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An Approach to Model and Control a Flexible Spacecraft

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Abstract: In this paper, an easy and effective methodology to model flexible structures, such as a spacecraft with flexible appendages, is provided, which is valid also in the hypotheses of structures having varying cross-sections and of large deformations. For the above flexible structures, it is also proposed a method to easily design a simple and efficient control law, considering as design specifications the gain margin, the maximum tracking error, the maximum control signal, and the maximum terminal deflection of the flexible parts, assuming the reference generic but with bounded first derivative. Moreover, it is shown that the robustness of the control system is assured even though the design of the controller is made with a low-order model of the flexible structure. Finally, the proposed approach is illustrated and validated in the case of a satellite with flexible appendages, for various references.

Keywords: Robust control design; Tracking control; Modelling; Flexible spacecraft; Majorant system.

1. INTRODUCTION

Flexible structures in the aerospace field have long been in use, since the modern spacecrafts (such as, e.g., communication satellites, earth observation satellites, and manned spacecrafts) are designed with large, lightweight, and low-stiffness appendages (solar panels, antennas, space manipulators, or debris capture systems). Moreover, lately, expandable and inflatable structures, based on flexible materials, in order to cut even more the spacecraft's mass (e.g., to reduce the launch costs) have also been proposed.

In view of this, the attitude tracking control problem for flexible spacecrafts is a basic and hard issue. Indeed, vibrations of the appendages and the effects of external disturbances deteriorate the performance in terms of rapid convergence and attitude control precision, which are essential requirements for such structures.

To deal with the above significant matter, first, the flexible structure has to be well modelled to the aim of its analysis and control, then, easy, robust and efficient control laws need to be provided.

Numerous approaches have been proposed for the control of a flexible spacecraft, including the ones based on the standard controllers (see e.g., Di Gennaro (2002), Liu et al. (2016), Malekzadeh et al. (2012), Meng et al. (2016), Tahmasebi et al. (2018), Wang et al. (2022)), linear quadratic regulator (LQR) approaches (see e.g., Mazzini (2016), Wang et al. (2022)), variable structure methods, such as sliding mode control (SMC) with its numerous variants (see e.g., Golestani et al. (2023), Hu (2012), Zhang et al. (2020), Wang et al. (2022)), H-infinity control (see e.g., Mazzini (2016), Mohsenipour et al. (2013), Zhang et al. (2021)), adaptive control strategies (see e.g., Li et al. (2009), Shahravi et al. (2006), Xiao et al. (2022)), fuzzy approaches (see e.g., Guan et al. (2005)), and so on.

However, most of the above control laws are designed on the basis of an approximate model and do not consider more than a single specification; moreover, some of these require the exact knowledge of the system parameters.

In the present paper, an easy and effective methodology to model flexible structures, such as a spacecraft with flexible appendages, is provided, which is valid also in the hypotheses of structures having varying cross-sections and of large deformations.

It is explicitly noted that the proposed modelling methodology can be used to model also other flexible structures and is based on the results presented in Celentano (2016), which have theoretically and experimentally been validated.

Afterwards, for the considered flexible structures, it is provided a method, via majorant systems in the frequency domain, to easily design a simple and efficient control law, considering as design specifications the gain margin, the maximum tracking error, the maximum control signal, and the maximum terminal deflection of the flexible parts, assuming the reference generic but with bounded first derivative, and also bounded second derivative (for the purpose of the deflection specification). Moreover, it is shown that the robustness of the control system is assured even though the design of the controller is made with a low-order model of the flexible structure. Lastly, the proposed approach is illustrated and validated in the case of a satellite with flexible appendages, for various references.

It is worth noting that the main peculiarities and advantages of the proposed approach can be summarized as follows:

- The used modelling method is easy, numerically stable, and computationally efficient.

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- The proposed control law is easy to design and realize, robust and effective. Moreover, it allows to satisfy more specifications, suitably choosing the parameters of the control law and appropriately bounding the first and second derivatives of the reference signal.

Finally, the ongoing research aims at extending the provided results considering the spatial model of a satellite.

2. MAIN RESULTS

2.1 System Modelling

There exist numerous flexible structures, especially in the aerospace field, such as spacecrafts with flexible appendages (solar panels – see Fig. 1a, antennas – see Fig. 1b), which need to be well modelled to the aim of the analysis and control.

