Final state interaction effects on the $\eta_b \to J/\psi J/\psi$ decay[*]

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We study the effects of final state interactions on the $\eta_b \to J/\psi J/\psi$ decay. In particular, we discuss the effects of the annihilation of η_b into two charmed meson and their rescattering into $J/\psi J/\psi$. We find that the inclusion of this contribution may enhance the short-distance branching ratio up to about 2 orders of magnitude.

Large efforts have been invested during the past thirty years to look for η_b but the evidence of its existence emerged very recently thanks to the BABAR collaboration [1]. In [1] is reported the first unambiguous evidence of η_b, with a $10\,\sigma$ significance, through the hindered magnetic dipole transition process $\Upsilon(3S) \to \eta_b\gamma$. The mass of η_b is also measured to be $m_{\eta_b} = 9388.9^{+3.1}_{-2.3}(\text{stat}) \pm 2.7(\text{syst})$ MeV. Apart from its mass and the branching ratio of the $\Upsilon(3S) \to \eta_b\gamma$, almost nothing is known regarding the decay pattern of η_b [2]. However, rough estimate of the branching ratios of some exclusive two and three-bodies hadronic decays can be found in [3].

Some *golden* modes have been proposed to observe η_b, such as $\eta_b \to J/\psi J/\psi$ [4] and $\eta_b \to J/\psi\gamma$ [5, 6]. Despite very clean signature due to the J/ψ in final state, these decay modes are estimated to have rather suppressed branching ratios. Regarding the $\eta_b \to J/\psi J/\psi$ decay mode, the original estimate [4], which was compatible with the discovery of η_b in Tevatron Run I, has been reconsidered [3, 7]. In particular, an explicit NRQCD calculation gives $\mathcal{B}r[\eta_b \to J/\psi J/\psi] = (0.5 \div 6.6) \times 10^{-8}$ [3][1] too small to be observed also in Tevatron Run II.

An interesting decay channel to observe η_b, $\eta_b \to D^{(*)}\overline{D^*}$, has been proposed in [7] where the range $10^{-3} < \mathcal{B}r[\eta_b \to D\overline{D^*}] < 10^{-2}$ and $\mathcal{B}r[\eta_b \to D^*\overline{D^*}] \approx 0$ were predicted. On the other hand, in Ref. [3], by doing reasonable physical considerations, the author obtained $\mathcal{B}r[\eta_b \to D\overline{D^*}] \sim 10^{-5}$ and $\mathcal{B}r[\eta_b \to D^*\overline{D^*}] \sim 10^{-8}$ which are at odds with the ones obtained in [7].

In [9] we assumed that the long distance contribution to the final state made of two J/ψ is dominated by the $D\overline{D^*}$ state and the subsequent rescattering of it into two J/ψ with a charmed meson in the $t-$channel as is shown in figure 1. The branching ratio of $\eta_b \to D\overline{D^*}$ is poorly known at present. However, as we already said there are two theoretical determinations we will use in considering the contribution to the $\eta_b \to J/\psi\ J/\psi$. Moreover, we will neglect the contribution coming from the annihilation of the η_b to $D^*\overline{D^*}$, in agreement with the results in [3, 7].

The dominance of $D\overline{D^*}$ intermediate state is a consequence of the large coupling of $D^{(*)}\overline{D^{(*)}}$ to J/ψ as a result of quark models and QCD Sum Rules calculations.

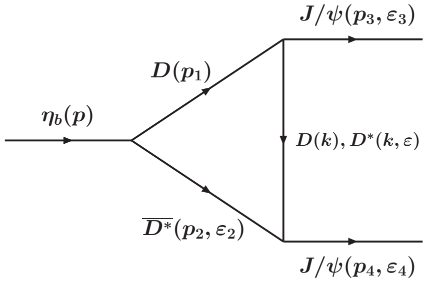

Figure 1: Long-distance $t-$channel rescattering contributions to $\eta_b \to J/\psi\ J/\psi$.

