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Capturing State-of-Charge-Dependent Efficiency in Battery Energy Storage Systems: A Bernoulli Model Approach

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ABSTRACT

Accurate modelling of battery energy storage systems (BESSs) is critical for optimising their integration into power grids with high penetration of renewable energy. Conventional stochastic models often assume constant charging and discharging efficiencies, an oversimplification that neglects the significant dependence of battery performance on its state-of-charge (SOC). This paper introduces a novel analytical model for BESS based on a discrete-time discrete-state Markov chain that explicitly incorporates SOC-dependent efficiencies and charge acceptance limitations. The BESS is modelled as a finite-state buffer subject to stochastic energy inflows and outflows, where the transition probabilities are functions of the current SOC. We derive a closed-form analytical solution for the steady-state probability distribution of the battery's charge levels. Through an illustrative numerical study, the comparative analysis demonstrates that, when charge acceptance is limited at high SOC (e.g., for State Of Health preservation), our model predicts a significant shift in operational behaviour compared to ideal or constant-efficiency models. The system's probability mass tends to concentrate in an intermediate range, a behaviour that simpler models often overlook. This has significant implications for BESS sizing and control strategies, indicating that the selected operating strategy greatly affects the appropriate usable capacity. The proposed model provides a computationally efficient and more realistic framework for analysing these dependencies.

1 | Introduction

Increasing the share of renewable energy (RE) sources may be an efficient and effective solution to oil depletion while simultaneously meeting global electricity demand and limiting climate change [1]. In addition, various governments, along with supranational organisations such as the European Union, have enacted policies and enabling frameworks to increase the penetration of RE in power grids [2]. Even if the integration of RE may provide us with undeniable environmental and economic

advantages, it still poses significant challenges to grid stability. In fact, the dependence of RE production on weather conditions makes these non-dispatchable sources also intermittent and stochastic [3].

As the penetration of RE increases, existing conventional power systems are becoming more and more challenging to accommodate the potential increase in RE generation ([4]). In the case of photovoltaic (PV) generation, overproduction during midday leads to the so-called 'duck' curve [5]. One of the possible

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solutions to this problem is to consider multiple sources of RE and use hybrid renewable energy systems (HRESs). In these systems, the low generation of one resource can be naturally compensated by other resources with high generation [6].

To maintain the stability and quality of the power supply and to ensure the reliable operation and economic dispatch of the supply systems in the presence of a high penetration of RE, another widely investigated solution is to combine generation with energy storage systems (ESSs) [7]. An ESS transfers energy from a primary form (often electricity) to a secondary form that can be temporarily stored and converted back to electricity. When power demand is low, an ESS leverages physical or chemical processes to charge energy and release it when needed. As various storage applications have different quality requirements and performance constraints, several types of ESS technology are available, such as batteries, supercapacitors, compressed air storage, flywheel energy storage and pumped hydroelectric energy storage [8]. Often, these technologies can be combined in the so-called hybrid energy storage systems (HESSs) [9].

Among the various storage technologies, battery energy storage system (BESS) is an electrochemical ESS that is receiving great attention due to its fast response, controllability and geographical independence [10]. In addition to having a wide scope of applications, battery storage technologies appear to be mature and reliable, which is a good reason for BESS to become a strong candidate as the main storage solution in the energy sector [11]. The relevance of accurate BESS modelling is further amplified by the emergence of new, decentralised energy paradigms. The transition towards smart grids has enabled more active participation from end-users, giving rise to concepts such as energy communities (ECs) and citizen energy communities (CECs), which are now formally recognised under EU legislation [12]. In these contexts, groups of citizens collectively invest in, manage, and benefit from local RE assets, with shared storage playing a distinctive role. Furthermore, the ambitious goal of developing positive energy districts (PEDs), that is, urban areas that generate more RE than they consume on an annual basis, critically depends on the effective integration of storage to balance local supply and demand [13]. In these distributed, user-centric applications, the consequences of inaccurate BESS sizing are borne directly by the community members. Undersizing leads to poor reliability and unmet expectations, while oversizing results in stranded investment. Therefore, the need for robust, physically grounded analytical models that can accurately predict storage performance under realistic operational constraints is more pressing than ever. However, when using BESS to improve the performance of the RE system, a critical decision for a profitable investment is the optimisation of the size of BESS [14, 15].

The capacity sizing problem of BESSs has been investigated for several battery applications [11, 16] and in the presence of various RE sources, also in combination [17], considering different performance technical and/or financial indicators and adopting diversified sizing techniques. Battery modelling represents a prerequisite in capacity-sizing problems, with stochastic models, particularly Markov chain processes, apt to characterise the stochastic nature of battery-powered systems as a whole [14, 18].

The first Markov chain energy storage model was introduced in [19] to evaluate the performance of a stand-alone PV solar energy system. A one-step Markov process models the energy capture, storage and discharge process. The model assumes constant load and stationarity in the day-to-day process, leveraging, at a minimum, knowledge of the variance of the probability density of the daily solar insolation and an estimate of the mean of the difference between the daily array output and the load. This model only considers the fully charged and empty storage states, and the details of the energy level changes during the charging and discharging processes are not taken into account. Later, [20] proposed an analogous Markov chain-based energy storage model to develop a framework for power supply availability in PV generation. The model attempts to examine the battery charge and discharge process more closely by discarding the constant load hypothesis and allowing for a discrete number of possible state changes at each time step. [21] developed an analytical method to estimate the power availability of a wind power system, using a discrete-time Markov chain model to describe the residual energy of the battery storage system in the charging and discharging process. The model considers a constant load and discretises the state-of-charge (SOC) value into a finite set. Observing that the existing models struggled to incorporate the fluctuation of load and PV generation within a day, [22] proposed a two-dimensional Markov chain model to derive the distribution of the SOC of the battery over time during the day.

All of these models adopted the crucial assumption of considering fixed round-trip efficiencies, that is, the hypothesis that the battery charging and discharging processes are independent of their actual state (which translates into constant probabilities in the Markov chain model). Although it is a practical choice, this hypothesis has not been verified in real-life conditions of BESS [23–25]. In fact, when the BESS is used to change energy in time from intermittent renewables or to handle sudden power transients, its operational profile can be highly irregular, involving frequent partial charges or discharges in disparate SOC regions, thus affecting energy throughput and power capabilities [26, 27]. Neglecting the variable nature of charging and discharging efficiencies, as will be shown in this paper, leads to inaccurate evaluations in BESS capacity sizing, especially when the BESS is expected to operate in particular SOC regions as a consequence of its operational strategies.

To address this limitation, we examine the stochastic dynamics of the BESS charging and discharge process, as well as its SOC-dependent efficiencies. Specifically, we consider energy conversion (charging and discharging) efficiencies and charge acceptance limitations associated with battery standard charging strategies. We model the BESS state using a Markov chain model whose transition probabilities explicitly consider state-dependent efficiencies. Moreover, unlike the models mentioned above, we adopted the Bernoulli process hypothesis for the BESS model. This assumption allows us to capture the battery's non-linear dynamics while preserving the model's analytical tractability. This is a key advantage of the proposed model, as it avoids the computationally intensive simulations required by methods like Monte Carlo while offering deeper insights than simplified deterministic models.

The main contributions of our work are as follows:

- We propose a novel, analytically tractable Markov chain model for a BESS where transition probabilities are modulated by realistic, SOC-dependent functions for efficiency and charge acceptance.
- We derive a closed-form analytical solution for the steady-state SOC distribution, providing a computationally efficient tool for performance evaluation without resorting to simulation.
- We demonstrate that incorporating realistic charge acceptance constraints fundamentally alters the predicted operational behaviour of the BESS, causing the SOC to concentrate in specific intermediate ranges—a critical insight missed by conventional models.
- We provide a quantitative framework to analyse how different operating strategies and battery characteristics interact to define the effective usable capacity, offering a more robust basis for BESS sizing and control decisions.

The remainder of this paper is organised as follows. Section 2 reviews the operational characteristics of BESS, with a focus on how charging and discharging efficiencies are modelled as a function of SOC. Section 3 presents the proposed stochastic model and derives its analytical solution, along with key performance metrics. Section 4 presents a numerical case study, compares the results with simplified models and discusses the practical implications of the findings. Finally, Section 5 concludes the paper and outlines future research directions.

2 | The BESS: Characteristics and Efficiency Models

2.1 | BESSs

A BESS is a technology capable of absorbing, storing and delivering electric energy on demand, with the potential to mitigate the variability of RE generation and the stochastic nature of load profiles [28]. Modern power grids, characterised by significant integration of PV and wind resources, often face fluctuating supply and demand conditions. Due to its rapid response and ability to modulate power flows, BESS has become a strategic component not only in large-scale applications (e.g., peak shaving, frequency regulation) but also in smaller, localised contexts such as microgrids or behind-the-metre installations [9].