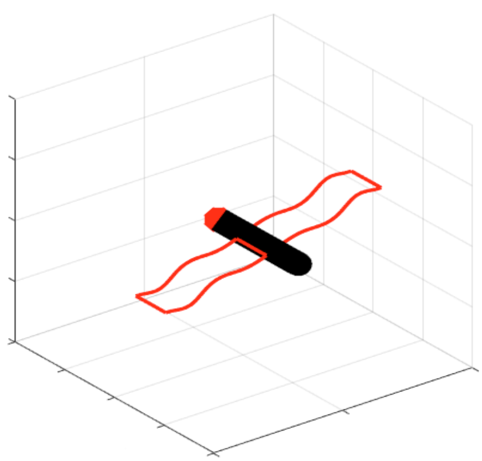

Fig. 1a

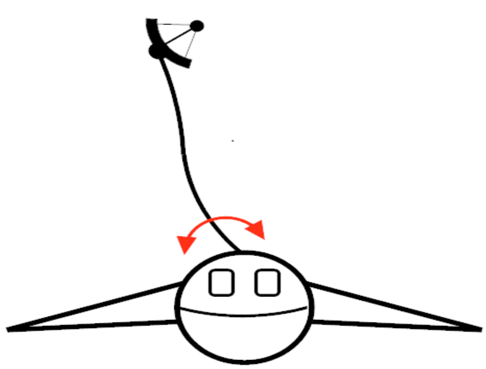

Fig. 1b

Fig. 1. a) Satellite with flexible solar panels. b) Satellite with flexible antennas.

An easy, numerically stable, and computationally efficient methodology to model flexible structures, also under the hypothesis of structures having varying cross-sections and of large deformations, has been presented and has theoretically and experimentally been validated in Celentano (2016).

In this work, for brevity, the above methodology is used to model only a satellite with two flexible appendages (henceforth called flexible wings).

Assuming a constant flight speed (which is a very recurring case), the model to control the orientation around the main rotation axis, supposed to be the principal axis of inertia, can be made using the method given in Celentano (2016), suitably and fictitiously subdividing each wing into n rigid sub-wings, to the aim of the computation of the inertia matrix M and the internal friction matrix K_a, and n-1 flexible sub-wings to compute the stiffness matrix.

Figure 2 shows the approximation of the wings with $2n = 8$ rigid sub-wings and $2n-2 = 6$ flexible sub-wings, and the terminal deflection γ of the right wing.

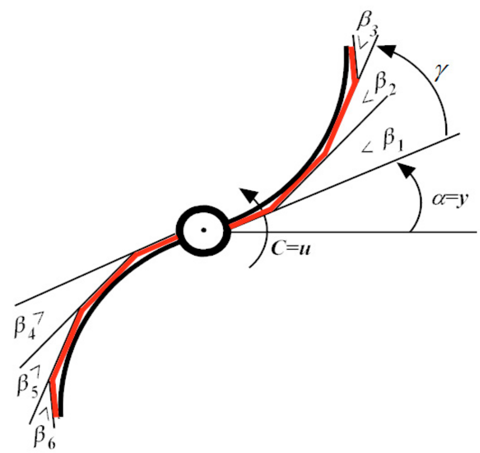

Fig. 2. Approximation of the flexible wings of a satellite with 8 rigid sub-wings and 6 flexible sub-wings.

Taking into account the expression of the inertia matrix and the centrifugal and Coriolis forces (see Celentano et al. (2006) and Celentano (2016)), in the hypothesis of small deformations, the structure of the linearized model is of the type

$$M\ddot{q} + K_a\dot{q} + K_e q = bu, \tag{1}$$

where $q = [\alpha\ \beta_1\ \beta_2\ \ldots\ \beta_{2n-2}]^T$, M is the inertia matrix for $\beta_i = 0,\ i = 1,\ldots,2n-2$, K_a and K_e are diagonal matrices with $K_a(1,1) = k_{a0}$, $K_e(1,1) = 0$, and $b = [1\ 0\ 0\ \ldots\ 0]^T$.

