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Corporate social responsibility and network externalities: a game-theoretic approach

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Abstract

This research revisits the pioneering work by Katz and Shapiro (Am Econom Rev 75:424–440, 1985) with network (consumption) externalities in a twofold way: first, it considers Corporate Socially Responsible (CSR), instead of profit-maximising, firms; second, it uses a game-theoretic approach and analyses the commitment decision game in which firms face the binary choice to credibly commit (C) or not to commit (NC) themselves to an announced output level in the first decision-making stage. Competition at the market stage occurs à la Cournot. Results show a rich spectrum of sub-game perfect Nash equilibrium (SPNE) outcomes, ranging from the prisoner's dilemma (self-interest and mutual benefit of output commitment conflict) to the anti-prisoner's dilemma or deadlock (self-interest and mutual benefit of output commitment do not conflict), passing from the coordination to the anti-coordination game. These outcomes depend on the intensity of the social concern in the firm's objective and the network size. The article also pinpoints the welfare outcomes corresponding to the SPNE and extends the analysis to a Stackelberg rivalry setting.

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1 Introduction

This article examines a quantity-setting (Cournot) duopoly with Corporate Socially Responsible (CSR) firms and network externalities in consumption,[1] representing two pillars of modern industries framed in a strategic context. It follows the pioneering work by Katz & Shapiro (1985), who exogenously distinguish the cases in which firms commit and do not commit to a publicly announced output level, and studies the commitment decision game with CSR instead of profit-maximising firms, which has recently been considered by Choi & Lim (2022). The article belongs to a literature on the topic of the timing of commitment and expectations formation mechanisms in strategic markets with network effects (e.g., Griva & Vettas, 2011; Hagiu & Halaburda, 2014; Leppänen, 2020; Nakamura, 2021; Suleymanova & Wey, 2012; Toshimitsu, 2019, 2021).

Engagement in CSR has recently emerged as a dominant global business practice and has been the result of two main forces. On one hand, the expectations of the public audience and the overall society for business are rising. As McKinsey (2019) reports, customer interest in being informed about companies' engagement in environmental and social issues has grown; customers also strongly believe that companies have a responsibility on those subjects. On the other hand, a global survey of 350 business leaders conducted by Deloitte (2019) (in collaboration with Forbes) reveals that business leaders are deeply convinced of role of firms as stewards of society, and they are planning to engage more on societal-impact issues in the next future.

CSR has also become an issue of burgeoning importance in the academic literature (e.g., Baron, 2001, 2009; Goering, 2007, 2008; García-Gallego & Georgantzís, 2009; Lambertini & Tampieri, 2010, 2015; Bénabou & Tirole, 2010; Kopel et al., 2014; Kopel & Lamantia, 2018). Amongst the industries that have experienced extraordinary progress in CSR activities in recent years, it is remarkable to find a leading position precisely in those producing network goods. For instance, according to a KPMG report, the technology, media & telecommunications sector presents 79 per cent of the companies surveyed reporting CSR activities, the highest level amongst the surveyed industries. Specifically, the telecommunication subsector shows the highest rate of CSR reporting, with 87 per cent of the companies (KPMG, 2015, 2017).

The remarkable development pace of network industries has led an increasing number of scholars to start studying how consumption externalities may alter the results of the standard models of imperfect competition framed in strategic contexts (Katz & Shapiro, 1985), focusing also on the role of monopoly (Cabral et al., 1990), R&D innovation (Buccella et al., 2022a; Cabral, 1990; Naskar & Pal, 2020; Shrivastav, 2021) strategic delegation (Bhattacharjee & Pal, 2014; Chirco & Scrimitore, 2013; Hoernig, 2012), and unionised labour market and codetermination (Fanti & Buccella, 2016; Fanti & Gori, 2019).

