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Eyes on Kepler Laws: a multimedia based on data/images by a planetarium

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This multimedia (MM) is in English and freely downloadable at http://people.na.infn.it/~smaldone
Total size of the file is about 49 Mb.
Once downloaded into the local computer, it can run without Internet connection. The MM has been realised on the basis of data and dynamic images from the Planetarium in Caserta (Italy) (scientific director L.A. Smaldone, http://www.planetariodicaserta.it). The three Kepler's laws are addressed in the case of the planets of the solar system. The visualization of the planets' motions, the surrounding stars and the Milky Way are based on real data and images. The polar projection (*fish-eye* format) allows to observe the full hemisphere of the sky. The time counters at the bottom of the images measure the time in Universal Time (U.T.).

The three laws by J. Kepler about the motion of the planets of the solar system are almost always taught by formulas and schemas. Often there are teaching-learning problems in relating the analytical expressions of these laws with the motions of the planets as perceived by a terrestrial observer. Difficulties are also encountered in understanding the meaning of these laws and their role in the historical and conceptual development of physics. At the beginning of the MM two historical portraits show Tycho Brahe (1546 – 1601) and Johannes Kepler (1571 – 1630) together with the first edition of *Quadrans Muralis* (1598), *Astronomia nova* (1609) and *Harmonices Mundi* (1619). These allude to the great power of abstraction of the work by Kepler, based on the analysis of very many and much accurate observations by Tycho Brahe.

The First Law has been formulated after many numerical analyses of the data collected about Mars and has been fed by Kepler's studies of conical sections. Since Kepler has worked only on planets' observations performed by Tycho Brahe, no open orbits (parabolas or hyperboles) have been hypothesised.

In the case of the Second Law the analysis has been based on Kepler's approach to calculus that later will be developed by Newton.

The work resulted in the Third Law has been the most complex one. Kepler calculated several ratios of planetary parameters in order to find regularities. The models he has been inspired by were linked to musical tones and semitones (*the music of the spheres*); therefore much numerical analysis have been made. Eventually he came up with the constancy of the ratio of the square of the planet revolution period to the cube of its orbit semi-major axis.

The First and Second laws have been formalised for the planet Mars and published in 1609 (*Astronomia nova*); the Third Law, published in 1619 (*Harmonices Mundi*), has been formalised on the basis of data about Mercury, Venus, Mars, Jupiter and Saturn.

We use the two didactical indicators Pedagogical Effectiveness and Transformation Potential to summarise the educational contributions offered by the MM in addressing some learning/teaching problems commonly encountered by students and teachers when the Kepler's laws are studied (e.g. meaning of each law, similarities and differences in the planets' orbits, estimation of orbits' eccentricity and periods, areal versus orbital velocity, etc.). The connection between the orbits of the eight planets in the solar system and their abstract representations as visualization of the orbits (dynamic display) can help the learners familiarise with the planetary motions. The used planetarium images together with the possibilities of various viewpoints and scale changes (zooming in/out) produce a context very close to reality and much richer then that of a simulation based only on mathematical models. In this way the learners are helped to familiarise with the characteristics of the orbits and to understand a key content of basic astronomy. The context realised allows discussions about power and limits of a model, approximations, graphic displays, etc…

Pedagogical Effectiveness, ex. Visualization of Orbits and Measurements

The Kepler First Law and some common difficulties in understanding the different eccentricity of elliptical orbits are addressed by visualising Mercury, Venus and Earth orbits, the last two being almost circular while the first is clearly elliptical The choice of Mercury is justified by the fact that the eccentricity of its orbit is 0.21 . After various viewpoints the orbit is looked at by an observer whose vision line points toward the Terrestrial South Pole (you can see the Southern Cross). Given a circle centred on the Sun, Mercury orbit appears clearly elliptical, with its two foci (Fig. 1, 2).
The orbits of Venus and Earth are therefore constructed and visualised to show the differences with respect to the Mercury one: both are almost circular, their eccentricity being respectively 0.01 and 0.02, their foci are very near (Fig. 3, 4).