From the model (1) it turns out to be

$$\dot{x} = Ax + Bu,\ y = Cx, \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix},\ y = \begin{bmatrix} \alpha \\ \dot{\alpha} \\ \gamma \end{bmatrix}$$
$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K_e & -M^{-1}K_a \end{bmatrix},\ B = \begin{bmatrix} 0 \\ M^{-1}b \end{bmatrix},\ C = \begin{bmatrix} c_1^T \\ c_2^T \\ c_3^T \end{bmatrix}, \tag{2}$$

where c_1, c_2, c_3 are appropriate vectors.

2.2 Control Design

First, consider the control system in Fig. 3. For this system the following theorem holds.

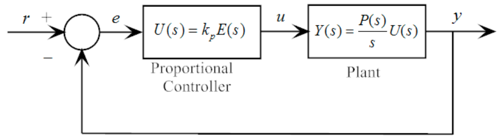

Fig. 3. Control scheme of a process $P(s)/s$, with $P(s)$ asymptotically stable and with positive static gain.

Theorem 1. Consider the control system in Fig. 3. Suppose that $P(s)$ is asymptotically stable, with positive static gain, and that its frequency response $P(s)_{|s=j\omega} = M(\omega)e^{j\varphi(\omega)}$ is increased by the frequency response of the system $\hat{P}(s) = \mu e^{-sT}$, where $\mu \ge M(\omega)$ and $T = \sup_{\omega>0} \frac{-\varphi(\omega}{\omega} \Rightarrow \varphi(\omega) \ge -\omega T$, said to be majorant system (in the frequency domain) of $P(s)$. Then, this control system is asymptotically stable $\forall k_p = \hat{k}_p/m$, $\hat{k}_p = \frac{\pi}{2\mu T}$, $m > 1$, with a gain margin greater than or equal to m. Moreover, the tracking error of a generic reference $r(t)$ with bounded first derivative $\dot{r}(t)$ satisfies the relation

$$|e(t)| \le H_e \max|\dot{r}(t)|,\ H_e = \int_0^\infty |s_{-1}(\sigma)|d\sigma, \tag{3}$$

where $s_{-1}(t)$ is the step response of the sensitivity function $S(s)$, i.e., $s_{-1}(t) = L^{-1}\left(\frac{1}{s + k_p P(s)}\right)$.

Proof. The proof of the first part of the theorem follows from the fact that the majorant control system in Fig. 4 is asymptotically stable with gain margin m and that $\mu \ge M(\omega)$, $\varphi(\omega) \ge -\omega T$. The proof of the second part easily follows taking into account that $S(s) = \frac{s}{s + k_p P(s)}$, $E(s) = S(s)L(r(t)) = (S(s)/s)sL(r(t)) = L(s_{-1}(t))L(\dot{r}(t))$ and, hence,

$$|e(t)| = \left|\int_0^t s_{-1}(\tau)\dot{r}(t-\tau)d\tau\right| \le \left(\int_0^\infty |s_{-1}(\tau)|d\tau\right)\max|\dot{r}(t)|. \tag{4}$$

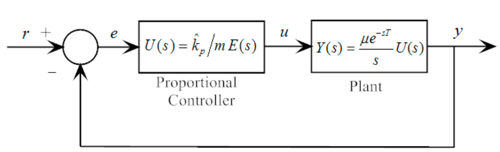

Fig. 4. Scheme of the majorant control system.

That being stated, a satellite with flexible wings modelled by (2) can be controlled with the control law (see also Fig. 5)

$$u = -k_{av}\dot{\alpha} + k_p(r - \alpha). \tag{5}$$

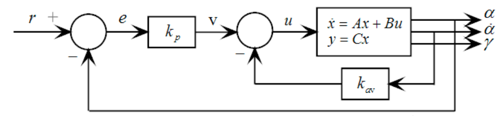

Fig. 5. The proposed control scheme.

Remark 1. Note that $\dot{\alpha}$ can be obtained directly measuring it with a velocity sensor or estimating it with a real derivative action or an optimal estimator (see e.g., Celentano et al. (2021)).

Since $K_e(1,1) = 0$, if $k_{at} = k_{a0} + k_{av} > 0$ it is easy to verify that

$$\frac{\alpha(s)}{V(s)} = c_1^T(sI - A_t)^{-1}B = \frac{P(s)}{s}, \tag{6}$$

where $A_t = A$ with $K_a(1,1) = k_{at}$, and $P(s)$ is asymptotically stable with positive static gain. Hence, using Theorem 1, it is possible to stabilize the control system in Fig. 5 and compute the constant H_e such that $|e(t)| \le H_e \max|\dot{r}(t)|$ for any reference signal $r(t)$ with bounded first derivative $\dot{r}(t)$.