The behaviour of firms under network consumption externalities is twofold (Katz & Shapiro, 1985): they can or cannot commit to a publicly known output level. This choice, in

[1] Network goods can generate positive (resp. negative) externalities, in turn, causing the so-called bandwagon (resp. snob) effect: the desire to have (resp. not to have) some goods because almost everyone else has them. A typical example is the positive (resp. negative) feature of fashion due to widespread brands.

turn, affects the size of the market, the consumer welfare, profits and social welfare. If firms cannot credibly commit themselves to an announced output level, each of them follows the standard Cournot assumption that the actual output of the rival is fixed and then it chooses its production by assuming that the consumers' expectations about the size of the network are given. In this case "a firm's announcement of its planned level of output does not affect consumer expectations" (Katz & Shapiro, 1985, p. 440). If firms can credibly commit themselves to an announced output level, consumers know the selected output and then expectations are equal to the committed production. The key difference between these cases is that under full commitment "firm i can directly influence consumers' expectations regarding its network size", thus getting "the standard Cournot equilibrium with demand-side economies of scale" (Katz & Shapiro, 1985, p. 440).

The present article extends Katz & Shapiro (1985) in a twofold way: (1) it considers CSR, instead of profit-maximising, firms, and (2) it uses a game-theoretic approach to analyse the CSR commitment decision game (CSR-CDG henceforth) in which firms face the binary choice to commit (C) or not to commit (NC) themselves to an announced output level in the first (decision-making) stage, and then determine the endogenous market configuration. These two scenarios (C and NC) were taken exogenously and analysed separately in the original contribution of Katz & Shapiro (1985). However, this choice has important strategic effects, with implications on the endogenous emergence of sub-game perfect Nash equilibrium (SPNE) and social welfare effects, which instead determine some possible policy implications. These effects have been pointed out only recently by Choi & Lim (2022) in a relevant contribution studying the CDG with profit-maximising firms. Dealing with CSR firms, the present contribution shows sharp differences in the prevailing SPNE compared the results found by Choi & Lim (2022). This is because of the strategic effects generated by the extent of the degree of social concern in the firm's objective.

If firms are profit-maximising (Choi & Lim, 2022), the outcomes of the CDG are the following:

- If the network externality is positive, the unique Pareto inefficient SPNE is (C, C). The CDG is a prisoner's dilemma in which self-interest and mutual benefit of output commitment conflict when the extent of the negative network effect is sufficiently low ($n < 0.75$), where $-1 < n < 1$ is the strength of the consumption network externality. The CDG becomes an anti-prisoner's dilemma (deadlock) in which self-interest and mutual benefit of output commitment do not conflict when the extent of the negative network effect is sufficiently high ($n > 0.75$). Under output commitment firms produce more than under output non-commitment due to the bandwagon effect generated by the positive network externality. In this case, the commitment strategy tends to incentivise firms to produce a higher output than the non-commitment strategy because of the larger (outward) shift in the market demand of the network goods generated by the positive externality that committing in advance, in turn, causes compared to non-committing in advance to a given output level. If the network effect is relatively low, C is a dominant strategy but firms are entrapped in a dilemma as they could jointly obtain a higher profit by playing NC, but neither would like to be the one that unilaterally obtains the lowest payoff through the NC strategy. Larger values of the network effect allow firms to solve the dilemma as the increase in the output sustained by the higher demand increases profits under the C strategy letting them when both firms play C go beyond the corresponding value if both played NC. Committing to an announced output level, therefore, has a relevant strategic value in a non-cooperative context as, e.g., the managerial delegation device.

- If the network externality is negative, the unique Pareto inefficient SPNE is (NC, NC) and the CDG is a prisoner's dilemma in which self-interest and mutual benefit of output non-commitment conflict. This holds irrespective of the extent of the negative network effect. Under output commitment firms produce less than under output non-commitment due to the snob effect generated by the negative network externality. In this case, the non-commitment strategy tends to incentivise firms to produce a higher output than the commitment strategy because of the larger (inward) shift in the market demand of the network goods generated by the negative externality that committing in advance, in turn, causes compared to non-committing in advance to a given output level. Given the snob effect, NC is a dominant strategy but firms are entrapped in a dilemma as they could jointly obtain a higher profit by playing C, but neither would like to be the one that unilaterally obtains the lowest payoff through the C strategy.

