nature physics

Matters arising

https://doi.org/10.1038/s41567-025-03062-w

Inadequacy of the Casimir force for explaining a strong attractive force in a micrometre-sized narrow-gap re-entrant cavity

Received: 11 April 2024

Accepted: 5 September 2025

Published online: 21 October 2025

Check for updates

Giuseppe Bimonte[1,2] ✉

ARISING FROM J. M. Pate et al. *Nature Physics* https://doi.org/10.1038/s41567-020-0975-9 (2020)

Pate et al.[1] investigated a macroscopic optomechanical system with a narrow-gap re-entrant cavity coupled to a silicon nitride (SiN) membrane resonator coated with gold (Au) or niobium (Nb). They observed a large increase in the membrane's effective spring constant $k_{\rm eff}$ for sub-2-μm gaps x. This increase scales roughly with x^{-4}, suggesting an attractive force pulling the membrane towards the re-entrant aluminium (Al) post, with an x^{-3} dependence. A calculation based on the proximity force approximation (PFA) reveals that the Casimir force, at the investigated gap sizes, is orders of magnitude weaker than the observed force. This large discrepancy necessitates an alternative explanation for the observed attraction.

Computation of the Casimir spring

The geometry of the re-entrant cavity is displayed in supplementary fig. 3 of ref. 1. The Casimir force $F_{\rm C}(x)$ between the Al post and the membrane can be estimated using the standard PFA[2,3], which involves decomposing the surfaces of the two bodies into pairs of small and parallel patches and then adding up the Casimir forces for all pairs of patches. The PFA remains a popular tool for interpreting Casimir force measurements due to its simplicity and effectiveness for objects in close proximity. This is particularly true for the narrow-gap cavity used by Pate et al.[1], where x is much smaller than the cap radius r_0 by a factor exceeding 50. It is important to remember that the experiment is primarily concerned with the Casimir spring constant $k_{\rm C}$, rather than the absolute force. The spring constant represents the rate of change of $F_{\rm C}(x)$ with respect to x:

$$k_{\rm C} = F'_{\rm C}(x). \tag{1}$$

Using the PFA, one finds:

$$k_{\rm C} = \pi r_0^2 F'_{\rm PP}(x) + \frac{2\pi(r_1-r_0)}{h}\left[r_1 F_{\rm PP}(x+h) - r_0 F_{\rm PP}(x)\right] + \frac{2\pi(r_1-r_0)^2}{h^2}\left[E_{\rm PP}(x+h) - E_{\rm PP}(x)\right], \tag{2}$$

where r_0 and r_1 represent the radii of the post's cap and base, while h is the height of the cavity (see supplementary fig. 3 of ref. 1). In the above formula, $E_{\rm PP}(a)$ represents the Casimir energy per unit area between two (infinite) plane-parallel slabs separated by a gap of width a, whereas $F_{\rm PP}(a) = -E'_{\rm PP}(a)$ is the corresponding Casimir force per unit area (negative forces represent attraction). The first term in the equation accounts for the contribution of the top flat surface of the post, while the remaining terms represent the contribution of the post's sidewalls. According to the Lifshitz formula[4], $E_{\rm PP}(a)$ has the expression:

$$E_{\rm PP}(a) = \frac{k_{\rm B}T}{2\pi}\sum_{l=0}^{\infty}{}' \int_0^{\infty} {\rm d}k_\perp k_\perp \sum_{\alpha={\rm TE,TM}} \log\left[1 - r_\alpha^{(1)}(i\xi_l, k_\perp) r_\alpha^{(2)}(i\xi_l, k_\perp)\, {\rm e}^{-2aq_l}\right], \tag{3}$$

