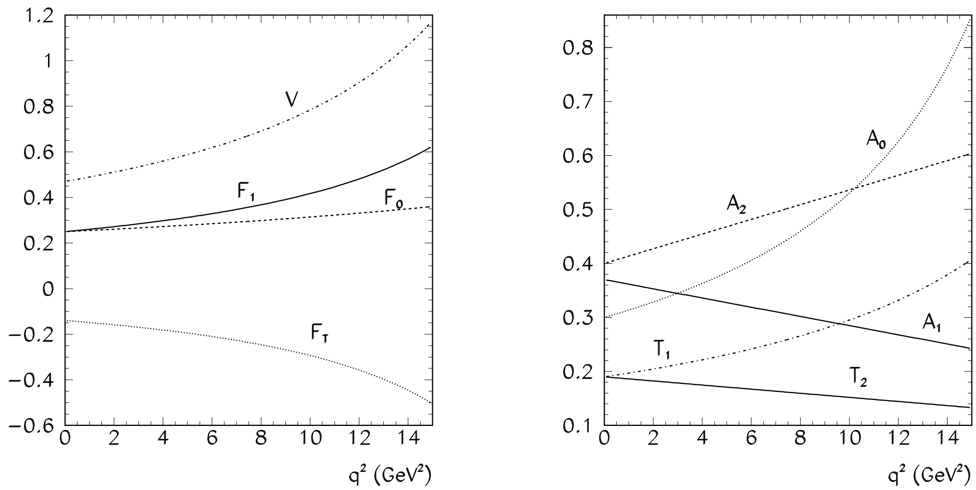

Figure 1: $B \to (K, K^*)$ form factors. The curves refer to the parameters in Table 1.

form factors in the range $[0, 15]$ GeV^2. The form factor $T_3(q^2)$ can be obtained using the quark equation of motion ◇

$$T_3(q^2) = -M_{K^*}(m_b - m_s)\frac{A_3(q^2) - A_0(q^2)}{q^2} \ . \qquad (6)$$

As discussed in Ref. [11], the same procedure can be successfully adopted for F_T, where the equation

$$F_T(q^2) = (M_B + M_K)\,(m_b + m_s)\frac{F_0(q^2) - F_1(q^2)}{q^2} \ , \qquad (7)$$

gives results in agreement with the direct calculation, but with a sensibly larger error. Our results agree at $q^2 = 0$ with similar calculations performed using the Light Cone Sum Rule (LCSR) approach [13]. It should be stressed, however, that the q^2 behaviour predicted for the form factors by LCSR is quite different, in particular for A_1 and T_2 which LCSR predict to be increasing functions of q^2, whereas in three-point QCD sum rules they are quite flat. Various possible sources for these differences have been advocated in the literature [14]; a discussion is beyond the aims of the present paper.

Predictions for $B \to (K, K^*)(\ell^+\ell^-, \nu\bar{\nu})$ processes

Using the computed $B \to (K, K^*)$ form factors, it is straightforward to determine the differential decay rates and the asymmetries for the $B \to (K, K^*)(\ell^+\ell^-, \nu\bar{\nu})$ processes. The various analytical expressions, together with the numerical results, can be found in Refs. [11, 15]. Here we only report our predictions for the branching ratios, without including, for the transition with $\ell^+\ell^-$ in the final state, the resonant long-distance contributions, which are important only in narrow q^2 regions centered at $q^2 = M^2_{J/\psi,\psi'}$. Using $|V_{ts}| = 0.04$, we predict the branching ratios reported in the following table.

$\mathrm{Br}(B \to K\ell^+\ell^-) = 3 \times 10^{-7}$	$\mathrm{Br}(B \to K \sum_i \bar{\nu}_i\nu_i) = (2.4 \pm 0.6) \times 10^{-6}$
$\mathrm{Br}(B \to K^*\ell^+\ell^-) = 1 \times 10^{-6}$	$\mathrm{Br}(B \to K^* \sum_i \bar{\nu}_i\nu_i) = (5.1 \pm 0.8) \times 10^{-6}$

◇However, the cancellation between the two terms in the Eq. (6) implies a large error in its determination. In any case, if we limit to consider light leptons (i.e. excluding the case of τ) in $B \to (K, K^*)\ell^+\ell^-$ the contribution of T_3 is negligible. A direct calculation of T_3 is in progress.