If firms are socially oriented (CSR), there are relevant changes in the prevailing SPNE of the CSR-CDG compared to Choi & Lim (2022). An increase in the weight of the social concern in the firm's objective tends to favour the emergence of the no-commitment scenario if the network externality is positive and the commitment scenario if the network externality is negative. This is because firms produce more under C (resp. NC) than under NC (resp. C) if the network externality is positive (resp. negative). For any given value of the positive network strength, the more firms are socially oriented, the larger the increase in the output they would get by choosing to commit in advance to an announced output level and the lower the price consumers are willing to pay. The former effect increases profits. The latter effect reduces profits. When the extent of the firm social concern is high enough, the negative price effect dominates the positive quantity effect and profits under C reduce, thus favouring the emergence of NC as a dominant strategy leading to a Pareto efficient SPNE in which self-interest and mutual benefit of output non-commitment do not conflict under positive network effects. Unlike this, for any given value of the negative network strength, the more firms are socially oriented, the larger the increase in the output they would get by choosing not to commit in advance to an announced output level and the lower the price consumers are willing to pay. The former effect increases profits. The latter effect reduces profits. When the extent of the firm social concern is high enough, the negative price effect dominates the positive quantity effect and profits under NC reduce, thus favouring the emergence of C as a dominant strategy leading to a Pareto efficient SPNE in which self-interest and mutual benefit of output commitment do not conflict under negative network effects.

To sum up, the more firms are socially oriented the more the CSR-CDG shows sharp different SPNE outcomes than the CDG with profit-maximising firms. The main differences are reported below by distinguishing between positive and negative network effects.

1.1 Positive network effects

- If the bandwagon effect is high enough, the SPNE would be the Pareto efficient (C, C), with the CSR-CDG being an anti-prisoner's dilemma in which self-interest and mutual benefit of output commitment do not conflict if the firm social concern is sufficiently low. An increase in the firm social concern allows for the emergence of two asymmetric, Pareto efficient SPNE, (C, NC) and (NC, C). Therefore, the CSR-CDG becomes an anti-coordination game with indeterminacy instead of an anti-prisoner's dilemma with a unique SPNE in which self-interest and mutual benefit of output commitment do not conflict.
- If the bandwagon effect is lower, the SPNE would be the Pareto inefficient (C,C) with the CSR-CDG being a prisoner's dilemma in which self-interest and mutual benefit of output

commitment conflict if the firm social concern is sufficiently low. An increase in the firm social concern changes the paradigm of the game by allowing for the emergence of (C,C) as the unique Pareto efficient SPNE, so that the CSR-CDG becomes an anti-prisoner's dilemma in which self-interest and$C$ mutual benefit of output non-commitment do not conflict.

1.2 Negative network effects

- The SPNE would be the Pareto inefficient (NC,NC) with the CSR-CDG being a prisoner's dilemma in which self-interest and mutual benefit of output non-commitment conflict if the firm social concern is sufficiently low. An increase in the firm social concern changes the paradigm of the game by allowing for the emergence of (C,C) as the unique Pareto efficient SPNE, so that the CSR-CDG becomes an anti-prisoner's dilemma in which self-interest and mutual benefit of output commitment do not conflict.

We pinpoint that choosing strategically C or NC is not mere theoretical speculation. Focusing on network industries, companies can make credible announcements about their production levels. For instance, in September 2020, Wistron, one of the three Apple top contract manufacturers in India, which assembled about 200,000 second-generation iPhone SEs per month, announced a plan to scale production up to 400,000 a month by the end of that year (Reuters, 2020). Moreover, even if a company does not announce precise output levels, consumers (and rival firms) can infer production using as a proxy the (very often released, as a simple search on the websites of, inter alias, reuters.com, bloomberg.com, ft.com, economist.com reveal) announcements concerning the value of investment plans related to (1) new assembly lines at the old location, or (2) new factories.

The rich spectrum of equilibrium results offers many testable implications to econometricians. For example, it should be found that whether firms are (1) slightly socially oriented they should publicly announce the output to which they aim at committing, and (2) highly socially oriented they should not commit to an announced output level when the extent of the network externality of products is small or they should commit when the extent of the network externality of products is large.

Before proceeding further, it may be convenient to clarify the logical timing of the events, which represents a theoretical modelling synthesis in a static one-shot environment of the historical timing of the events of the CSR-CDG, which in turn affects the behaviour of consumers and firms. The logical structure used in the present work resembles those adopted by Choi & Lim (2022), describing the CDG with profit-maximising firms that compete simultaneously à la Cournot in the market stage of the game. This is because the reader may wonder whether the strategic use of output commitment in a network industry affects the consumers' expectations[2] and the nature of the competition in the output market.