Although BESS has achieved commercial viability in many regions, further improvements are needed in terms of cost, usable life and overall system efficiency. In particular, repeated cycling at high charging rates or in suboptimal SOC ranges can induce significant performance degradation. SOC is defined as the ratio between the remaining available capacity of the battery and its nominal full capacity, expressed as a percentage [29]. The resulting loss of capacity and energy losses are direct barriers to achieving the economic objectives associated with the operation of BESS [9, 30]. Moreover, when a BESS is used to time-shift energy from intermittent renewables or to handle sudden power transients, its operational profile can be highly irregular, involving frequent partial charges or discharges in disparate SOC regions. These complex duty cycles exacerbate the stress on the battery electrodes and

the associated solid-electrolyte interphase, thus affecting energy throughput and power capabilities [26, 27].

In the literature, considerable attention has been devoted to two parallel research avenues aimed at improving BESS performance. One avenue focuses on *battery materials and cell design*, investigating novel active materials, advanced electrolytes, or electrode architectures to enhance high-rate charge acceptance and thermal stability [31, 32]. The other avenue emphasises *system-level modelling and control*, seeking to develop effective operating policies—such as deciding when to start or stop charging/discharging and at what current level—while preventing harmful phenomena like lithium plating or extreme temperature excursions [9, 23, 24].

However, a distinctive challenge arises when a system-level perspective attempts to account for the intricate electrochemical effects that vary with SOC. Traditional queueing or flow-based representations of energy storage often assume fixed round-trip efficiencies or constant probabilities for charging and discharging. Such simplifications, however, can limit the accuracy of predicting real-world performance. This is because key operational characteristics of lithium-ion cells, particularly their energy conversion efficiencies, are not constant but vary significantly with the SOC value, and are also influenced by temperature and operating current [24, 25, 33, 34]. This variability is especially pronounced under dynamic conditions like fast-charging or partial SOC cycling, common in smart-grid applications [27, 35, 36].

To address these gaps, this work proposes a stochastic modelling approach in which the battery's effective charging and discharge efficiencies explicitly modulate the transitions between discrete charge levels. By capturing the SOC-dependent nature of these efficiencies, the model more accurately reflects the battery's behaviour as a 'buffer' in the energy flow. In addition, the analytical tractability of the proposed approach makes it suitable for long-term performance evaluations, parametric studies and the design of control policies. The following subsection provides a thorough discussion of the role of SOC in defining lithium-ion battery efficiencies, referencing the most salient studies and experimental results in the field.

2.2 | SOC and Efficiency

A key factor governing lithium-ion cell performance, safety and degradation is the SOC. Beyond its role in battery management, SOC significantly influences operational characteristics, including the energy efficiency of the charging and discharging processes. These efficiencies are not constant but vary systematically with SOC, current rate (C-rate), temperature (T), and even the State-of-Health (SOH) of the battery [24, 25, 27, 37]. Quantifying these losses through appropriate efficiency metrics is fundamental for accurate system modelling, thermal management and the development of optimal operating strategies. The literature distinguishes several key definitions of efficiency relevant to battery performance.

First, the Coulombic efficiency (η_{Coul}) quantifies the effectiveness of charge transfer over a given cycle. It is defined as the ratio of the total charge extracted during discharge (Q_{dis}) to the total charge supplied during the preceding charge (Q_{ch}), that is, $\eta_{\text{Coul}} = Q_{\text{dis}}/Q_{\text{ch}}$. Losses reflected in η_{Coul} being less than unity

are primarily due to irreversible side reactions consuming charge carriers (lithium ions) or active material, such as the formation and continued growth of the solid electrolyte interphase (SEI) or lithium plating [34, 38, 39]. While η_{Coul} is often very high (typically > 99%) for healthy lithium-ion cells under nominal operating conditions [40], it is not constant and can be subtly influenced by factors including C-rate, temperature and SOC [38].

Second, the voltage efficiency (η_V) accounts for energy losses arising from the voltage difference between charge and discharge cycles. Defined as the ratio of the average voltage during discharge ($V_{\text{avg,dis}}$) to the average voltage during charge ($V_{\text{avg,ch}}$) over a comparable SOC range, $\eta_V = V_{\text{avg,dis}}/V_{\text{avg,ch}}$ is always less than unity. This is because the charging voltage must be higher than the open-circuit voltage (OCV) (making $V_{\text{ch}} > V_{\text{OCV}}$), and an under-potential occurs during discharging (making $V_{\text{dis}} < V_{\text{OCV}}$) to drive the current against the cell's internal impedance. This impedance encompasses ohmic resistance, charge transfer resistance at electrode interfaces and mass transport limitations, collectively causing voltage drops that dissipate energy, primarily as heat [29, 37]. Voltage hysteresis effects also contribute to $\eta_V < 1$ [37].

Consequently, the overall energy efficiency (η_E), representing the net useful energy recovered from the storage system, combines charge and voltage losses. Defined over a complete cycle as the ratio of total energy delivered during discharge (E_{dis}) to the total energy supplied during charge (E_{ch}), $\eta_E = E_{\text{dis}}/E_{\text{ch}}$, it can be directly related to the previous metrics by $\eta_E = \eta_{\text{Coul}} \cdot \eta_V$ [37]. This energy efficiency, often referred to as the round-trip efficiency (RTE), is the most critical metric for evaluating the overall energy performance and predicting the thermal behaviour of a BESS. It is highly sensitive to operating conditions, especially C-rate and temperature, as these significantly impact the internal impedance and thus the voltage efficiency component [28, 37].

While RTE provides a useful aggregate measure, modelling the distinct charge and discharge phases dynamically, as required by the stochastic framework developed in this paper, necessitates considering one-way energy efficiencies: η_c for charging and η_d for discharging. Consistent with the general behaviour of battery efficiencies outlined above, both η_c and η_d are confirmed to be non-constant, exhibiting a primary dependence on the battery's instantaneous SOC, alongside influences from C-rate, temperature and SOH [24, 25, 27, 29, 37]. Therefore, capturing the influence of SOC on η_c and η_d is essential for enhancing the realism of BESS models.

Focusing on the charging energy efficiency, η_c , recent experimental investigations reveal complex behaviour [24]. Studying NMC cells, observed a distinct 'M-shaped' profile for η_c as a function of SOC, exhibiting lower efficiencies at both low and high SOC extremes compared to the mid-SOC range. This non-monotonic characteristic was strongly correlated with a 'W-shaped' profile of the measured internal impedance versus SOC, suggesting that variations in voltage losses (related to η_V) dominate the observed η_c (SOC) trend [24]. Formulating simple, universally applicable analytical functions for η_c (SOC, C, T, SOH) remains challenging; high-fidelity simulations often employ detailed electrochemical models or empirically derived look-up tables (LUTs) [25, 37]. However, representative mathematical functions can be adopted for system-level analyses aiming for analytical tractability, as in this study. In this preliminary investigation, we therefore choose a polynomial function to represent

the non-monotonic SOC-dependency of η_c suggested by experimental findings:

$$\eta_c(x) = c_4x^4 + c_3x^3 + c_2x^2 + c_1x + c_0 \quad (1)$$

Here, $x = i/N$ represents the normalised SOC, where $i \in \{0, 1, \dots, N\}$ is the discrete charge level index and N is the total number of discrete energy states defining the BESS capacity (formally introduced in Section 3.1). This normalisation ensures $x \in [0, 1]$. The coefficients c_k are parameters that must be chosen to reflect the characteristic shape (e.g., 'M-shape' or 'U-shape') for the specific battery chemistry and operating conditions (average C-rate, T) considered while ensuring $0 < \eta_c(x) \leq 1$.

Similarly, the discharging energy efficiency, η_d , likewise shows a notable dependence on SOC, among other operating factors [25, 27, 37]. A more fundamental understanding of η_d can be derived by considering the energy losses due to internal battery characteristics. Assuming that Coulombic losses during discharge are minimal for healthy Li-ion cells (i.e., $\eta_{\text{Coul,dis}} \approx 1$), the discharging energy efficiency can be well approximated by the voltage efficiency [37]. The instantaneous terminal voltage (V_t) during discharge at a current I is determined by the open-circuit voltage (V_{OCV}) and the voltage drops across the internal impedances:

$$V_t(\text{SOC}, I, T) = V_{\text{OCV}}(\text{SOC}, T) - I \cdot R_{\text{int}}(\text{SOC}, I, T) - V_{\text{hyst}}(\text{SOC}, d, h) \quad (2)$$

where $R_{\text{int}}(\text{SOC}, I, T)$ represents the effective internal resistance (combining ohmic, charge-transfer and diffusion-related contributions), and $V_{\text{hyst}}(\text{SOC}, d, h)$ accounts for the voltage hysteresis, which is dependent on the SOC, the direction of current flow (discharge in this case), and the recent operational history of the cell [25, 37].

From this, the instantaneous discharging energy efficiency η_d can be expressed as:

$$\eta_d(\text{SOC}, I, T) \approx \frac{V_t(\text{SOC}, I, T)}{V_{\text{OCV}}(\text{SOC}, T)} = 1 - \frac{I \cdot R_{\text{int}}(\text{SOC}, I, T) + V_{\text{hyst}}(\text{SOC}, d, h)}{V_{\text{OCV}}(\text{SOC}, T)} \quad (3)$$

The terms $V_{\text{OCV}}(\text{SOC}, T)$, $R_{\text{int}}(\text{SOC}, I, T)$ and $V_{\text{hyst}}(\text{SOC})$ are complex, non-linear functions specific to the cell chemistry and design, typically obtained from experimental characterisation and often represented by LUTs or fitted empirical functions [25, 37].