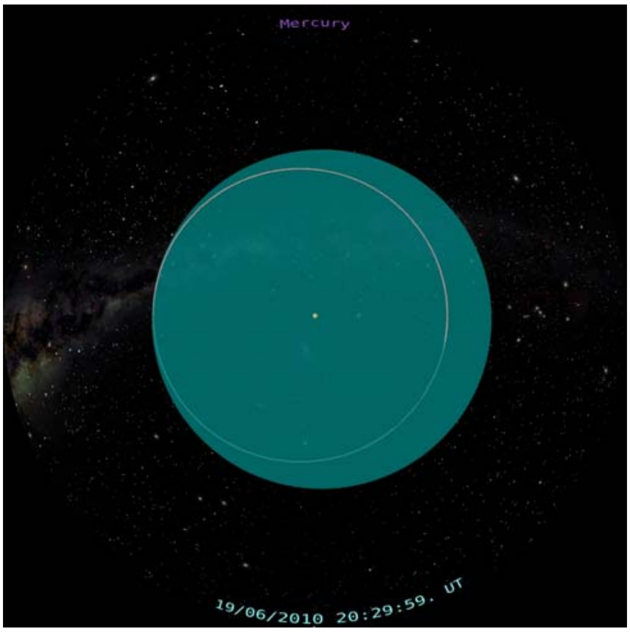

Fig 1: Mercury orbit (white ellipse)

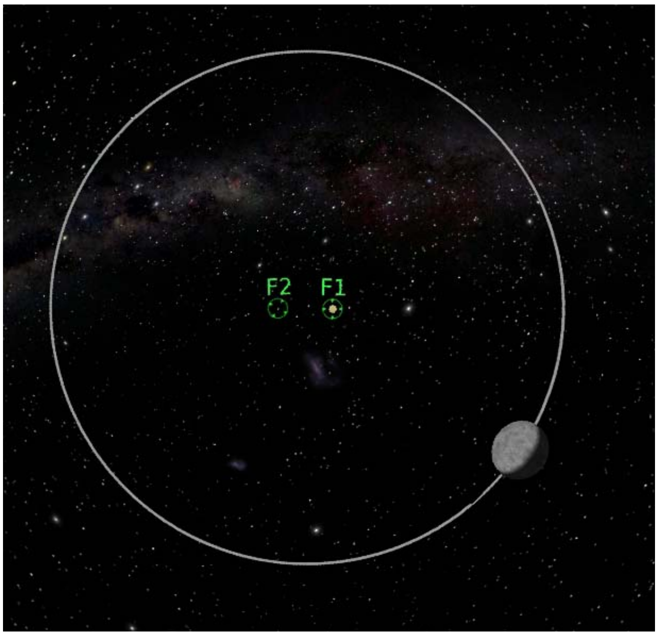

Fig 2: The two foci of the Mercury orbit

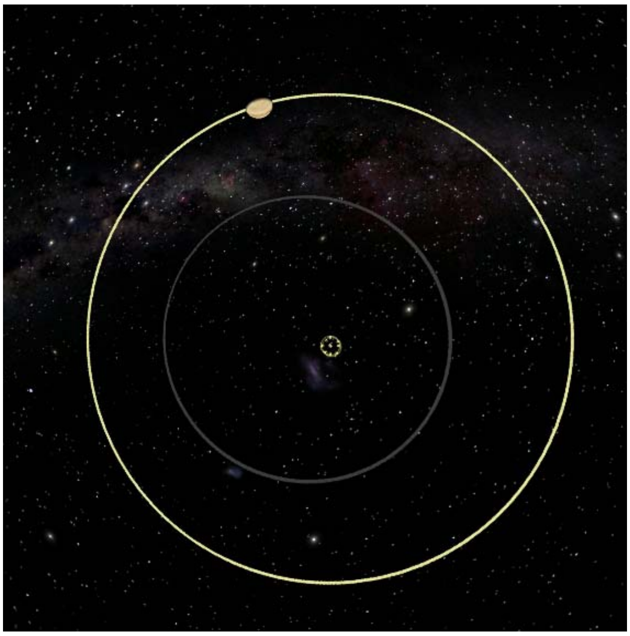

Fig 3: Venus orbit (yellow curve), the two foci almost coincide.