Now, suppose that the control system in Fig. 5 is asymptotically stable. Then, if $r(t) = 1(t)$, at steady-state, $u(t)$ is null, since $K_e(1,1) = 0$; if, instead, $r(t) = 1(t)t$, also $\gamma(t)$ is null at steady-state, since the inertial forces acting on the wings are null. Therefore,

$$\frac{U(s)}{R(s)} = W_u(s) = W_{u-1}(s)s,\quad \frac{\Gamma(s)}{R(s)} = W_\gamma(s) = W_{\gamma-2}(s)s^2, \tag{7}$$

i.e., $W_u(s)$ has a zero at the origin and $W_\gamma(s)$ has a double zero at the origin.

Thence, if $\ddot{r}(t)$ is bounded it is

$$\begin{aligned} |u(t)| &\le H_u \max|\dot{r}(t)|,\ H_u = \int_0^\infty |w_{u-1}(\sigma)|d\sigma \\ |\gamma(t)| &\le H_\gamma \max|\ddot{r}(t)|,\ H_\gamma = \int_0^\infty |w_{\gamma-2}(\sigma)|d\sigma. \end{aligned} \tag{8}$$

Based on the above results, the controller design can be made as follows:

- The gain margin m is fixed (e.g., $m = \sqrt{2}, 2 \Leftrightarrow m_{dB} = 3dB, 6dB$) on the basis of the parametric uncertainties of the satellite.
- Using Theorem 1 and relations (7), (8), as k_{at} varies it is computed $k_p(k_{at})$ and the values of the gains $H_e(k_{at}), H_u(k_{at}), H_\gamma(k_{at})$.
- On the basis of the considered class of references, especially of the allowed values $\max|\dot{r}(t)|$ and $\max|\ddot{r}(t)|$, and of an appropriate compromise between the maximum allowed tracking error $\max|e(t)|$, the maximum acceptable and/or available control signal $\max|u(t)|$, and the

maximum acceptable terminal deflection $\max|\gamma(t)|$, k_{at} is computed and, hence, the parameters k_{av} and k_p of the controller are determined.

Remark 2. As it is highlighted by the example reported in Section 3, the values of μ and T of the majorant system can be computed also with a low value of n, since these values turn out to be invariant as n increases, i.e., with respect to the precision of the considered model of the satellite. Therefore, the robustness of the control system is assured even though a low value of n to design the controller is chosen.

3. CASE STUDY

As application of the modelling and control approach proposed in the previous section consider the satellite in Fig. 6 with its body and wings made of aluminium ($\rho = 2750 Kg/m^3$, $E = 6.6e10 N/m^2$), and the following values of the parameters:

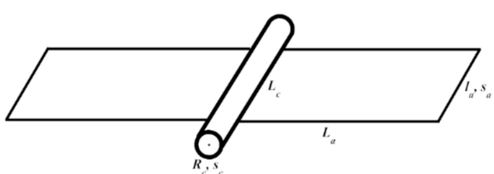

Fig. 6. Scheme of a satellite with two flexible wings.

$L_c = 1.00m, R_c = 0.060m, s_c = 0.002m, L_a = 0.80m, l_a = 0.30m,$
$s_a = 0.002m,\ k_{a0} = 0\ Nms/rad$.

Fictitiously subdividing the wings in an optimal way as in Celentano (2016), it is $K_e(i,i) = k_e = EI/(L_a/(n\text{-}1))$, $i = 2,...,2n-2$, where n is the number of sub-wings of each wing, and $I = l_a s_a^3/12$. In the hypothesis of $K_a(i,i) = k_a = k_e/1e5$, $i = 2,...,2n-2$, and $k_{av} = 1.5\ Nms/rad$, in Figs. $7 \div 10$ the Bode diagrams of $P(s) = L(\dot{\alpha})/L(\text{v})$ for $n = 2,4,7,10$ and of the corresponding majorant systems are reported. The corresponding values of the parameters of the majorant systems are independent of n and turn out to be $\mu = 0.6667$, T=0.4531.

Remark 3. It is explicitly noted that, due to the symmetric structure of the satellite, the reduced transfer function $P(s)$ does not contain $n-1$ modal frequencies.