A key observation from experimental studies is the non-monotonic behaviour of R_{int} with SOC. It often exhibits a 'U-shape', with higher resistance values at very low and very high SOC and lower values in the mid-SOC range [24, 37]. This characteristic of $R_{\text{int}}(\text{SOC})$, combined with the profile of $V_{\text{OCV}}(\text{SOC})$, typically results in an $\eta_d(\text{SOC})$ profile that is also non-monotonic. For a constant discharge current I_{ref} and temperature T_{ref} , $\eta_d(\text{SOC})$ often displays an inverted 'U-shape' (or a 'bell shape'), where efficiency is maximised at intermediate SOC levels and decreases towards the SOC extremes [29]. This is because the term $I_{\text{ref}} \cdot R_{\text{int}}(\text{SOC})$ becomes more significant relative to $V_{\text{OCV}}(\text{SOC})$ at SOC extremes, where $R_{\text{int}}(\text{SOC})$ is higher. The impact of $V_{\text{hyst}}(\text{SOC})$ further modulates this profile. The resulting characteristic non-monotonic relationship between η_d

and SOC, often observed in experimental studies, is depicted in Figure 1, illustrating a peak efficiency at intermediate SOC levels.

To capture the characteristic non-monotonic behaviour of $\eta_d(x)$ illustrated in Figure 1 with an analytically tractable function, we adopt the following quadratic form as follows:

$$\eta_d(x) = A_d x^2 + B_d x + C_d \quad (4)$$

where $x = i/N$ represents the normalised SOC. The coefficients $A_d < 0$, B_d and C_d are selected to ensure that $\eta_d(x)$ shows a peak efficiency at an intermediate SOC and has realistic values across the entire operational range (typically $0 < \eta_d(x) \leq \eta_{d,peak} \leq 1$). For instance, based on fitting to representative target points (e.g., $\eta_d(0.1) \approx 0.88$, $\eta_d(0.55) \approx 0.96$, $\eta_d(0.9) \approx 0.87$), illustrative coefficients can be determined (e.g., $A_d \approx -0.544$, $B_d \approx 0.531$ and $C_d \approx 0.832$). The precise parameters for Equation (4) should ideally be derived from experimental characterisation of the specific battery technology under the relevant average operating conditions (current and temperature) for the study. This quadratic representation is employed in our subsequent stochastic model for the discharge efficiency.

Beyond energy conversion efficiency, the charge acceptance rate of the battery is also a critical state-dependent factor. Standard charging protocols, notably constant current–constant voltage (CC–CV) [33], impose inherent limitations on the charging rate, especially at high SOC. The initial CC phase applies a constant current (I_{CC}) until a predefined voltage limit (V_{limit}) is reached, typically occurring around $SOC_{sw} \approx 80\% - 85\%$. Subsequently, in the CV phase, the voltage is held constant, necessitating a gradual decrease in the charging current (I_{CV}) as the battery's OCV increases and approaches V_{limit} [27, 41].

More sophisticated charging strategies may dynamically adjust the current based on estimated internal states to prevent detrimental

conditions, such as lithium plating or excessive temperature rise [42]. However, the characteristic current decay during the CV phase of a standard protocol is a well-established phenomenon linked to the cell's electrochemical response at high states of charge [43]. For our stochastic model, which assumes operational dynamics analogous to standard charging behaviour, we introduce the parameter $\eta_A^{(i)}$ to represent this charge acceptance limitation as a modulation factor. We employ a functional form capturing the essential CC–CV characteristic: full acceptance in the CC phase, followed by diminishing acceptance in the CV phase. The current decay in the CV phase is often approximated by an exponential function [41]. Accordingly, we propose:

$$\eta_A(x) = \begin{cases} 1 & \text{if } x < SOC_{sw} \\ \exp\left(-k \frac{x - SOC_{sw}}{1 - SOC_{sw}}\right) & \text{if } x \geq SOC_{sw} \end{cases} \quad (5)$$

where $x = i/N$, SOC_{sw} is the threshold (e.g., 0.8), and the parameter $k > 0$ determines the rate of decay.

The proposed functional forms presented in Equations (1), (4) and (5) are illustrated graphically in Figure 2. This figure illustrates the characteristic shapes generated by these equations using representative parameters, providing a visual summary of the SOC-dependent behaviours incorporated into our model.

These functions provide specific mathematical descriptions, informed by the literature [24, 33, 37, 40–42], for the state-dependent parameters $\eta_c^{(i)}$, $\eta_d^{(i)}$ and $\eta_A^{(i)}$. It is acknowledged that these represent simplifications, particularly regarding the coupled dependencies on C-rate, T and SOH, and the complex nature of charge acceptance limits. However, they enable an analytically tractable investigation into the impact of state-dependent performance characteristics on BESS behaviour, suitable for this preliminary study. For quantitative application, the parameters within these functions (c_k , $\eta_{d,max}$, $\eta_{d,min}$, SOC_{sw} , k) would require careful determination through experimental characterisation of the specific battery

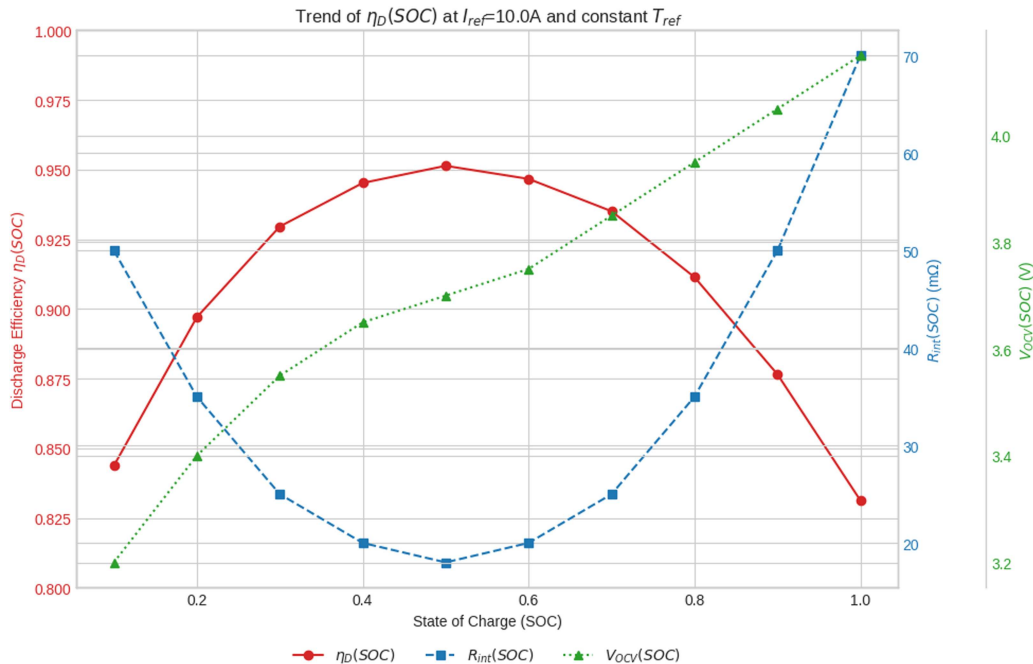


FIGURE 1 | The typical non-monotonic behaviour of the discharging energy efficiency η_d as a function of SOC. The efficiency profile is influenced by the interplay of internal resistance and voltage hysteresis effects, exhibiting a peak efficiency at intermediate SOC levels and decreasing towards the SOC extremes.