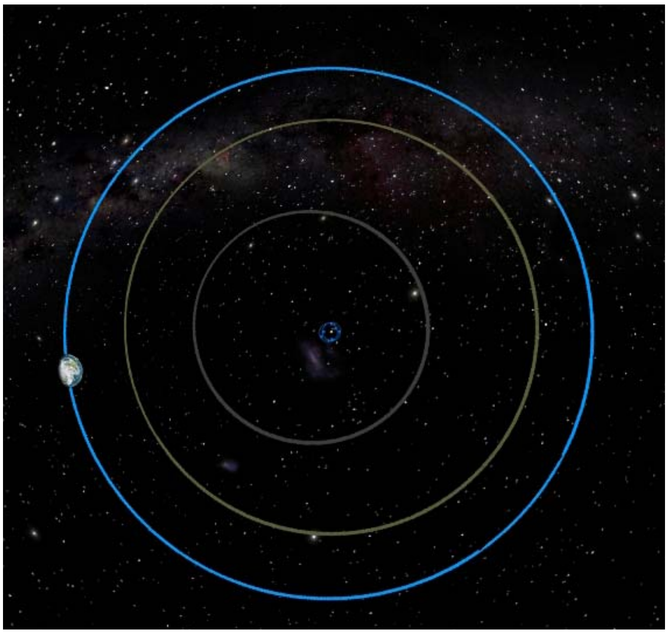

Fig 4: Earth orbit (blue curve), the two foci almost coincide.

The First Law is then expressed as "The orbits of the planets are ellipses, the Sun is at one focus of the ellipse".

An emblematic task for the learners may be to ask them to describe how the orbits of Mercury, Venus and Earth relate to each other. A discussion about the features of a bound system of two bodies (Sun and a planet) can clarify why:

- the orbits are contained in a plane (identified by the initial position of the planet and its initial velocity);
- the solar planets move along elliptic orbits (the orbits of bound systems are elliptic because the gravitational interaction Sun-planet depends on their distance as $1/r^2$). The Halley comet moves along an ellipse with high eccentricity (e = 0.967 !), its visit to the internal solar system is periodic (P = 75.3 years), only a small fraction of its orbit belongs to the internal solar system;
- in the focus of the elliptic orbit, not occupied by the Sun, no other massive body is needed. The orbits is completely determined by the only Sun-planet gravitational interaction.

The Second Law about the areal velocity of a planet is clarified by analysing the motion of Mercury, with special focus on its position and velocity near perihelion and aphelion. The visualisation of the orbit and an accurate observation of the motion in two time intervals, near perihelion and aphelion, shows clearly that the angular velocity is not constant. Some measures can be made using the Pause functionality of the dynamic image (Fig. 5, 6, 7). The constancy of areal velocity can be inferred by measuring the area swept off in four days of Mercury revolution (P_M = 88 days) A plane divided in small squares, as a red graph paper, is superimposed to the sky scenario to facilitate the observation of the orbit features (the squares are deformed by the polar projection).

As in any Keplerian motion the areal velocity is constant, this implies that at the perihelion the orbital velocity of the planet is greater than at the aphelion.