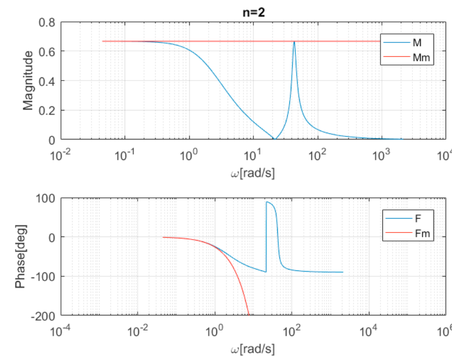

Fig. 7. Bode diagrams of $P(s)$ and of the majorant system for n=2.

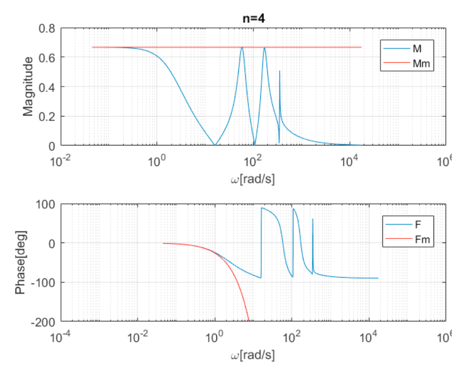

Fig. 8. Bode diagrams of $P(s)$ and of the majorant system for n=4.

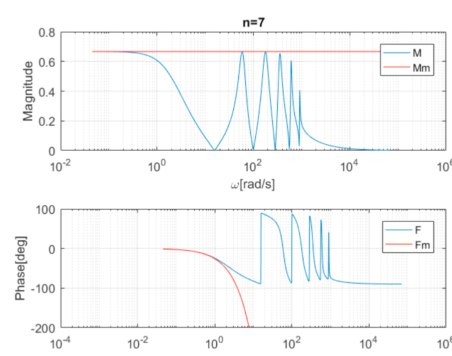

Fig. 9. Bode diagrams of $P(s)$ and of the majorant system for n=7.

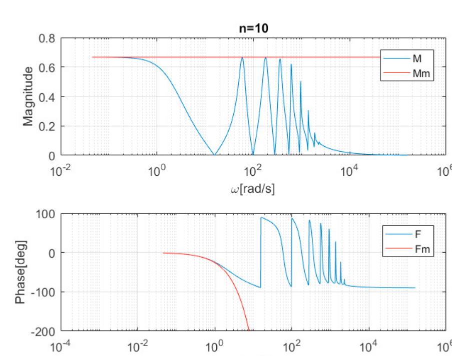

Fig. 10. Bode diagrams of $P(s)$ and of the majorant system for n=10.

Figs. 11a,b show the behaviours of the gains $H_e(k_{at}),\ H_u(k_{at}),\ H_\gamma(k_{at})$ for $k_{at} \in [0.5,\ 2.5]$ and $m_{dB} = 3dB,\ 6dB,$ respectively. As it can be noted, when k_{at} increases (or m_{dB} decreases - e.g., $m_{dB} = 3$ instead of $m_{dB} = 6dB$) then the tracking error decreases, but both the control signal and the terminal deflection increase.

If $m_{dB} = 6dB$ and $k_{av} = 1.5$ it turns out to be $u = 2.50e - 1.50\dot{\alpha},\ H_e = 0.879,\ H_u = 1.572,\ H_\gamma = 0.0089;$ while if $m_{dB} = 3dB$ and $k_{av} = 1.5$ it is $u = 3.54e - 1.50\dot{\alpha},\ H_e = 0.795,\ H_u = 2.212,\ H_\gamma = 0.0103.$