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[*] To the memory of Giuseppe (Beppe) Nardulli

[1] See also very recent calculation in NRQCD at NLO in α_s $\mathcal{B}r[\eta_b \to J/\psi J/\psi] = (2.1 \div 18.9) \times 10^{-8}$ [8].

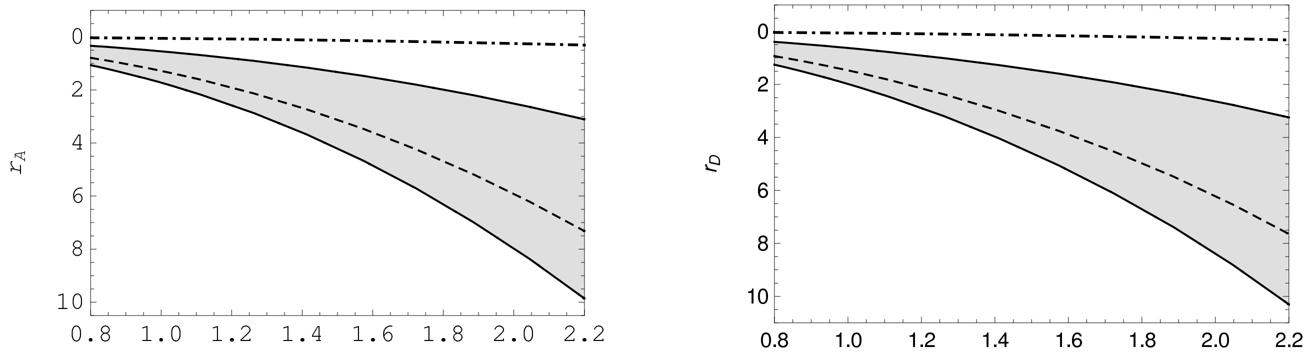

Figure 2: The contributions coming from the loop graphs (for definitions see text). The contributions are plotted for $g_{\eta_b DD^*}/g_{\eta_b JJ} \approx 1$ (dashed-dotted lines) and $g_{\eta_b DD^*}/g_{\eta_b JJ} \approx \{11, 35\}$ (solid lines). The dashed lines correspond to $g_{\eta_b DD^*}/g_{\eta_b JJ} \approx 26$.

The full amplitude which takes into account the short distance part and the contribution coming from the evaluation of the graphs in figure 1 can be written as

$$\mathcal{A}_f(\eta_b(p) \to J/\psi(p_3,\varepsilon_3)\; J/\psi(p_4,\varepsilon_4)) \;=\; \imath\, \frac{g_{\eta_b JJ}}{m_{\eta_b}}\, \varepsilon_{\alpha\beta\gamma\delta}\, p_3^{\alpha} p_4^{\beta} \epsilon_3^{*\gamma} \epsilon_4^{*\delta} \left[1 + \frac{g_{\eta_b DD^*}}{g_{\eta_b JJ}} \Big(\imath\, A_{LD} + D_{LD} \Big)\right], \tag{1}$$

where A_{LD} and D_{LD} represent the absorbitive and the dispersive part of the graphs in figure 1, respectively. For details about the calculation of the previous quantities we refer to [9, 10]. The coupling $g_{\eta_b JJ}$ is obtained by using the results in [3] while $g_{\eta_b DD^*}$ from the estimate of the $\mathcal{B}r[\eta_b \to D\overline{D^*}]$ and so

$$\frac{g_{\eta_b DD^*}}{g_{\eta_b JJ}} \begin{cases} = 1 & \text{for } \mathcal{B}r[\eta_b \to D\bar{D}^*] \approx 10^{-5} \quad [3] \\ \\ \in [11, 35] & \text{for } 10^{-3} \leq \mathcal{B}r[\eta_b \to D\overline{D^*}] \leq 10^{-2} \quad [7]. \end{cases} \tag{2}$$