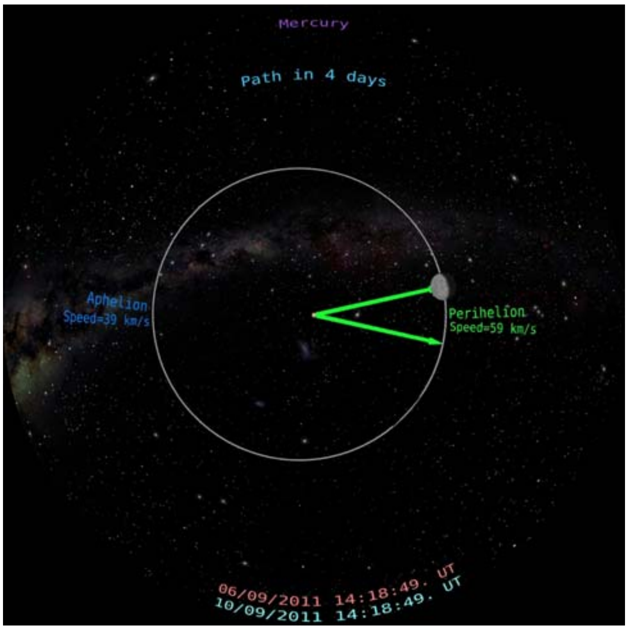

Fig 5: Mercury orbit: sector at perihelion (green)

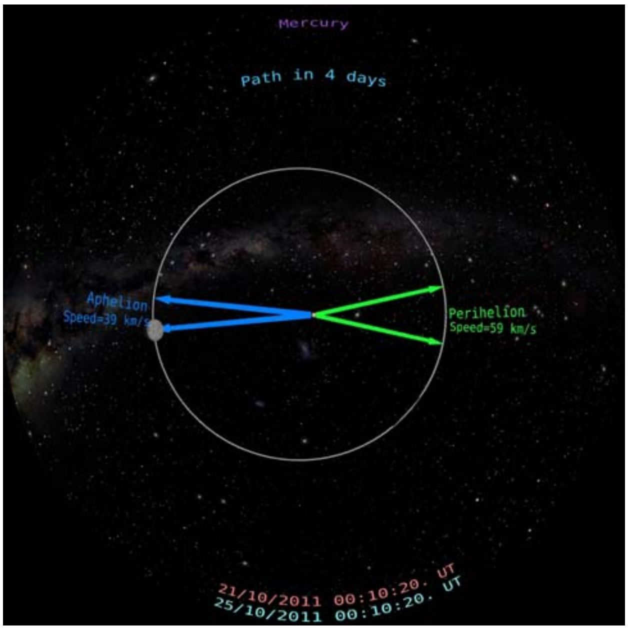

Fig 6: Mercury orbit: sector at perihelion (blue)

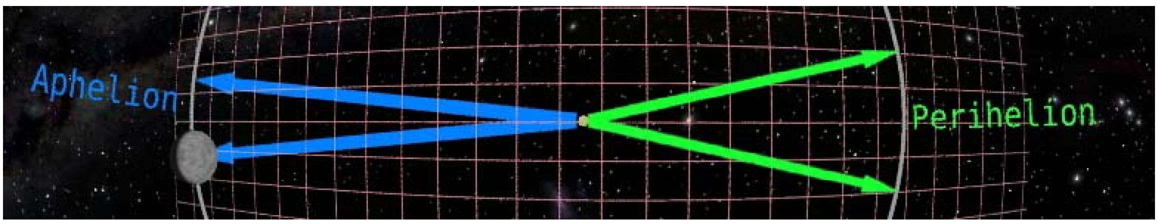

Fig 7: To clarify the II Kepler law by measuring the equal swept area in four days.

The Second Law is then expressed as " A line joining a planet and the Sun sweeps out equal areas in equal times as the planet moves along its elliptic orbit".

Emblematic questions for the learners can be: "According to your knowledge and ideas, how do you justify the constancy of the areal velocity?" "Why the angular velocity ω is not constant ?" "Why at the aphelion, where in the near focus there is not an attracting massive body does the planet move slower than at perihelion? "
The last question allows deepen the previous discussion about the First Law and address the naïve idea that in both foci of the orbit should be a massive attracting object. Because of the constant areal velocity the orbit's arc described, in the same time interval, is shorter at the aphelion than at perihelion; the planet at aphelion is moving at lower angular speed: the Sun is far and the attractive force is smaller than at perihelion.

