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Systematic Bifurcation Analysis of a Planar Diffusion Flame Model with Radiative Heat Losses

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Abstract. We analyse in a systematic way the dynamics of a model describing a planar diffusion flame with radiative heat losses. In particular we construct the full bifurcation diagram with respect to the Damköhler coefficient including the branches of oscillating solutions. Based on this analysis we found several interesting nonlinear phenomena including global bifurcations (homoclinic bifurcations) that mark the abrupt extinction of the nonlinear oscillations.

Keywords: Combustion, Diffusion Flames, Bifurcation Analysis, Homoclinic Bifurcation
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INTRODUCTION

The interest in detecting *unstable* combustion conditions is continuously increasing together with the need to reduce pollutant emissions. Indeed, this achievement is pursued by moving the operative conditions of combustion appliances towards conditions that, while being beneficial with respect to the emissions, are often more prone to the loss of stability of a stationary behavior, sometimes leading to complete extinction, e.g. blow-off or quenching.

It is clearly emerged from previous investigations that an oscillating behavior and flame instabilities can emanated from the competition between the thermal and mass diffusion [1, 2]. Several examples of this behavior are reported in the literature for the case of blow-off of anchored diffusion flames [3, 4] or, extinction due to heat losses [1, 5, 6]. Experimental evidence of diffusion flames instabilities have been reported in a variety of configurations like candle flames and jet flames (see also the recent review of Matalon [7]).

However, despite the evidence a rich nonlinear dynamic behavior, diffusion flames have been much less theoretically studied and in particular the analysis of the nonlinear dynamics following the onset of instabilities. A systematic identification of the conditions of stable oscillating regimes has been rarely applied in the combustion science [8, 9, 10], while it is much more usual in the chemical engineering science for similar test models, as reported in [11].

One reason is that in all the above mentioned experiments the fluid dynamics are generally non-trivial and as a consequence the comparison with theoretical predictions is quite difficult [7].

Previous investigations reported in [14] were based on numerical computations that identified supercritical Hopf bifurcation points. In a successive analysis [15], a more complex pattern was found, with the possibility to distinguish different paths leading to extinction and the occurrence of limit cycle behavior associated with the existence of a double Hopf bifurcation.

A linear stability analysis was adopted in [16], making possible to describe the stability map of the system varying both the Damköhler number and the radiative heat release coefficient. The adoption of a more rigorous approach, namely the nonlinear time series analysis in combination with the method of mutual information, was the basis of a clearer description of the transition to chaos in [17].

In this paper we focus on the systematic analysis of the nonlinear dynamics of a planar counterflow flame model with radiative heat losses, already well investigated by several authors [14, 15, 16, 17, 18]. By applying a computational approach we managed to construct the full bifurcation diagram including the branches of limit cycles. By systematically tracing the branches of oscillating solutions we found a rich nonlinear behaviour including homoclinic bifurcations which mark the abrupt disappearance of the oscillating solutions.

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MATHEMATICAL MODEL

The equations describing the evolution of flames can be written as [16, 18, 17]:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + w - RD\left(T^4 - T_o^4\right) \tag{1}$$

$$L\frac{\partial Y_o}{\partial t} = \frac{\partial^2 Y_o}{\partial x^2} - w \tag{2}$$

$$L\frac{\partial Y_f}{\partial t} = \frac{\partial^2 Y_f}{\partial x^2} - w, \tag{3}$$

where $T = T(x,t)$ is the temperature, Y_o is the oxidizer mass fraction and Y_f denotes the fuel mass fraction of the mixture. L is the Lewis number (same for both the fuel and the oxidizer), and R is the ratio of the characteristic chemical and radiation time scales. The nondimensional reaction term w for the one-step reaction is given by:

$$w = DY_oY_f e^{-T_a/T}. \tag{4}$$

Above D is the Damköhler number, and T_a is the activation temperature. The following boundary conditions are assumed at the porous walls:

$$\text{at} \quad x = -1: \quad T = T_o \quad Y_f = 1, \quad Y_o = 0 \tag{5}$$

$$\text{at} \quad x = +1: \quad T = T_o \quad Y_f = 0, \quad Y_o = 1. \tag{6}$$

COMPUTATION OF LIMIT CYCLES

Limit cycles are computed as fixed points of a Poincare map using a shooting formulation. We employ a well-tested initial value solver (LSODE) [19], for which numerical accuracy for the time integration of the system, as well as variational equations (which are important for bifurcation detection and continuation), can be easily adjusted [20]. If the evolution of the system is described through an autonomous non-linear system:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(x(t), \lambda), \tag{7}$$

with λ a system parameter, which serves as the bifurcation parameter, to compute a periodic solution one seeks for solutions which satisfy:

$$\mathbf{R} \equiv x(0) - x(T) = \mathbf{0}, \tag{8}$$

where T the period of oscillation. Newton-Raphson iteration is employed to solve the non-linear set of equations above, and the Jacobian matrix is:

$$\frac{\partial \mathbf{R}}{\partial x(0)} = \mathbf{I} - \frac{\partial x(T)}{\partial x(0)}, \tag{9}$$

where $\mathbf{I}$ is the identity matrix and $\partial x(T)/\partial x(0)$ is the state transition matrix of the system, describing the sensitivity of the “final” state $x(T)$ with respect to the “initial” state x(0). The state transition matrix is computed from the variational matrix $\mathbf{V}(t) \equiv \partial x(t)/\partial x(0)$, the evolution of which is given by:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{V} = \left[\frac{\partial f}{\partial x}\right] \cdot \mathbf{V}, \tag{10}$$

with initial conditions:

$$\mathbf{V}(0) = \left.\frac{\partial x(t)}{\partial x(0)}\right|_{t=0} = \mathbf{I} \tag{11}$$