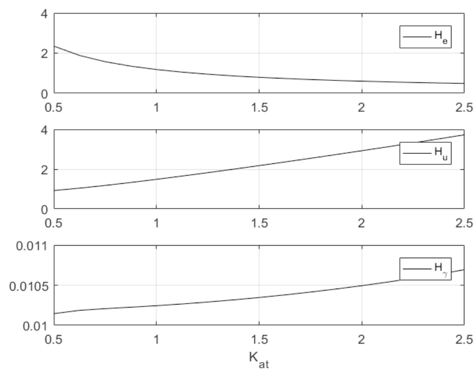

Fig. 11a

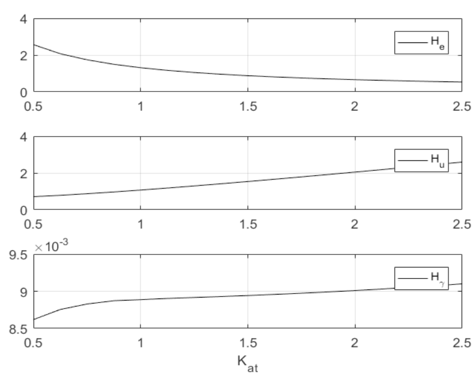

Fig. 11b

Fig. 11. a) $H_e(k_{at}),\ H_u(k_{at}),\ H_\gamma(k_{at}),\quad m_{dB} = 3dB.$
b) $H_e(k_{at}),\ H_u(k_{at}),\ H_\gamma(k_{at}),\quad m_{dB} = 6dB.$

If $u = 2.50e - 1.50\dot{\alpha}$ and as reference signal $r(t)$ is considered the pseudo-trapezoidal one at minimum time (see Celentano et al. (2018)) with $\max|\dot{r}(t)| = 0.2$ and $\max|\ddot{r}(t)| = 0.1$ shown in Fig. 12a then the time histories of $e(t),\ u(t),$ and $\gamma(t)$ are the ones reported in Fig. 12b. Moreover, the maximum values of $|e(t)|,\ |u(t)|,\ |\gamma(t)|$ are as follows:

$\max|e(t)| = 0.132,\ \max|u(t)| = 0.143,\ \max|\gamma(t)| = 0.000723$,

in accordance with the theoretical inequalities

$|e(t)| \le H_e \max|\dot{r}(t)| = 0.879 \times 0.2 = 0.176,$

$|u(t)| \le H_u \max|\dot{r}(t)| = 1.572 \times 0.2 = 0.314,$

$|\gamma(t)| \le H_\gamma \max|\ddot{r}(t)| = 0.0089 \times 0.1 = 0.00089.$

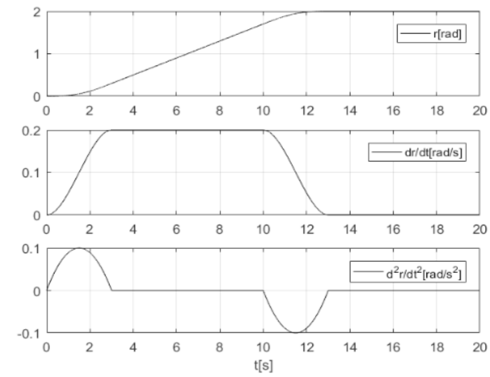

Fig. 12a

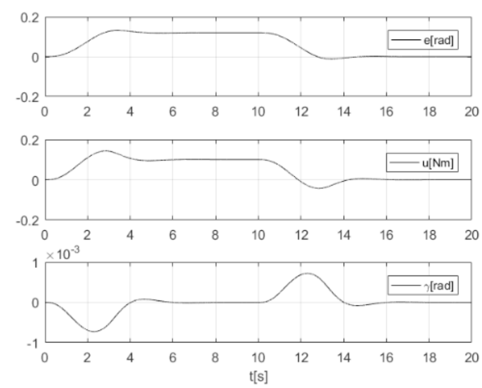

Fig. 12b

Fig. 12. a) Time histories of $r(t)$, $\dot{r}(t)$ and $\ddot{r}(t)$, with $\max|\dot{r}(t)| = 0.2$ and $\max|\ddot{r}(t)| = 0.1$.
b) Time histories of $e(t),\ u(t),$ and $\gamma(t)$ with $\max|\dot{r}(t)| = 0.2$ and $\max|\ddot{r}(t)| = 0.1$.

If, instead, as reference signal $r(t)$ is considered the pseudo-trapezoidal one at minimum time with $\max|\dot{r}(t)| = 0.5$ and $\max|\ddot{r}(t)| = 10$ shown in Fig. 13a then the time histories of $e(t),\ u(t),$ and $\gamma(t)$ are the ones shown in Fig. 13b.