The Third Law establish a relation between the periods and the semi-major axes of the orbits of all solar planets. In this MM it is addressed via the examples of Mercury and Venus that move on the real background of a steady Milky Way and of the moving Earth and Mars. The periods and semi-axes of their dynamically shown orbits are measured and related. The clock starts when Sun, Mercury and Venus are aligned (blue line, this event happens 4-5 times per year). It is immediately evident that Mercury moves with greater velocity with respect to Venus. Measurements of periods, together with geometrical data about the orbits, allow collect data for clarifying the Third Law. It is more convenient to use the Earth's quantities as measure units (time: Earth revolution period, the year; distances: Earth orbit semi-major axis, the Astronomical Unit).
The Third Law is then expressed as "The square of the orbital revolution period of a planet is directly proportional to the cube of the semi-major axis of its orbit". Using Earth's units, the ratio of the square of the revolution period and the cube of the semi-major axis is 1!
The MM allows to implement the well acknowledged Predict – Observe - Explain (POE) Learning Cycle (also Prediction, Experiment, Comparison (PEC); for example see White, R.T., & Gunstone, R.F. (1992). Probing understanding. London: Falmer Press).
In the Prediction phase of POE the learners can be asked to express their ideas and reasoning about the motion of various solar planets and to predict features of orbits and periods. Then, the MM is used (Observe phase) and the learners are asked to comment the results of their observations. In the Comparison phase the learners discuss amongst themselves (peer learning) and with the teacher similarities or differences between the previous two phases and possible learning difficulties are addressed. The iteration of the POE cycle facilitates to spiral around Kepler's Laws at diverse depths according to the context and the learning objectives.

Example tasks:

a) Estimate the period of Mars by measuring the time it needs to move around the Sun by 90^o. To find the final position it is enough to put two "post-it" on the PC screen: the first aligned with Mars and Sun at the starting time (when Mercury and Venus are aligned, blue line); the second orthogonal to the first and aligned with the Sun (the small spot at the center of the image). The MM is paused when Mars reaches the second "post-it". Fig. 8 shows the initial and final position of Mars that has moved along a quarter of its orbit.
The elapsed time (within few minutes) is Δt = 04/05/2012-16/10/2011 = 202 days. The resulting period of Mars is $4 \times 202 = 808$ days, greater than 687 days, the Mars period. An error of about 18% has to be taken care of; it is not due to uncertainties in positioning the two "post-it" but it derives from the II Kepler Law: the areal velocity is constant, not the angular one. The eccentricity of Mars orbit is $e = 0.09$, large enough to produce a significant difference in the times needed to span an orbit quadrant.