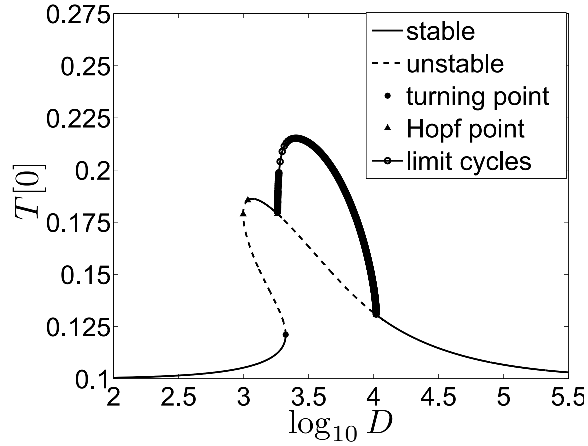

FIGURE 1. Parametric dependence of temperature at $x = 0$ with respect to the Damköhler parameter. Parameter set values: $R = 0.2$, $T_a = 1.0$.

In its current form the Newton-Raphson problem is ill-posed. Due to the translational (in time) invariance of the periodic solutions any point on the limit cycle satisfies Eq.8; it is possible to single out one point on the limit cycle by applying an additional algebraic constraint (also called pinning condition), which has the general form:

$$g(x, \lambda, T) = 0, \tag{12}$$

and allows the computation of the unknown period of oscillation T.

Furthermore, one can examine the dependence of the obtained periodic solutions on the system parameter λ, through the application of parameter continuation techniques. Here we report the pseudo arc-length continuation method, which traces a branch of periodic solutions, using a linearized arc-length condition:

$$N(x, \lambda, T) \equiv \frac{x_1 - x_0}{\delta s}^T \cdot (x - x_1) + \frac{T_1 - T_0}{\delta s} \cdot (T - T_1) + \frac{\lambda_1 - \lambda_0}{\delta s} \cdot (\lambda - x_1) - \delta s = 0, \tag{13}$$

where δs is the pseudo arc-length continuation step, and (x_0, λ_0, T_0), (x_1, λ_1, T_1) represent to two already computed periodic solutions.

RESULTS

In past investigations, the complete characterization of branches of limit cycles solutions has never been obtained. Oscillations have been computed through numerical simulation for only a few selected points. A typical example is reported in Fig.14 of paper [16], for the case $R = 0.2$ and $T_a = 1$ which is reproduced in Fig.1.

Here, some new features are computed and identified. In particular, the full branch of stable periodic solutions between the Hopf points at $D = 1823.9$ and $D = 10471$ has been successfully traced. Stable and unstable branches of steady state solutions are depicted with solid and broken lines respectively. The periodic solution branch is represented by the solid line with empty circles. Circles and triangles are used to point out turning and Hopf points respectively. This diagram clearly reports that no extinction occurs along this interval.

By increasing the value of R while keeping constant the value of $T_a = 1$, the region between the Hopf bifurcation point and the turning point reduces up to completely disappearance at $R = 0.233$. The dependence of $T[0]$ on D for $T_a = 1$ and $R = 0.233$ is detailed in Fig.2. The corresponding bifurcation diagram shows again the existence of a region of D values, where stable periodic solutions can be admitted.

This region is located within a homoclinic bifurcation point at $D = 2742$ and a supercritical Hopf point at $D = 5957$.

The homoclinic bifurcation is a global bifurcation which occurs when a periodic orbit collides with a saddle point. At the bifurcation point the period of oscillations tends to infinity and becomes a homoclinic orbit. In the vicinity of $D = 2742$ the period tends to infinity signaling the existence of a homoclinic bifurcation and the abrupt loss of oscillations.

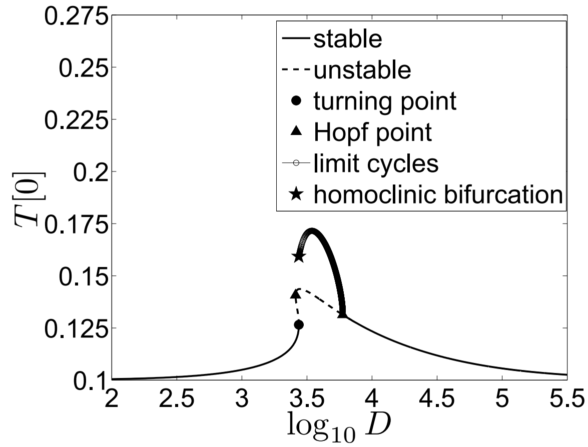

FIGURE 2. Parametric dependence of temperature at $x = 0$ with respect to the Damköhler parameter. Stable and unstable branches of steady state solutions are depicted with solid and dashed lines respectively. The periodic solution branch is represented by a solid line with empty circles. Full circles and triangles are used to point out turning and Hopf points respectively. The homoclinic bifurcation point is illustrated with a star. Parameter set values: $R = 0.233$, $T_a = 1.0$.

CONCLUSIONS

We constructed the complete bifurcation diagram (including the branches of limit cycles) of the dynamics of a planar counterflow flame model with radiative heat losses with respect to the Damköhler coefficient which served as our continuation parameter. Based on the computational approach, bifurcation and limiting points have been accurately determined. We showed that for certain values of the parameter space there are homoclinic bifurcations that are responsible for the sudden disappearance of flame oscillations. To our knowledge this is the first time that these results are reported for the particular model using the full set of the governing partial differential equations. As the model is rich of nonlinear behavior, the proposed approach can be efficiently used to reveal the mechanisms responsible for the abrupt loss of stability, which is crucial in combustion engineering. Future research is targeted to more detailed chemical kinetics, thus obtaining the bifurcation diagram of more realistic models.

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