where $k_{\rm B}$ is the Boltzmann constant, T is temperature, $k_\perp$ is the in-plane momentum, the prime symbol in the sum indicates that the $l = 0$ term is taken with weight one-half, $\xi_l = 2\pi l k_{\rm B}T/\hbar$ $(l = 0, 1, 2, ...)$ are the imaginary Matsubara frequencies, $q_l = \sqrt{\xi_l^2/c^2 + k_\perp^2}$, the index α = TE, TM labels the two independent states of polarization of the electromagnetic field (that is, transverse magnetic (TM) and transverse electric (TE)) and $r_\alpha^{(k)}(i\xi_l, k_\perp)$ denote the Fresnel reflection coefficients of the kth slab for polarization α. We modelled the Al post (slab 1) as infinitely thick, while the membrane (slab 2) is a composite of a 500 nm SiN substrate and a 300 nm metallic (Au or Nb) coating. The force $F_{\rm PP}(a)$ and its derivative $F'_{\rm PP}(a)$ relevant to the Casimir force are obtained from equation (3). The experimental set-up in Pate et al.[1] allows a simplification. The gap size x is much smaller than characteristic dimensions of the post (r_0, r_1 and h), and the post is also thin ($h \gg r_0, r_1$). This implies that the post's lateral surface makes a negligible contribution to $k_{\rm C}$ compared with its top face. Therefore, in equation (2), only the first term is important. Consequently, $k_{\rm C}$ can be approximated by a simpler formula:

$$k_{\rm C} = \pi r_0^2 F'_{\rm PP}(x). \tag{4}$$

[1]Dipartimento di Fisica E. Pancini, Universitá di Napoli Federico II, Naples, Italy. [2]INFN Sezione di Napoli, Naples, Italy. ✉e-mail: bimonte@na.infn.it

https://doi.org/10.1038/s41567-025-03062-w

Table 1 | Drude parameters for Au, Nb and Al

Parameter	Value (eV$\hbar^{-1}$)
Parameters for Al	
Ω_{Al}	13
γ_{Al}	0.1
Parameters for Au	
Ω_{Au}	9.0
γ_{Au}	0.035
Parameters for Nb	
Ω_{Nb}	9.9
γ_{Nb}	0.2

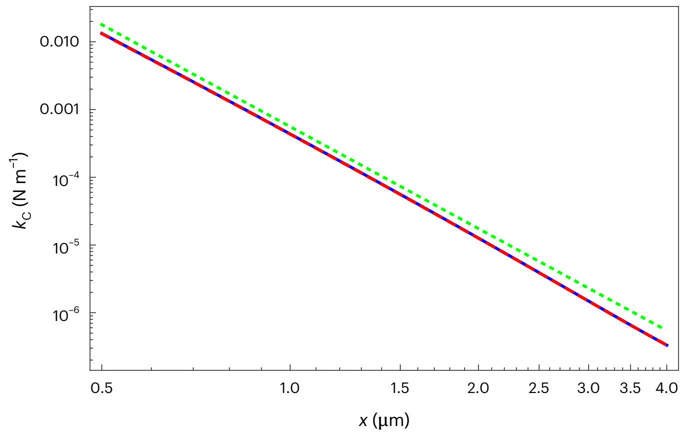

Fig. 1 | Casimir spring constant as a function of gap size. The spring constant is shown for a Au-coated membrane (blue solid line) and a Nb-coated membrane (red dashed line), both computed for $T = 300$ K (room temperature). The green dashed line shows $k_C^{(pc)}$ in the limit of a perfectly conducting cavity at $T = 0$ (see equation (8)).

Another key simplification emerges from the properties of relevant Casimir force contributors. The Lifshitz theory (equation (3)) indicates that crucial Matsubara modes have imaginary frequencies near $\omega_c = c/(2x)$ (c is the speed of light) determined by x. For the experiment's gap range (0.59 µm to 3.3 µm), the penetration depth δ of these modes in Au, Nb and Al is limited to tens of nanometres. As this depth is much thinner than the metallic coating (300 nm thick) on the SiN membrane, the membrane behaves essentially like an infinitely thick slab of either Au or Nb for Casimir force calculations. This allows us to model both the post and the membrane as infinitely thick planar slabs (Al and Au/Nb, respectively) when evaluating equation (4). Consequently, the following well-known expressions for Fresnel coefficients can be employed:

$$r_{TE}^{(k)}(i\xi_l, k_\perp) = \frac{q_l - s_l^{(k)}}{q_l + s_l^{(k)}}, \quad (5)$$

$$r_{TM}^{(k)}(i\xi_l, k_\perp) = \frac{\epsilon_l^{(k)} q_l - s_l^{(k)}}{\epsilon_l^{(k)} q_l + s_l^{(k)}}, \quad (6)$$

where $s_l^{(k)} = \sqrt{\epsilon_l^{(k)} \xi_l^2 / c^2 + k_\perp^2}$ and $\epsilon_l^{(k)} \equiv \epsilon^{(k)}(i\xi_l)$ is the permittivity of the material constituting the slab. In the range of frequencies that is relevant to the Casimir force, the optical properties of Au, Nb and Al can be described by a simple Drude model:

$$\epsilon(i\xi) = 1 + \frac{\Omega^2}{\xi(\xi + \gamma)}, \quad (7)$$

where Ω is the plasma frequency and γ is the relaxation frequency. The parameters are provided in Table 1.