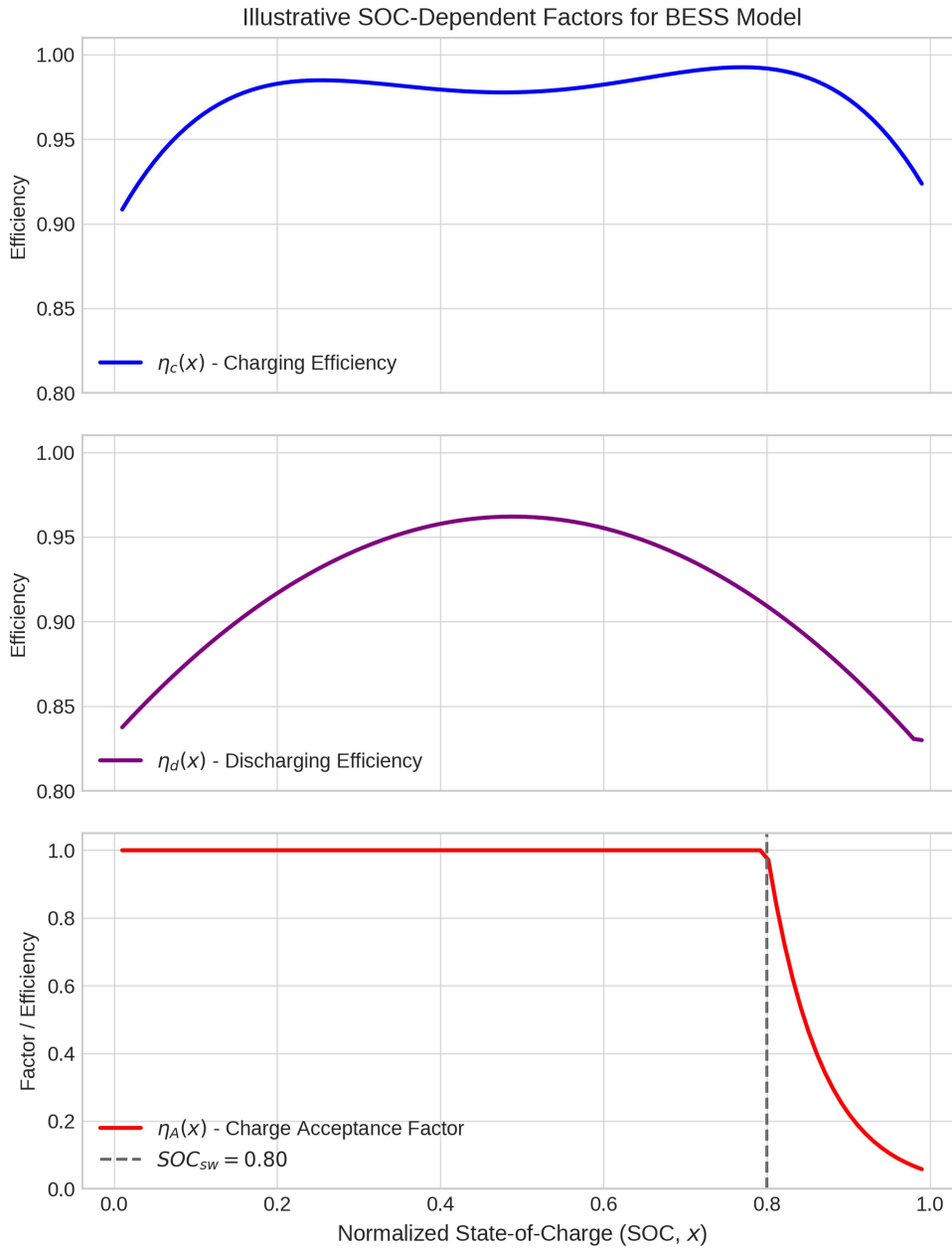


FIGURE 2 | Illustrative representation of the proposed state-of-charge (SOC)-dependent functions, plotted against the normalised SOC (x): (Top) Charging energy efficiency $\eta_c(x)$ (Equation 1), modelled as a fourth-order polynomial derived from experimental data for an NMC cell (qualitatively based on [24]). (Middle) Discharging energy efficiency $\eta_d(x)$ (Equation 4), represented by a quadratic function with coefficients $A_d = -0.54365$, $B_d = 0.53115$ and $C_d = 0.8323215$. (Bottom) Charge acceptance factor $\eta_A(x)$ (Equation 5), characterised by a switching threshold $SOC_{sw} = 0.8$ and decay constant $k = 3.0$.

technology under study. Building on these detailed SOC-dependent efficiency and charge acceptance models, the following section develops a comprehensive stochastic framework to analyse the BESS operational behaviour and performance under uncertain energy flows.

3 | Stochastic Modelling of the BESS

This section presents the comprehensive stochastic modelling approach developed to evaluate the performance of a BESS subject to state-dependent operational characteristics. We begin by establishing a conceptual framework that describes the BESS as an energy buffer within a system of stochastic energy generation and consumption (Section 3.1). Based on this conceptualisation, we

then formulate a discrete-time Markov chain model to capture the probabilistic dynamics of the battery's state of charge (Section 3.2). Finally, leveraging the analytical solution of this model, we define a set of key performance metrics to quantify various aspects of the BESS operation (Section 3.3).

3.1 | Conceptual Flow Model and Problem Statement

The foundation of the proposed stochastic model is a conceptual representation of the BESS as an energy buffer interacting with stochastic generation and load. This conceptualisation, illustrated in Figure 3, outlines the primary energy flows and the storage dynamics. Since the proposed stochastic model operates with a

discrete-time discrete-state approach, the ESS capacity is divided into a predefined number, N , of energy quanta, e_s . In this model, the basic discrete energy quantum (e_s) represents the smallest indivisible energy unit considered in the system dynamics, typically chosen in Ampere-hour (Ah) or kilowatt-hour (kWh), depending on the battery scale and the intended application [9, 26]. This discretisation facilitates an analytical stochastic representation of charging and discharging processes.

This defines our discrete energy storage level (DESL) concept, ranging from 0, that is, the empty (or minimum charge level allowed) state, to N , that is, the maximum charge, with $\text{DESL} = 0, 1, 2, \dots, N$.

The determination of the time unit's length and the derivation of the other parameters will be discussed later. We will focus now on explaining the energy flows, referring to what can happen in a generic time unit.

In Figure 3, the energy flow is depicted from left to right. The generator G represents the stochastic process able to produce an amount of energy e_s in a generic time unit; it is assumed to operate as a memoryless process with parameter p_1 , with the latter being the constant generation probability. On the opposite side, the block U represents the consumption process, which operates in a memoryless manner, and is characterised by a parameter p_2 representing the constant probability of consuming an amount of energy e_s in a generic time unit.

In the middle, the storage unit S compensates for the differences between the generation and the consumption flows. Specifically, when the generation and the consumption occur contemporarily in the same time unit, with probability $p_1 p_2$, the energy flow skips the storage system. As a consequence, when the storage system is neither full nor empty, the probability that, in a generic time unit, an amount of energy e_s is sent to the storage is $p_1(1 - p_2)$, while, in the opposite situation, the probability that an amount of energy e_s is withdrawn from the storage is $p_2(1 - p_1)$.

Additionally, the behaviour of the ESS's charge controller must also be considered. From a technical point of view, it determines the current that can be sent to the storage based on some state parameters of the storage itself. For example, in the case of lithium batteries, the performance mainly depends on the temperature. In general, as also shown in Section 2.2, it is

possible to obtain a suitable curve expressing the fraction of the current that can be sent into the battery, with respect to the maximum one, as a function of the SOC (the DESL in our case). This is the meaning of the parameter η_A in Figure 3, which actually represents a reduction of the probability to store the amount of energy e_s in a time unit due to the behaviour of the charge controller. As a direct consequence, the model is assigned a probability $(1 - \eta_A)p_1(1 - p_2)$ of sending the energy amount e_s outside the system (i.e., to the grid).

Finally, when the storage system is full or empty, that is, in its limits, a probability $p_1(1 - p_2)$ of sending the amount of energy e_s to the grid and a probability $p_2(1 - p_1)$ of taking the same amount of energy from the grid should be considered, respectively.

In Figure 4, the effect of charging and discharging losses is added to the model. Following the definition of η_A , η_c and η_d represent the charging and discharging yields, respectively, as a function of the energy storage level DESL. Hence, the probability of sending an energy amount e_s in the storage system in a generic time unit is reduced η_c , while on the other hand, the probability of withdrawing the same amount e_s has to be increased by the factor $1/\eta_d$ to compensate discharge losses. The amount of losses during a time unit can also be easily represented in probabilistic terms, as reported in Figure 4.

3.1.1 | Practical Parameter Estimation

To facilitate practical application, we now outline a procedure for estimating the model parameters from real-world data, such as historical time series of power generation and consumption. The first step is to select an appropriate time unit, Δt , which should be short enough to capture the system's dynamics but long enough to ensure that state changes are typically limited to one energy quantum per step. A common choice may be 1 h. Next, the energy quantum e_s should be chosen based on the desired model resolution and the BESS capacity; for instance, $e_s = C_{\text{nom}}/N$, where C_{nom} is the nominal capacity and N is the desired number of discrete states. With these choices fixed, the Bernoulli parameters can be estimated directly from the data. Let $P_{\text{gen}}(t)$ and $P_{\text{load}}(t)$ be the instantaneous power generation and load at time t .