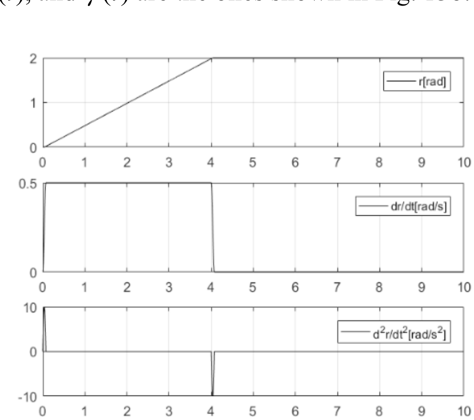

Fig. 13a

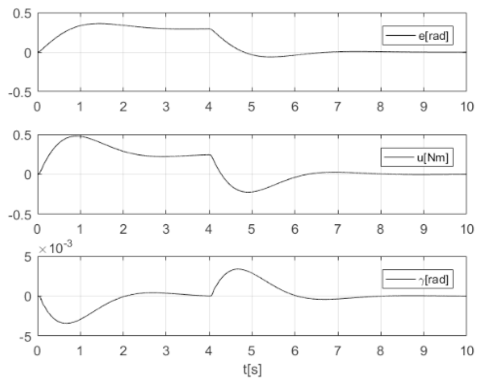

Fig. 13b

Fig. 13. a) Time histories of $r(t)$, $\dot{r}(t)$ and $\ddot{r}(t)$, with $\max|\dot{r}(t)| = 0.5$ and $\max|\ddot{r}(t)| = 10$.
b) Time histories of $e(t)$, $u(t)$, and $\gamma(t)$ with $\max|\dot{r}(t)| = 0.5$ and $\max|\ddot{r}(t)| = 10$.

Remark 4. It is worth noting that

$$u = k_p(r-\alpha) - k_{av}\dot{\alpha} = k_p(r-\alpha) + k_{av}(\dot{r}-\dot{\alpha}) - k_{av}\dot{r} = k_p e + k_{av}\dot{e} - k_{av}\dot{r}.$$

Hence, taking into account that the robustness of the control system does not depend on the reference, also a *PD* controller can be used. In such a case, it can be proven that the tracking error decreases, but the control signal and the terminal deflection amplitudes increase. If the reference signal has some discontinuities (e.g., the initial discontinuity in the case of a step reference) then a pre-filter can be used (see Fig. 14). Concerning this, if a pre-filter of the type $PF = 1/(\tau s + 1)$ is used then it is easy to prove that $|\dot{r}_f| \le \max|r|/\tau$.

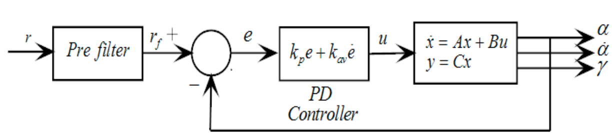

Fig. 14. Control scheme with a *PD* controller and a pre-filter.

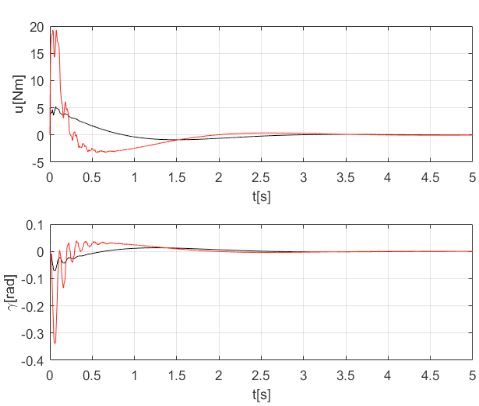

Fig. 15. Time histories of $u(t)$ and $\gamma(t)$, case $r(t) = 2\cdot 1(t)$, $u(t) = 2.5e(t) - 1.5\dot{\alpha}(t)$ (black), case $r(t) = 2\cdot 1(t)$, $u(t) = 2.5e(t) + 1.5\dot{e}(t)$, pre-filter $PF = 1/(0.1s+1)$ (red).

Fig. 15 shows the time histories of $u(t)$ and $\gamma(t)$ in the hypothesis that $r(t) = 2\cdot 1(t)$, when $u(t) = 2.5e(t) - 1.5\dot{\alpha}(t)$, and in the case of $u(t) = 2.5e(t) + 1.5\dot{e}(t)$ and a pre-filter $PF = 1/(0.1s+1)$.

Remark 5. It is worth noting that the provided simulations about the performance of the control system, which have been obtained using the linearized model of the satellite and with $n = 7$, practically coincide with the simulations made with $n > 7$ and using the nonlinear model.

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