In Fig. 1 we show a plot of k_C (in N m^{-1}) for a Au-coated membrane (blue solid line) and for a Nb-coated membrane (red dashed line) versus the gap size x (in µm) computed using equation (4) for $T = 300$ K (room temperature). The green dashed line in Fig. 1 shows the PFA value $k_C^{(pc)}$ of the spring constant equation (4) in the limit of a perfectly conducting (pc) cavity at $T = 0$:

$$k_C^{(pc)} = \frac{\pi^3 \hbar c r_0^2}{60 x^5}. \quad (8)$$

Discussion

Our calculations (Fig. 1) reveal that k_C is much weaker—orders of magnitude lower—than the fundamental spring constant (k_S) of the membranes (572 N m^{-1} for Au and 949 N m^{-1} for Nb, as shown by the horizontal orange and grey lines in fig. 2 of ref. 1). This discrepancy eliminates the need for more computationally expensive exact methods like those described in ref. 5 to refine our calculations. Given that k_C is so much weaker than k_S, the Casimir force cannot be the primary explanation for the substantial increase in k_{eff} observed by Pate et al.[1] for gaps smaller than 2 µm. As the Casimir force cannot explain the observed attraction, alternative explanations must be explored. Assuming that the experiment is accurate, one possible explanation lies in electrostatic interactions between the Al post and the membrane. Previous research[6] suggests that variations in the surface potential across these surfaces can lead to an important electrostatic force. This electrostatic force might exhibit a similar distance dependence (scaling with the gap size) as the Casimir force, but with a much larger magnitude. This aligns with observations from Sushkov et al.[7], who proposed that electrostatic forces due to surface potential variations could explain forces ten times stronger than the Casimir force in a torsional balance experiment with Au-coated surfaces. To investigate the possibility of electrostatic effects, a Kelvin probe measurement[8] could be a valuable tool. By mapping the electrostatic potential across the surfaces used in Pate and colleagues' experiment[1], this technique could reveal the presence or absence of important potential variations.

Online content

Any methods, additional references, Nature Portfolio reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41567-025-03062-w.

References

1. Pate, J., Goryachev, M., Chiao, R., Sharping, J. & Tobar, M. Casimir spring and dilution in macroscopic cavity optomechanics. *Nat. Phys.* **16**, 1117–1122 (2020).
2. Parsegian, V. A. *Van der Waals Forces: A Handbook for Biologists, Chemists, Engineers, and Physicists* (Cambridge Univ. Press, 2006).
3. Bordag, M., Klimchitskaya, G. L., Mohideen, U. & Mostepanenko, V. M. *Advances in the Casimir Effect* (Oxford Univ. Press, 2009).
4. Lifshitz, E. M. The theory of molecular attractive forces between solids. *Sov. Phys. J. Exp. Theoret. Phys.* **2**, 73–83 (1956).
5. Emig, T. & Bimonte, G. Multiple scattering expansion for dielectric media: Casimir effect. *Phys. Rev. Lett.* **130**, 200401 (2023).
6. Speake, C. C. & Trenkel, C. Forces between conducting surfaces due to spatial variations of surface potential. *Phys. Rev. Lett.* **90**, 160403 (2003).

https://doi.org/10.1038/s41567-025-03062-w

7. Sushkov, A. O., Kim, W. J., Dalvit, D. A. R. & Lamoreaux, S. K. Observation of the thermal Casimir force. *Nat. Phys.* **7**, 230–233 (2011).
8. Behunin, R. O. et al. Kelvin probe force microscopy of metallic surfaces used in Casimir force measurements. *Phys. Rev. A* **90**, 062115 (2014).

https://doi.org/10.1038/s41567-025-03062-w

Data availability
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Acknowledgements
I acknowledge discussions with A. Cassinese.

Competing interests
The author declares no competing interests.

Additional information
Correspondence and requests for materials should be addressed to Giuseppe Bimonte.

Peer review information *Nature Physics* thanks Jeremy Munday and Salvatore Savasta for their contribution to the peer review of this work.