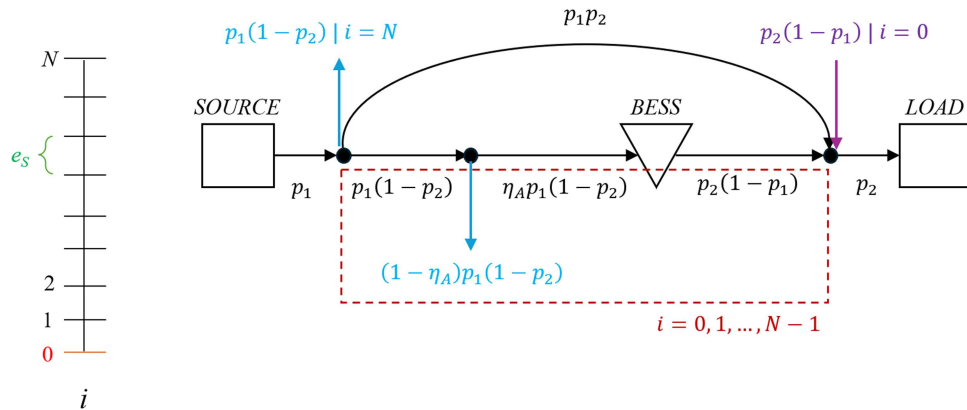


FIGURE 3 | Conceptual macro flow scheme of the BESS as an energy buffer. The model considers a generator (G) and a user (U) with stochastic operation probabilities p_1 and p_2 . The storage unit (S) absorbs or releases energy quanta (e_s) based on the net flow. The parameter η_A represents the SOC-dependent charge acceptance factor, which can divert incoming energy from the storage when it is near full capacity. The probabilities for boundary conditions ($i = 0$ and $i = N$) are also shown.

a stored quantum to meet a given demand. It is this state-dependent modulation of the transition probabilities that ultimately shapes the steady-state SOC distribution, causing it to differ qualitatively from models that assume constant efficiencies. In the case where generation and consumption occur simultaneously (with probability $p_1 p_2$), the BESS is bypassed, and its charge level remains unchanged. All transition probabilities are evaluated based on the state at the beginning of a time unit, with the state change occurring at the end.

These state-dependent transition rules define a birth-death process, whose dynamics are visualised in the state transition diagram shown in Figure 5.

To find the steady-state probability $\mathbf{P}_i$ of finding the system in any state i , we solve the set of balance equations that equate the probability flow into and out of each state:

$$\beta^{(1)} \mathbf{P}_1 = \alpha^{(0)} \mathbf{P}_0, \quad (10)$$

$$\alpha^{(i-1)} \mathbf{P}_{i-1} + \beta^{(i+1)} \mathbf{P}_{i+1} = (\alpha^{(i)} + \beta^{(i)}) \mathbf{P}_i, \quad (11)$$

for $i = 1, \dots, N-1$

$$\alpha^{(N-1)} \mathbf{P}_{N-1} = \beta^{(N)} \mathbf{P}_N. \quad (12)$$

These equations can be solved consecutively to express all probabilities in terms of the unknown $\mathbf{P}_0$. From the boundary condition at state 0, Equation (10), we directly find $\mathbf{P}_1$:

$$\mathbf{P}_1 = \frac{\alpha^{(0)}}{\beta^{(1)}} \mathbf{P}_0. \quad (13)$$

We can then proceed recursively. By rearranging the general balance equation, Equation (11), we can solve for $\mathbf{P}_{i+1}$. Applying this for $i = 1$, we use our result for $\mathbf{P}_1$ to find $\mathbf{P}_2$:

$$\begin{aligned} \mathbf{P}_2 &= \frac{1}{\beta^{(2)}} ((\alpha^{(1)} + \beta^{(1)}) \mathbf{P}_1 - \alpha^{(0)} \mathbf{P}_0) \\ &= \frac{1}{\beta^{(2)}} \left((\alpha^{(1)} + \beta^{(1)}) \frac{\alpha^{(0)}}{\beta^{(1)}} \mathbf{P}_0 - \alpha^{(0)} \mathbf{P}_0 \right) = \frac{\alpha^{(1)} \alpha^{(0)}}{\beta^{(2)} \beta^{(1)}} \mathbf{P}_0. \end{aligned} \quad (14)$$

Continuing this process for $i = 2$, we find $\mathbf{P}_3$:

$$\mathbf{P}_3 = \frac{1}{\beta^{(3)}} ((\alpha^{(2)} + \beta^{(2)}) \mathbf{P}_2 - \alpha^{(1)} \mathbf{P}_1) = \frac{\alpha^{(2)} \alpha^{(1)} \alpha^{(0)}}{\beta^{(3)} \beta^{(2)} \beta^{(1)}} \mathbf{P}_0. \quad (15)$$

A clear pattern emerges from these steps. We can generalise this result to find the expression for any state probability $\mathbf{P}_i$ in terms of $\mathbf{P}_0$:

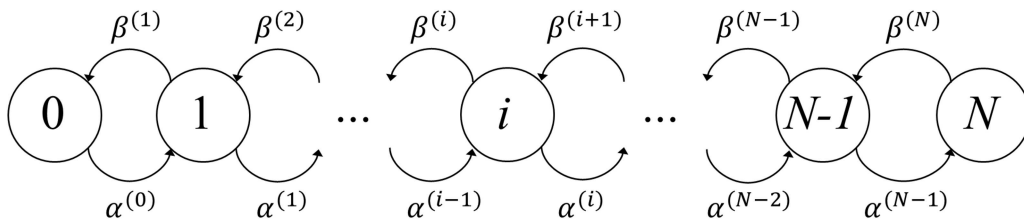


FIGURE 5 | State transition diagram for the proposed discrete-time Markov chain model. Each state i represents the BESS's discrete charge level (from 0 to N). The upward transition probability $\alpha^{(i)}$ and downward transition probability $\beta^{(i)}$ are explicit functions of the current state i , incorporating SOC-dependent efficiencies.

$$\mathbf{P}_i = \frac{\prod_{j=0}^{i-1} \alpha^{(j)}}{\prod_{j=1}^i \beta^{(j)}} \mathbf{P}_0. \quad (16)$$

For notational convenience, let us define an intermediate term $\delta^{(i)}$ that represents this recursive product:

$$\delta^{(i)} = \begin{cases} 1, & \text{for } i = 0, \\ \frac{\prod_{j=0}^{i-1} \alpha^{(j)}}{\prod_{j=1}^i \beta^{(j)}}, & \text{for } i = 1, \dots, N. \end{cases} \quad (17)$$

The probability of any state can now be written compactly as $\mathbf{P}_i = \delta^{(i)} \cdot \mathbf{P}_0$. To find the final unknown $\mathbf{P}_0$, we apply the normalisation condition that the sum of all state probabilities must equal unity: $\sum_{i=0}^N \mathbf{P}_i = 1$. This leads to the definition of the normalisation constant $\Delta = \sum_{k=0}^N \delta^{(k)}$, which allows us to write the final closed-form solution for the steady-state probability distribution:

$$\mathbf{P}_i = \frac{\delta^{(i)}}{\Delta}, \quad \text{for } i = 0, \dots, N. \quad (18)$$

This solution provides a complete, analytical description of the BESS's long-term behaviour. Its closed-form nature makes it a powerful and computationally efficient tool for system analysis, forming the basis for the performance metrics discussed in the next section.

3.3 | Performance Metrics

The analytical solution for the steady-state probability distribution of BESS charge levels, as provided by Equation (18), serves as a robust foundation for quantifying various critical aspects of the system's operational performance. These performance metrics offer valuable insights into the BESS's efficiency, its interaction with the grid, its typical operating states and the dynamics of its charge and discharge cycles.

A key set of metrics relates to the energy flows within and around the BESS. The average amount of energy sent to the grid per time unit, E_{out} , is defined as:

$$\begin{aligned} E_{\text{out}} &= e_s \left(\sum_{i=0}^{N-1} (1 - \eta_A^{(i)}) p_1 (1 - p_2) \mathbf{P}_i \right. \\ &\quad \left. + p_1 (1 - p_2) \mathbf{P}_N \right) = \\ &= e_s \frac{p_1 (1 - p_2)}{\Delta} \left(\sum_{i=0}^{N-1} \delta^{(i)} (1 - \eta_A^{(i)}) + \delta^{(N)} \right) \end{aligned} \quad (19)$$

This metric quantifies the energy that could not be stored, either due to charge acceptance limitations ($\eta_A^{(i)}$) when the BESS is not full, or because the BESS has reached its maximum capacity (with probability $\mathbf{P}_N$). Conversely, the average amount of energy drawn from the grid per time unit, E_{in} , indicates the system's reliance on external supply when the BESS is depleted:

$$E_{in} = e_S p_2 (1 - p_1) \mathbf{P}_0 = e_S \frac{p_2 (1 - p_1)}{\Delta} \quad (20)$$

Internal energy losses are captured by the average amount of energy dissipated during charging and discharging processes per time unit, E_{dis} :

$$\begin{aligned} E_{dis} &= e_S \left(\sum_{i=0}^{N-1} (1 - \eta_c^{(i)}) \eta_A^{(i)} p_1 (1 - p_2) \mathbf{P}_i \right. \\ &\quad \left. + \sum_{i=1}^N \frac{1 - \eta_d^{(i)}}{\eta_d^{(i)}} (1 - p_1) p_2 \mathbf{P}_i \right) = \\ &= e_S \left(\frac{p_1 (1 - p_2)}{\Delta} \sum_{i=0}^{N-1} (1 - \eta_c^{(i)}) \eta_A^{(i)} \delta^{(i)} \right. \\ &\quad \left. + \frac{(1 - p_1) p_2}{\Delta} \sum_{i=1}^N \frac{1 - \eta_d^{(i)}}{\eta_d^{(i)}} \delta^{(i)} \right) \end{aligned} \quad (21)$$

This metric is crucial for evaluating the overall energy efficiency of the BESS, accounting for SOC-dependent charging ($\eta_c^{(i)}$) and discharging ($\eta_d^{(i)}$) inefficiencies.