Branching ratios of this order of magnitude should be measured at the future B factories, where 10^9 $B\bar{B}$ pairs *per* year are expected to be produced. In particular, the processes with neutrinos in the final state are theoretically interesting, due to the absence of long-distance resonant contributions and due to the fact that they are induced, in SM, by only one operator; therefore, they represent good candidates for testing the SM predictions and for probing the effects of possible new interactions. Another important quantity is represented by the forward-backward asymmetry in $B \to K^* \ell^+ \ell^-$ transition, which plays a fundamental role in testing the Standard Model predictions. It is expected that such an asymmetry should be about -0.60% for large q^2, and therefore it should be measured at B factories. Its importance come from the fact that it depends not only on the magnitudes but also on the relative signs of the Wilson coefficients, allowing to test their values as they are predicted by SM. So any observed deviation from the predictions, assuming a theoretical error of about 30%, would be interpreted as a signal of New Physics.

Acknowledgments

I thank P. Colangelo, F. De Fazio and E. Scrimieri for their collaboration on the topics discussed here.

References

[1] S. Bertolini, F. Borzumati and A. Masiero, in *B decays*, edited by S. Stone, World Scientific, Singapore, 1992, 458.

[2] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B 147** (1979) 385.

[3] B. L. Ioffe and A. V. Smilga, Phys. Lett. **B 114** (1982) 353; Nucl. Phys. **B 216** (1983) 373; V.A. Nesterenko and A. V. Radyushkin, Phys. Lett. **B 115** (1982) 410; JETP Lett. **39** (1984) 707.

[4] M. Neubert, Phys. Rept. **245** (1994) 259 and references therein; P. Colangelo, G. Nardulli, A. A. Ovchinnikov and N. Paver, Phys. Lett. **B 293** (1992) 207; P. Colangelo, G. Nardulli and N. Paver, Phys. Lett. **B 269** (1991) 204; P. Ball, V. M. Braun and H. G. Dosch, Phys. Rev. **D 44** (1991) 3567; P. Ball, Phys. Rev. **D 48** (1993) 3190.

[5] P. Colangelo, C. A. Dominguez, G. Nardulli and N. Paver, Phys. Lett. **B 317** (1993) 183; P. Ball, report TUM-T31-43/93; S. Narison, Phys. Lett. **B 327** (1994) 360.

[6] B. Grinstein, M.J. Savage and M. B. Wise, Nucl. Phys. **B 319** (1989) 271.

[7] A. Buras and M. Münz, Phys. Rev. **D 52** (1995) 186.

[8] A. Ali, T. Mannel and T. Morozumi, Phys. Lett. **B 273** (1991) 505; A. Ali, G. Giudice and T. Mannel, CERN-TH/7346-94.

[9] P. J. O' Donnell and H. K. K.Tung, Phys. Rev. **D 43** (1991) R2067; N. Paver and Riazuddin, Phys. Rev. **D 45** (1992) 978.

[10] T. Inami and C.S. Lim, Prog. Theor. Phys. **65** (1981) 287; G. Buchalla and A.J. Buras, Nucl. Phys. **B 400** (1993) 225.

[11] P. Colangelo, F. De Fazio, P. Santorelli and E. Scrimieri, Phys. Rev. **D 53** (1996) 3672.

[12] P. Colangelo, F. De Fazio and P. Santorelli, Phys. Rev. **D 51** (1995) 2237.

[13] T.M. Aliev, H. Koru, A. Ozpineci and M. Savci, Phys. Lett. **B 400** (1997) 194; T.M. Aliev, A. Ozpineci and M. Savci, Phys. Rev. **D 56** (1997) 4260.

[14] P. Ball and V. Braun, Phys. Rev. **D 55** (1997) 5561; see also D. Melikhov, these proceedings [hep-ph/9710446], and references therein.

[15] P. Colangelo, F. De Fazio, P. Santorelli and E. Scrimieri, Phys. Lett. **B 395** (1997) 339.