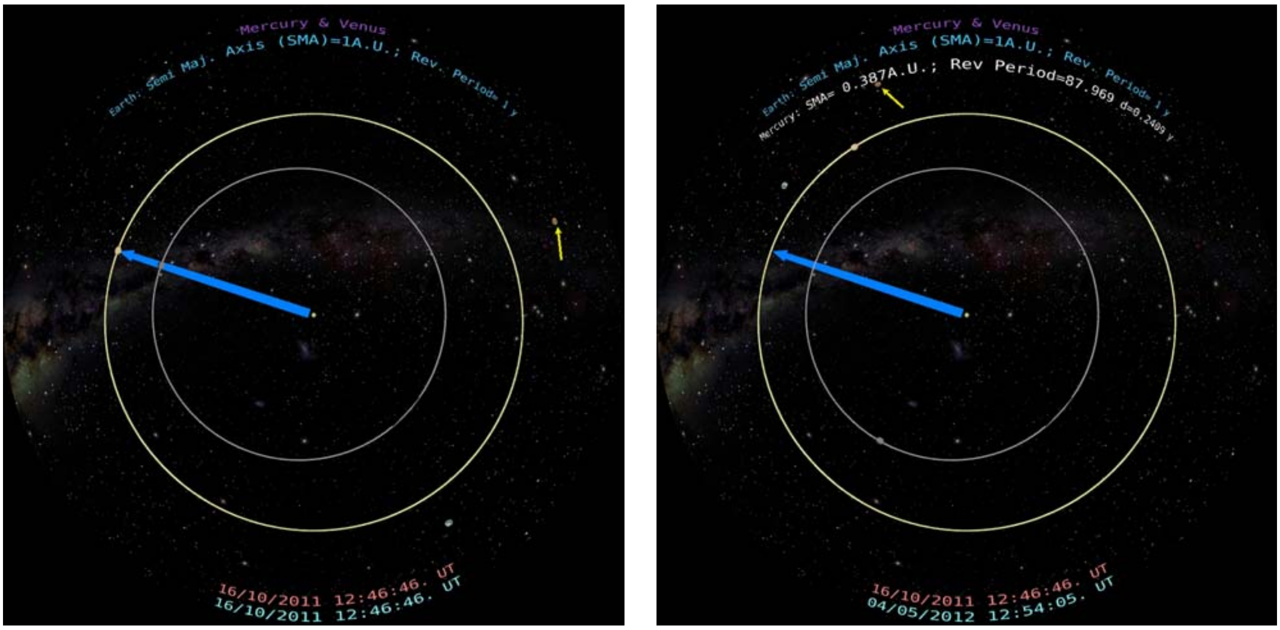

Fig 8: Mars (indicated by the yellow arrow) has moved 90° around the Sun.

b) Observe the features of the motions of the planets of internal solar system (from Mercury to Mars) and write a short essay about what have you noticed in terms of similarities and differences, …. and which questions you would like to be answered.

Transformation Potential, ex. Variational Approach: what happens if .. changes ?

A variational approach of the type "what happens if "such and such" changes ?" can be practised.. For instance the user can change the period of a planet and calculate, by the Third Law, how this change affects the orbit. This approach refers to "paper and pencil" problems, not being possible to change the MM images. The observation of how these changes affect the motion of solar planets can beneficially influence the learning process of the individual learner or the class dynamics when the tasks are part of a class activity. In this latter case the teacher-learners group can more easily transform into a learning community of practice.

Example of paper and pencil task:

a) Invent a "My Planet" choosing its distance with respect to the Sun (or the orbital period) and find out its orbital period (or the distance from the Sun). The Third Law is valid for all the bodies orbiting around the Sun. The orbital elements, for some popular comets and the dwarf planets (Table 1) can be used to calculate one orbital element from another one in case of "real" small bodies of the solar system.

Table 1: Orbital elements for the solar system dwarf planets and some popular comets.

Object	Semi-major axis (A.U.)	Orbital Period (years)	(days)
1P/Halley comet	17.834	75.32	27509.1
109P/Swift-Tuttle comet	26.092	133.28	48681.2
C/1995 O1 Hale-Bopp comet	185.864	2533.97	925534.2
C/1996 B2 Hyakutake comet	2295.540	109985.55	40172221.3
Ceres (dwarf planet)	2.765	4.60	1679.7
Pluto (dwarf planet)	39.445	247.74	90487.3
Haumea (dwarf planet)	42.995	281.93	102974.7
Makemake (dwarf planet)	45.344	305.34	111526.0
Eris (dwarf planet)	68.049	561.35	205034.4

Printed copies[1] of the MM images allow exercises that can answer some questions commonly asked by the learners.
b) Estimate the eccentricity of the orbits of Venus and Earth and compare.
Solution: select an image where the orbits of the inner planets fill the field of view (on the last 30 seconds of the MM), grab it. The image and its negative printed copy are shown in Fig. 9.
Two perpendicular diameters of the Venus and Earth orbits have been drawn on the printed copy. The measured values, within the experimental errors, indicate that the two orbits are almost circular. The line of sight of this image is orthogonal to the orbital plane of Mercury (inclined by 3.6 ^{o} with respect to the one of Venus and by 7.0 ^{o} with respect to the one of Earth). Because of these small values the orbits of Venus and Earth do not appear as very circular by eye.
c) Using the same printed image than before, estimate the eccentricities of Jupiter and Mars orbits and compare.