Understanding the distribution of the BESS's state of charge is also vital. The average SOC ($\overline{\text{SOC}}$) quantifies the expected normalised charge level of the battery over time and is computed as:

$$\overline{\text{SOC}} = \frac{1}{N} \sum_{i=0}^N i \mathbf{P}_i = \frac{1}{N \Delta} \sum_{i=0}^N i \delta^{(i)} \quad (22)$$

More specifically, the probability that the SOC lies within a certain range $[x_1, x_2]$ can be calculated as:

$$P(x_1, x_2) = \sum_{i=[x_1 \cdot N]}^{[x_2 \cdot N]} \mathbf{P}_i = \frac{1}{\Delta} \sum_{i=[x_1 \cdot N]}^{[x_2 \cdot N]} \delta^{(i)} \quad (23)$$

This allows for assessment of the time spent in desirable or critical SOC zones.

The probability of the BESS operating at its charge extremes is captured by the probability that the BESS is full and the probability that the BESS is empty, given respectively by:

$$P_{full} = \mathbf{P}_N = \frac{\delta^{(N)}}{\Delta}, \quad (24)$$

$$P_{empty} = \mathbf{P}_0 = \frac{1}{\Delta}. \quad (25)$$

Furthermore, metrics related to the frequency of reaching extreme states provide insights into the cycling behaviour of the BESS. The probability of entering the fully charged state in a time unit is:

$$\begin{aligned} P_{enter N} &= \eta_A^{(N-1)} \eta_c^{(N-1)} p_1 (1 - p_2) \mathbf{P}_{N-1} = \\ &= \eta_A^{(N-1)} \eta_c^{(N-1)} \delta^{(N-1)} \frac{p_1 (1 - p_2)}{\Delta} \end{aligned} \quad (26)$$

From this, the mean time between complete charges (MTBCCs), representing the average number of time units between two consecutive full charges of the battery, can be derived:

$$\text{MTBCC} = \frac{1}{P_{enter N}} \quad [\text{t.u.}] \quad (27)$$

Similarly, the probability of entering the empty state is:

$$P_{enter 0} = \frac{1}{\eta_d^{(1)}} (1 - p_1) p_2 \mathbf{P}_1 = \frac{\delta^{(1)} (1 - p_1) p_2}{\eta_d^{(1)} \Delta} \quad (28)$$

This leads to the mean time between complete discharges (MTBDs), which is the average number of time units between two complete discharges of the battery:

$$\text{MTBD} = \frac{1}{P_{enter 0}} \quad [\text{t.u.}] \quad (29)$$

Collectively, these performance metrics, derived from the steady-state probabilities, allow for a comprehensive evaluation of the BESS's behaviour, efficiency and reliability under the influence of SOC-dependent characteristics and stochastic energy flows.

4 | Results and Discussion

The analytical framework developed in Section 3 moves the discussion from a theoretical basis to a quantitative exploration of the BESS's operational characteristics. It is important to note that the numerical case study presented herein is primarily illustrative. Its purpose is to demonstrate the qualitative behaviour and diagnostic capabilities of the proposed stochastic method, rather than to provide a validated analysis for a specific battery chemistry. The SOC-dependent efficiency functions used are representative simplifications informed by the literature. For a rigorous, application-specific study, these functions would require careful parameterisation through experimental characterisation. We contend that acknowledging this limitation highlights the core novelty of this work: the provision of an analytically tractable framework that incorporates SOC-dependence, which is particularly valuable for early-stage design exploration and sensitivity analysis, even before precise chemical parameters are available. While assuming constant efficiencies simplifies modelling, it risks overlooking the nuanced behaviours dictated by the battery's state-dependent physics. This section, therefore, presents a detailed investigation of these dynamics, first through a direct numerical analysis and then by placing the findings in the broader context of system design methodologies.

The initial part of the analysis is a comparative case study, which contrasts the performance predictions of the proposed SOC-dependent model with those of ideal and constant-mean efficiency benchmarks. This comparison serves to quantify the degree to which realistic operational constraints, particularly charging limitations at high SOC, alter the system's steady-state behaviour and key performance metrics. Building on these results, the discussion explores the broader implications for the system's reliability. A critical examination of conventional

availability models reveals how ignoring the temporal and physical realities of battery charging can lead to optimistic conclusions, highlighting the need for more physically grounded approaches to robust BESS sizing.

4.1 | Numerical Case Study and Analysis

To illustrate the practical implications and diagnostic capabilities of the proposed modelling framework, we conduct a comprehensive numerical analysis. The study compares the performance predictions of the proposed SOC-dependent model against two crucial benchmarks under three distinct operational scenarios.

Before presenting the results, we specify the key model parameters used throughout this case study. The BESS capacity is discretised into $N = 100$ energy states. Each discrete state corresponds to a fundamental energy quantum of $e_s = 1$ kWh. The time unit is interpreted as 1 h. Under this parameterisation, the nominal BESS capacity is $N \cdot e_s = 1$ kWh. The generation and consumption probabilities (p_1 and p_2) represent, respectively, the likelihood that a net energy surplus or deficit of 1 kWh occurs within a given hour. This choice of scale is representative of a medium-sized residential or small commercial installation paired with a PV system. However, the model is inherently scalable: for a larger system, one could choose $e_s = 10$ kWh and $N = 100$ to represent a 1 MWh BESS, with the time unit remaining 1 h but now representing larger power flows.

The SOC-dependent efficiency functions are parameterised as follows. For the charging efficiency $\eta_c(x)$, we adopt the fourth-order polynomial of Equation (1) with coefficients derived from a qualitative fit to experimental data for NMC cells [24]: $c_4 = -0.02$, $c_3 = 0.16$, $c_2 = -0.42$, $c_1 = 0.30$ and $c_0 = 0.91$. These values produce an efficiency profile exhibiting lower values at the SOC extremes (approximately 0.91 at $x = 0$ and 0.93 at $x = 1$) and a peak of approximately 0.97 in the mid-SOC range, consistent with experimental observations. The constraint $0 < \eta_c(x) \leq 1$ is verified over the entire range $x \in [0, 1]$. For the discharging efficiency $\eta_d(x)$, the quadratic form of Equation (4) is used with coefficients $A_d = -0.544$, $B_d = 0.531$ and $C_d = 0.832$, as stated in Section 2.2. For the charge acceptance factor $\eta_A(x)$, the exponential form of Equation (5) is adopted with $\text{SOC}_{\text{sw}} = 0.8$ and $k = 3.0$.

This comparative structure allows us to isolate and quantify the impact of state-dependent efficiencies. The models under comparison are:

- *Ideal model*: A theoretical baseline assuming lossless and unconstrained operation. In this model, all efficiencies are set to unity ($\eta_c(x) = \eta_d(x) = \eta_A(x) = 1$ for all x), representing a perfect energy buffer.
- *Constant mean model*: A representation of the simplified approach common in the literature, where efficiencies are assumed to be constant. For a fair comparison, we use constant efficiency values derived by averaging the SOC-dependent functions over the entire SOC range (i.e., $\bar{\eta}_c = \int_0^1 \eta_c(x) dx$, and similarly for η_d). This is a reasonable baseline as it corresponds to the average RTE one might measure experimentally over many full cycles, a common

parameterisation approach in the literature. This model captures overall system losses but neglects their dependency on the battery's state. It is worth noting that alternative constant values could be chosen depending on the expected operating window (e.g., averaging only over a 20%–80% SOC range), but the mean over the full range provides the most unbiased general-purpose comparison.

- *SOC-dependent model (proposed)*: The full model developed in this paper, which utilises the state-dependent functions for charging efficiency ($\eta_c(x)$), discharging efficiency ($\eta_d(x)$) and charge acceptance ($\eta_A(x)$) as detailed in Section 2.2.

The analysis is based on a representative BESS with a capacity discretised into $N = 100$ energy states. The three operational cases are defined by the following probabilities of generation (p_1) and consumption (p_2):

- *Case A: Generation-dominant condition* ($p_1 = 0.6$, $p_2 = 0.5$). It simulates a system with a significant net-positive energy balance, typical of a well-sized renewable plant during peak generation.
- *Case B: Balanced condition* ($p_1 = 0.51$, $p_2 = 0.5$). It represents a system where generation and consumption are closely matched, creating minimal net energy flow.
- *Case C: Load-dominant condition* ($p_1 = 0.4$, $p_2 = 0.5$). It models an undersized or off-peak system where energy demand consistently exceeds generation.

The results for all cases and models are consolidated in Table 1.

In the first operational case, the strong charging pressure should intuitively result in a fully charged battery. The top panel of Figure 6 shows the steady-state probability distributions for this case. As expected, the ideal and constant mean models predict that the BESS will spend most of its time at or near maximum capacity. The probability distribution increases monotonically, with the highest probability at $i = 100$ (see Figure 6). The BESS is predicted to be full for 33% (ideal) and 25% (constant mean) of the time, respectively.