First, we pinpoint that becoming a Stackelberg leader is not necessarily related to playing C or NC at the decision-making stage of the CDG, but rather because one of the two firms

[2] The expectations formation mechanisms have been extensively debated in the economic literature. The subject is extremely relevant in different kinds of dynamic models to tackle the issue of (exogenous or endogenous) fluctuations. This has been done, e.g., in cobweb models (Gori et al., 2015; Hommes, 1994), OLG models (Fanti & Gori, 2013; Gori & Sodini, 2020, 2021) or dynamic oligopolies (Bischi et al., 1998, 1999). In a context of a static oligopoly, consumers' expectations become relevant in network industries. In this regard, some recent contributions pointed this question out of the attention, e.g., Griva & Vettas (2011), Suleymanova & Wey (2012), Hagiu & Halaburda (2014), Toshimitsu (2019, 2021), Leppänen (2020), Nakamura (2021).

can or cannot produce before the rival in the logical timing of events.[3] In the first case, it becomes the leader at the market stage of the game, with the rival producing only after observing the rival's output. In the second case, firms produce simultaneously, and each firm is only able to expect the rival's production. The CSR-CDG can eventually be played sequentially (Stackelberg) or simultaneously (Cournot) at the market stage. The complete logical sequence of events in each of the two cases is as follows.

- *Cournot competition in the market stage of the CDG*. In the first decision-making stage, each firm chooses to credibly commit or not to commit to an announced output level. This is a binary choice that happens *before* consumers make their purchase decisions and then affects their expectations about the prevailing network size. Consumers are rational and have rational expectations. If at least one of the two firms is committing, the consumers' expectations about the production of the committing firm(s) are realised immediately, i.e., at the same logical time as the announcement without the need to know the prevailing equilibrium. If at least one of the two firms is not committing, consumers' expectations become binding, and they are realised in equilibrium as consumers are rational so that they can know in advance (in the logical timing of the events) the prevailing equilibrium value of the output. This hold in an intermediate stage between the decision-making stage and the market stage. In the market stage, competition between firms holds simultaneously, i.e., each of them maximises its objective given the expectations about the production of the rival. Indeed, according to the Cournot rules, market competition in the historical timing of the events can be represented by using a model in which firms make production decisions simultaneously. In this case, each firm has expectations about the production of the rival. These expectations will be realised in equilibrium (i.e., when the output reaction function cross each other). We pinpoint that the non-commitment scenario is equivalent to the case in which firms are committing but their announcement is not credible. In this case, consumers behave as if they formed expectations before the commitment and they are self-fulfilling, in turn, responding to equilibrium quantity. "Compared to the case of commitment, this implies that a low output level arises in the network market" (Choi & Lim, 2022, p. 665) when the network externality is positive.
- *Stackelberg competition in the market stage of the CDG*. The timing of the events of the firms' behaviour in the first decision-making stage and the consumers' behaviour exactly replicate those detailed in the previous point. In the market stage, competition between firms holds sequentially (instead of simultaneously), i.e., one firm moves first and becomes the leader; the rival chooses its production after observing the production of the leader and then becomes the follower. The leader considers the output reaction function of the follower. Indeed, according to the Stackelberg rules, market competition in the historical timing of the events can be represented by using a model in which firms make production decisions sequentially, but the choice about output commitment does not affect the kind of competition in the output market. This implies that the committing firm can be the leader or the follower. The structure of the four sub-games of the CSR CDR à la Stackelberg (which is reported in the appendix) is the following: if both firms are (resp. are not) committing, the leader chooses to commit (resp. not to commit) before the follower.

[3] An anonymous reviewer pinpoints some doubts on the fact firm i publicly announces its quantity, but the rival, firm j, seems ignoring this information, and then suggests considering Stackelberg competition when only one firm is unilaterally committing (the C-firm is the leader and the NC-firm is the follower). In the last section of the Appendix, we explain the reasons why we believe that both approaches are logically consistent and develop the CSR CDG by considering Stackelberg competition in the asymmetric sub-games C/NC and NC/C. Remarkably, also under this assumption the SPNE solutions of the game are qualitatively similar.

If the leader is committing and the follower does not, the leader chooses to play C before the follower makes production decisions by playing NC. If the leader is not committing and the follower is committing, the leader chooses to play NC before the follower makes production decisions by playing C.