The numerical values of the on-shell strong couplings g_{JDD}, g_{JDD^*} and $g_{JD^*D^*}$[2] are taken from QCD Sum Rules [11], from the Constituent Quark Meson model [12] and from relativistic quark model [13] findings which are compatible each other. We used $(g_{JDD}, g_{JDD^*}, g_{JD^*D^*}) = (6, 12, 6)$. To take into account the off-shellness of the exchanged $D^{(*)}$ mesons in figure 1 we have introduced the $t-$dependance of these couplings by means of the function

$$F(t) = \frac{\Lambda^2 - m_{D^{(*)}}^2}{\Lambda^2 - t}\,. \tag{3}$$

No first-principles calculation of Λ exists, so, following the authors of [14], we write $\Lambda = m_R + \alpha\Lambda_{QCD}$, where m_R is the mass of the exchanged particle (D or D^*), $\Lambda_{QCD} = 220\ MeV$ and $\alpha \in [0.8, 2.2]$ [14]; with this values, the allowed range for Λ is given by: $2.1 < \Lambda < 2.5\ GeV$.

In figure 2, left panel (right panel) the ratio $r_A = A_{LD}\ g_{\eta_b DD^*}/g_{\eta_b JJ}$ ($r_D = D_{LD}\ g_{\eta_b DD^*}/g_{\eta_b JJ}$) is plotted as a function of α for the allowed value and the range of couplings ratio. Moreover, the dashed lines are for $g_{\eta_b DD^*}/g_{\eta_b JJ} \approx 26$ which correspond to the central value in the allowed range for $\eta_b \to D\overline{D^*}$ estimated in Ref. [7]. It is clear that for $g_{\eta_b DD^*}/g_{\eta_b JJ} \approx 1$ the effects of the final state interactions are negligible independently of α.

Very different is the case in which the annihilation of η_b into $D\overline{D^*}$ is large [7]. The effects of final-state interactions could be large and depend strongly on the value of α (cfr gray bands in figure 2).

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[2] We use dimensionless strong coupling constants in all cases. In particular we use the ratio $g_{JDD^*}/m_{J/\psi}$ instead of the dimensional G_{JDD^*} (GeV^{-1}) usually found in literature.

Starting from the estimate of the short-distance part in [3] we are able to give the allowed range for the full branching ratio

$$\mathcal{B}r[\eta_b \to J/\psi\ J/\psi] = 0.5 \times 10^{-8} \div 1.2 \times 10^{-5}, \tag{4}$$

where the lower bound corresponds to the corresponding one in [3], while the upper bound is obtained using the upper value in [3] and for $\alpha = 2.2$, $g_{\eta_b DD^*}/g_{\eta_b JJ} = 35$. The wide range for $\mathcal{B}r[\eta_b \to J/\psi\ J/\psi]$ in Eq. (4) depends on the large theoretical uncertainty of the estimate of $\mathcal{B}r[\eta_b \to D\overline{D}^*]$ and on the dependence on α parameter. It should be observed that in [14] the preferred value for α is $\alpha \approx 2.2$ for diagrams with D and D^* in t-channel, whereas a direct calculation or measurement of the $\eta_b \to D\overline{D^*}$ process is in order.

Finally we give an estimate of the discovery potential of the decay mode in the LHC experiments. Each J/ψ in the final state can be reconstructed by means of its muonic decay mode which represents about 6% of the total width, so we have $\mathcal{B}r[\eta_b \to J/\psi\ J/\psi \to 4\mu] \approx 2 \times 10^{-11} \div 4 \times 10^{-8}$. Moreover, assuming, as in [3], that i) the η_b production cross section at LHC is about 15 μb and ii) the integrated luminosity (per year) is about 300 fb^{-1}, the theoretically expected events are between 80 and 2×10^5. Experimentally we have to consider also the product of acceptance and efficiency for detecting J/ψ decay to $\mu^+\mu^-$ which is of the order of 0.1 [4], so we expect between 0.8 and 2000 observed events per year. Further, if we loose the constraint that J/ψ must be tagged by $\mu^+\mu^-$ pair and also allow its reconstruction through e^+e^- mode, we can have $3 \div 8000$ observed 4-lepton events per year. These results seem to indicate that the chance of observing η_b at LHC through the 4-lepton mode exists.

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