The proposed SOC-dependent model presents a significantly different scenario. It shows a 16% reduction in the average SOC if compared to the constant mean case, but more importantly, the very nature of the distribution changes. The probability of finding the battery full is virtually zero ($P_{\text{full}} \approx 0$), and the MTBCC is infinite. Instead of accumulating at the upper boundary, the probability mass concentrates in a sharp peak around 82% SOC.

This shift in the predicted operational behaviour is a direct consequence of the charge acceptance factor, $\eta_A(x)$, which imposes a kinetic bottleneck at high states of charge. As the BESS approaches the 80% SOC threshold, the model shows that its declining ability to accept charge effectively prevents the system from reaching full capacity under strong generation conditions. This phenomenon, which is not captured by models assuming constant or ideal efficiencies, has a direct impact on system sizing. It demonstrates that the *effective usable capacity* of the BESS is not its nominal rating but a narrower, dynamically defined operational window. Consequently, a design based on conventional models would lead to a significant overestimation of the usable energy reserve for a given nominal capacity.

The diagnostic power of the proposed model becomes even more evident when examining other operational cases. In a balanced system ($p_1 = 0.51, p_2 = 0.5$), where charging pressure is minimal, discharging inefficiencies become dominant. The middle panel of Figure 6 shows a dramatic shift: the mean SOC predicted by our model falls to 13.5%, and the system is expected to be empty for more than 13% of the time, leading to frequent grid imports (E_{in}) and deep discharge cycles (MTBD = 9.4 t.u.). For a system load-dominant ($p_1 = 0.4, p_2 = 0.5$), our model acts as a robust diagnostic tool, predicting a near-50% probability of the BESS being empty and quantifying its high reliance on the grid (see Table 1 and bottom panel of Figure 6).

The significant impact of these state-dependent parameters underscores the limitations of constant-efficiency models. Although computationally convenient, such approaches fail to capture the natural tendency of real batteries to operate within physical constraints, a behaviour that emerges organically from the proposed formulation. This has direct consequences for system design and longevity. For example, a designer sizing a BESS based on traditional models would expect regular operation near full capacity under favourable conditions. Our model reveals that the effective operating window is physically limited. Acknowledging this reality would lead the designer to oversize the nominal capacity to guarantee the required usable energy reserve, a stark contrast to conventional approaches.

This insight is also crucial in deploying effective control strategies. The attempt to maximise stored energy by pushing the BESS to 100% SOC confronts a harsh physical reality: beyond 80%–85% SOC, the battery's resistance to charging increases. The marginal capacity gain is often not worth the extended charging time and increased thermal stress. Our model quantifies this trade-off, providing a tool to design controllers that operate with the battery's physics, not against them.

Furthermore, these operational shifts have significant economic ramifications. Energy arbitrage models often assume predictable,

deep cycling [42]. Our analysis suggests a reality of more frequent, shallower partial cycles under strong charging, or rapid deep cycles under stress. This alters revenue projections and impacts degradation models, which are highly sensitive to cycle depth and frequency. Financial projections calibrated on simplified models could significantly misrepresent project profitability.

Finally, the proposed framework provides a direct method for assessing the system-level impact of specific battery characteristics. For instance, an improvement in cell chemistry that alters the charge acceptance threshold can be modelled by updating the parameters of the $\eta_A(x)$ function. The model can then compute the resulting changes in key performance metrics, such as the average SOC and the effective usable capacity. This offers a tool for quantifying the value of materials-level innovations in an operational context. The analytical tractability of the framework is a key advantage, as it enables rapid performance of these sensitivity analyses, facilitating design exploration without the need for computationally intensive simulations.

4.2 | Comparison With Conventional Availability Models

A central objective in designing BESS is to accurately estimate energy availability, particularly when paired with intermittent renewable sources. A common approach in the literature involves modelling the net energy flow, $\Delta E = E_{gen} - E_{load}$, over discrete time intervals and using its aggregated statistical distribution to size the BESS [20, 44]. Power supply availability is then assessed by calculating the probability that the BESS cannot cover a negative energy balance.

However, this methodology often relies on the critical assumption that successive ΔE values can be treated as independent samples from a single, time-invariant distribution and that if there is a sufficient net energy surplus during the

TABLE 1 | Performance metrics comparison across models and operational cases.

Operational case	Metric	Ideal	Constant mean	SOC-dependent
Generation-dominant ($p_1 = 0.6, p_2 = 0.5$)	Average SOC	0.980	0.970	0.814
	$P_{full} (P_{100})$	0.3333	0.2520	≈ 0
	$P_{empty} (P_0)$	≈ 0	≈ 0	≈ 0
	MTBCC (t.u.)	3.0	4.0	∞
	E_{dis}/e_S	0.0000	0.0244	0.0237
Balanced ($p_1 = 0.51, p_2 = 0.5$)	Average SOC	0.773	0.128	0.135
	$P_{full} (P_{100})$	0.0399	≈ 0	≈ 0
	$P_{empty} (P_0)$	0.0007	0.0724	0.1363
	MTBD (t.u.)	1.9e6	18.2	9.4
	E_{in}/e_S	0.0002	0.0177	0.0334
Load-dominant ($p_1 = 0.4, p_2 = 0.5$)	Average SOC	0.020	0.015	0.011
	$P_{full} (P_{100})$	≈ 0	≈ 0	≈ 0
	$P_{empty} (P_0)$	0.3333	0.4058	0.4897
	MTBD (t.u.)	3.0	2.5	2.0
	E_{in}/e_S	0.1000	0.1217	0.1469

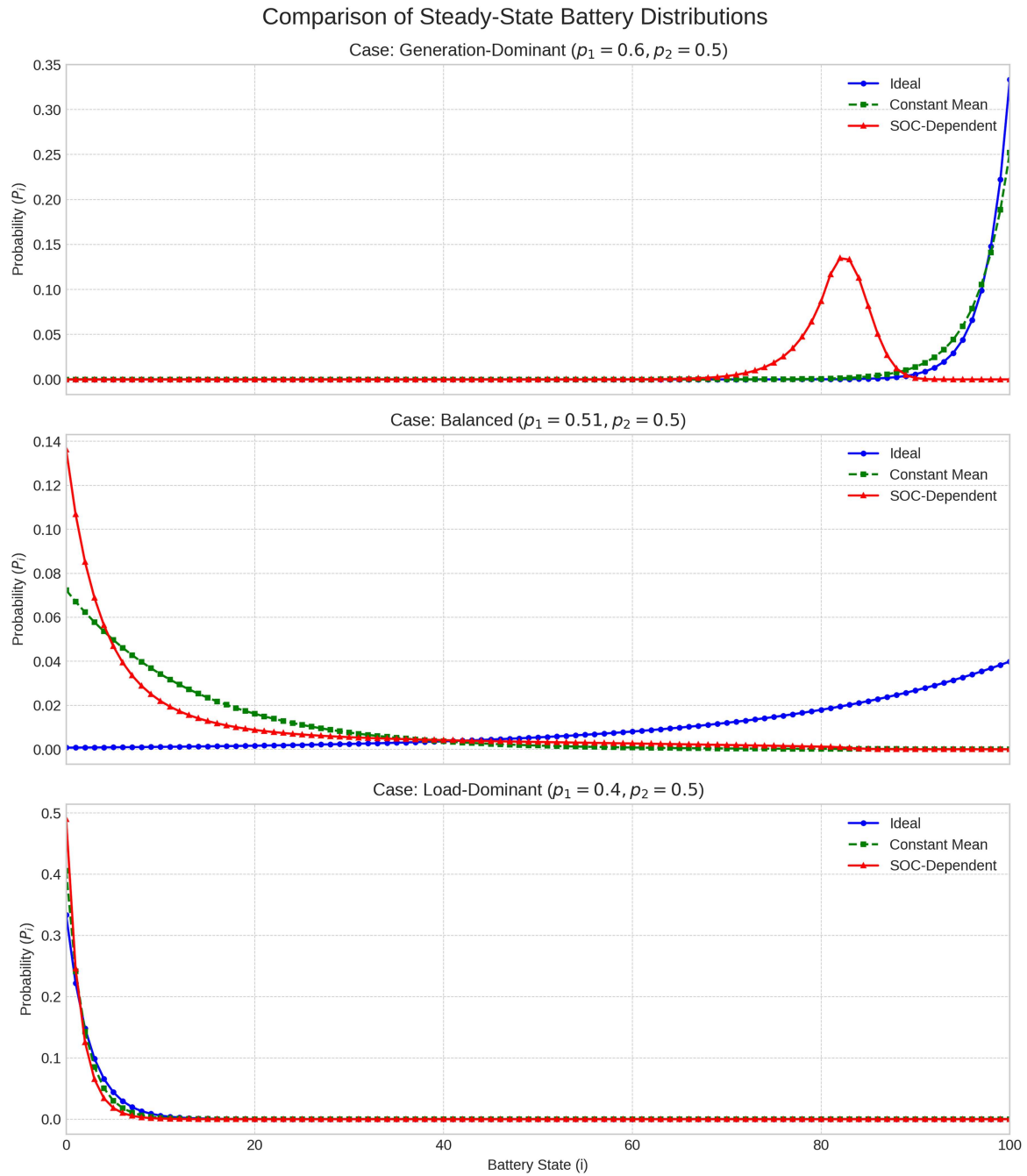


FIGURE 6 | Steady-state probability distributions for the three operational cases. Each panel compares the predictions of the SOC-dependent (red), ideal (blue) and constant mean (green) models, highlighting the qualitative and quantitative inaccuracies of simpler approaches across all conditions.

generation period (e.g., daytime for PV), this energy can be stored without constraints. This approach is blind to the physical feasibility and kinetics of the charging process, especially at high values of SOC. An aggregated statistical model might correctly predict the total quantity of energy available for storage, but it cannot assess whether the battery can physically accept that energy within the available time. This can lead to availability estimates that appear overly optimistic. For instance, in the analysis by [20], a BESS sized to cover just 0.4 days of average consumption is shown to achieve 99.9% availability. This result stems from a model in which the probability of charge is constant, regardless of the battery's SOC.