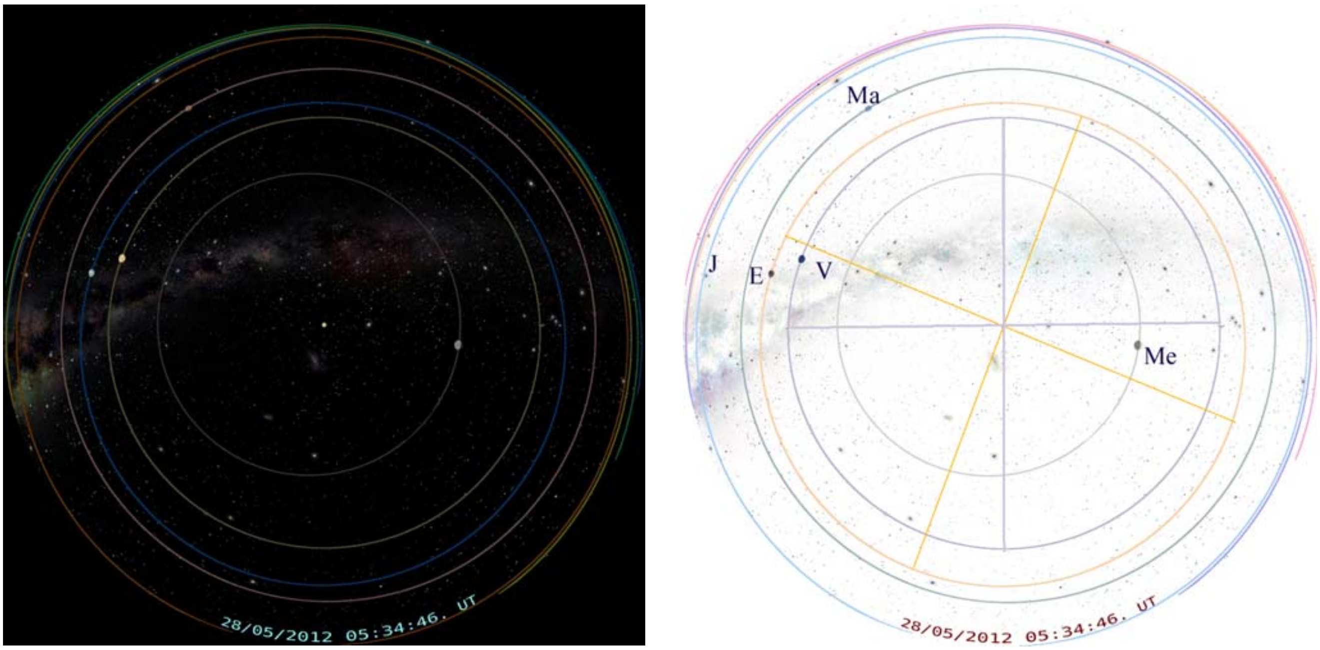

Fig 9: Orbits of the first five solar planets: Mercury (Me), Venus (V), Earth (E), Mars (Ma) and Jupiter (J)

Eyes on Kepler Laws can be used for class activities, home work, tasks and problem-based learning. It is also appropriate in teacher's education programs. In 2010-2011 it is being experimented with two types of audience: secondary school last year students and their physics teachers. These activities are held in the framework of an Italian National Plan aimed at:

- offering the students elements for deciding about University curricula;
- improving the scientific education at secondary school level;
- improving the preparation of in-service teachers through experiential activities done together with their classes.

This Piano Lauree Scientifiche (PLS, Plan for University Scientific Degrees) has been started by the Ministry of Education and Research in 2005; it involves many Italian Universities, Regional Education Authorities and Confindustria (Federation of National Companies) and is articulated in various sections, one particularly dedicated to Physics.
The University of Naples "Federico II" is active in PLS since its beginning, each year many secondary schools are involved (about 250 students and about 30 physics teachers).

[1] To make a printed copy use ***Pause*** option; ***Ctrl-Alt-Print*** allow to export the image in **Paint**; select the image **Invert Color** … and print.