We argue that a more physically grounded approach should ask a different question. Instead of simply balancing the total energy budget, it should evaluate the feasibility of reaching a critical target SOC required for a specific operational phase (e.g., surviving the night). The proposed model represents the appropriate tool for this assessment. It operates using the parameters representative of the charging period (i.e., daytime) to determine if reaching the required end-of-day SOC is a realistic outcome, given the battery's non-linear behaviour.

We can demonstrate this with a concrete example, using the numerical data provided in the case study by [20].

First, let's establish the energy demand for nighttime. The authors consider a load of 300 households, which they state has an average nighttime power consumption of 202.14 kW. Assuming a 12-h night (from 7:00 PM to 7:00 AM, as per their definition), the total energy required to cover the entire night is straightforward to calculate:

$$E_{\text{night}} = 202.14 \text{ kW} \times 12 \text{ h} \approx 2426 \text{ kWh, or } 2.43 \text{ MWh}$$

Next, let's translate this energy requirement into the language of their BESS model. Their system has a total capacity of 3 MWh, which they discretise into $N = 151$ states. This means each discrete state, or 'quantum' of energy, corresponds to an energy amount Δ of 20 kWh (3000 kWh/150, since there are 150 transitions). To supply the 2430 kWh needed for the night, the BESS must therefore be able to discharge a total of:

$$\text{Quanta}_{\text{needed}} = \frac{2430 \text{ kWh}}{20 \text{ kWh/quantum}} \approx 121 \text{ quanta}$$

This leads us to the critical requirement for the end-of-day charge. To be able to discharge 121 energy quanta, the battery must have at least that many quanta stored at sunset. In terms of SOC, this translates to a minimum required SOC of approximately 80%:

$$\text{SOC}_{\min} = \frac{121 \text{ quanta required}}{151 \text{ total states}} \approx 80.1\%$$

Herein lies the fundamental difference between the two modelling philosophies. An approach based on aggregated statistics, like that of [20], implicitly assumes that if sufficient net energy is generated during the day, achieving the 80% target SOC is simply a matter of accumulation. Our model, however, confronts this assumption with physical reality. As we demonstrate in our analysis (see Figure 7), the battery's charge acceptance, $\eta_A(x)$, begins to decrease precisely as the SOC

approaches this 80% threshold. This means that charging the battery from 80% to 90% is significantly slower and less efficient than charging it from 30% to 40%. The aggregated statistical method is blind to this non-linear charging constraint. Our model, on the contrary, reveals that reaching the required target of 80% SOC is a challenge in itself, forcing a designer to consider not only *if* the daily energy surplus is sufficient on average, but *if* it can be effectively stored in the battery within the available time, given the physical limitations at high states of charge.

To provide a concrete quantification of this impact for system designers, we can extend the preceding analysis in a simplified manner. Using the data from [20], a conventional model predicts that a nominal capacity covering approximately 0.4 days of autonomy is sufficient to achieve near-perfect availability. Our model, by accounting for the charge acceptance limit at high SOC, reveals that reaching the required 80% end-of-day SOC is physically constrained. If we define the 'effective usable capacity' as the SOC range the battery can reliably cycle within (e.g., from 10% to the charge acceptance threshold of 80%), this represents only 70% of the nominal capacity. Consequently, to guarantee the same usable energy reserve and thus the same level of availability in this illustrative scenario, a designer using our physically-grounded model would need to specify a nominal capacity that is on the order of 40%–45% larger (i.e., $1/0.7 \approx 1.43$) than what a conventional, constant-efficiency model would suggest.

5 | Conclusion

This paper presented a stochastic model for BESSs that successfully incorporates battery operation's critical, state-dependent nature. By moving beyond the common simplification of constant efficiencies and representing the charging efficiency (η_c), discharging efficiency (η_d) and charge acceptance (η_A) as realistic functions of the SOC, our analytically tractable Markov chain model provides a more physically faithful representation of BESS behaviour.

The primary contribution of this work is not the proposal of a universal law for battery operation, nor the presentation of a

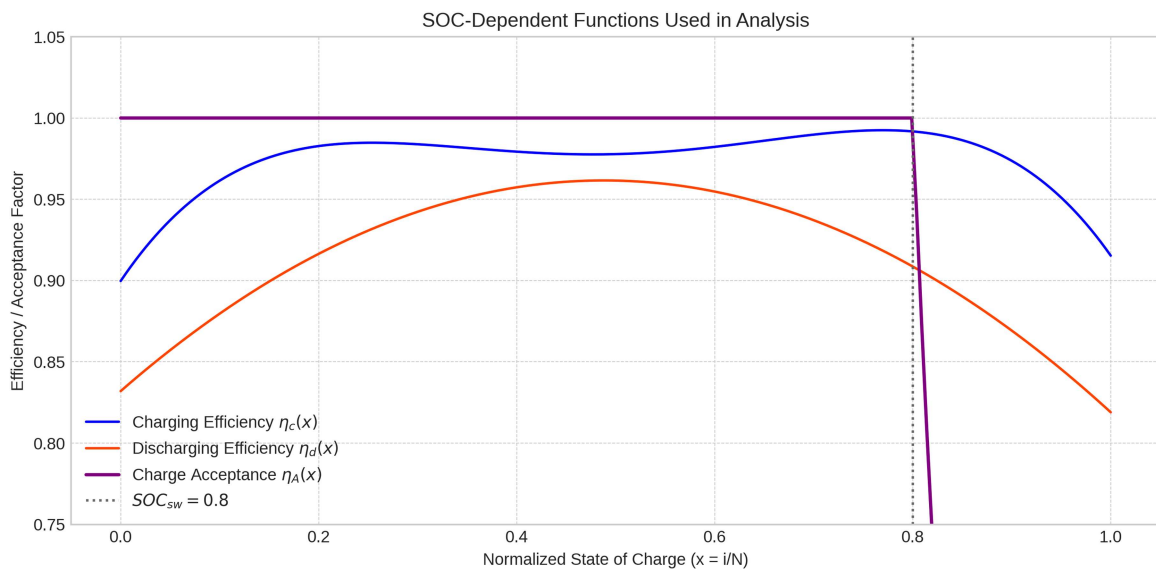


FIGURE 7 | Illustrative representation of the SOC-dependent functions used in the analysis. The charge acceptance factor $\eta_A(x)$, which declines exponentially beyond a switching threshold of $\text{SOC}_{\text{sw}} = 0.8$, is the dominant factor shaping the operational boundary at high SOC.

validated case study for a specific chemistry, but rather the development of an enhanced, analytically tractable tool. The numerical examples presented are illustrative, designed to demonstrate the method's diagnostic capabilities. We have demonstrated how this tool can reveal complex dynamics that simpler models would obscure. Specifically, our analysis shows that the interplay between external stochastic energy flows and the battery's internal, non-linear characteristics can lead to the emergence of preferential operating regions. As illustrated in our numerical case study, factors such as declining charge acceptance at high SOC can create a 'soft cap', making it inefficient and difficult to reach a full charge. This highlights that a battery's adequate usable capacity is an emergent property of the entire system, not just a nominal specification.

These findings have significant practical implications. Our comparison with conventional availability models shows that approaches that ignore the temporal correlation of renewable sources and the physical constraints of charging can lead to overly optimistic reliability estimates and potentially flawed BESS sizing. Our framework, by forcing a designer to consider the net energy balance and the feasibility of achieving a required state of charge, provides a more robust foundation for system design. This prevents underestimating the nominal capacity necessary to guarantee a usable energy reserve.

Despite the insights offered, we acknowledge the model's current limitations, which define clear pathways for future research. The present work assumes static probabilities for generation and consumption and does not explicitly model long-term battery degradation (ageing) or thermal effects, which are known to influence performance.

Future work will proceed along several key avenues. A primary extension will be the integration of SOH dynamics, allowing the model's transition probabilities to evolve over the battery's operational lifetime, thus capturing the effects of ageing. Another promising direction is the development of a non-homogeneous Markov chain to account for time-varying (e.g., seasonal or daily) energy flow patterns. Finally, the analytical tractability of the framework makes it an ideal candidate for integration into advanced energy management systems, enabling the design of optimal control policies that are aware of the battery's underlying physical constraints.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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