The article proceeds as follows. Section 2 presents the basic elements of the non-cooperative CSR-CDG based on Cournot rivalry. Section 3 (resp. 4) studies the endogenous market configuration emerging in this environment by considering profits (the firm's utility) as the main decision variable in the decision-making stage. Section 5 outlines the main conclusions. The Appendix shows (i) the microeconomic foundations of the market demand under C and NC, (ii) some analytical details of the model presented in the main text, (iii) the CSR-CDG in which the strategic interaction, in the market stage of the game, between the two firms occurs sequentially (Stackelberg) instead of simultaneously (Cournot) by assuming that firm i is always the leader, (iv) the CSR-CDG à la Cournot-Stackelberg in which the committing firm is always the leader, and (v) a comparison between Cournot and Stackelberg scenarios showing, depending on the parametric set, that both kinds of competition settings can emerge endogenously.

2 The model

Consider a Cournot duopoly in which firms produce homogeneous network goods (Katz & Shapiro, 1985). The network effect (consumption externality) can be positive (e.g., mobile communications, software, internet-related activities, online social networks, fashion, etc.) or negative (e.g., traffic congestion or network congestion over limited bandwidth). Under a positive (resp. negative) externality an increasing number of users increases (resp. reduces) the individual utility and the value of the goods for each consumer, in turn, causing the so-called bandwagon (resp. snob) effect. As in Katz & Shapiro (1985), there exist two polar cases: (1) firms can commit credibly themselves to an announced output level, given the network size, before consumers maximise their utilities (C); (2) firms cannot (are unable to) commit themselves to a given output level, so that consumers form their expectations on total sales, which are fulfilled in equilibrium according to the rational expectations hypothesis (NC). These two alternatives have been taken exogenously in Katz & Shapiro (1985) and endogeneised by Choi & Lim (2022), who build on the non-cooperative CDG with complete information profit-maximising firms.

The main aim of the present article is to add the decision-making stage to a Cournot duopoly à la Katz & Shapiro (1985) and follows Choi & Lim (2022) by assuming socially oriented, instead of profit-maximising, firms, in line with the aim of modern firms framed in strategic competitive markets (as clearly reported by McKinsey, 2019, and discussed in the introduction), and building on the non-cooperative CSR-CDG.

The microeconomic foundations of the (linear) market demand with network externality and full product compatibility follow the recent article by Buccella et al. (2022a), providing details (reported in the appendix) of how the commitment scenario and the non-committing scenario affect the demand of consumers of the product of network i.

Under the alternative C for both firms, the (normalised) inverse market demand of firm i is:

$$p_i^{C/C} = 1 - q_i - q_j + n(q_i + q_j) \tag{1}$$

where $p_i^{C/C}$ is the marginal willingness to pay towards products of network i ($i, j = \{1, 2\}$, $i \neq j$) under output commitment, q_i denotes the quantity of the goods produced by firm i and $n \in (-1, 1)$ is the strength of the network effect (the higher the absolute value of n, the stronger the effect of the network). Positive (resp. negative) values of n refers to the case of positive (resp. negative) network goods.

Under the alternative NC for both firms, the inverse demand functions (see, e.g., Hoernig, 2012; Chirco & Scrimitore, 2013; Bhattacharjee & Pal, 2014; Fanti & Buccella, 2016; Buccella et al., 2022a) are as follows:

$$p_i^{NC/NC} = 1 - q_i - q_j + n(y_i + y_j) \tag{2}$$

where y_i ($i, j = \{1, 2\}$, $i \neq j$) denotes the consumers' expectations about the equilibrium output produced by firm i, and $p_i^{NC/NC}$ is the marginal willingness to pay towards products of network i under output non-commitment.[4]

The generic firm i's profit function is given by:

$$\Pi_i = (p_i - w)q_i \tag{3}$$

where $0 \leq w < 1$ is the constant average and marginal cost (i.e., the technology has constant returns to scale), which is set to zero for analytical tractability and without loss of generality.

Therefore, the profit functions in the symmetric scenarios C/C and NC/NC read respectively as follows[5]:

$$\Pi_i^{C/C} = [1 - q_i - q_j + n(q_i + q_j)]q_i \tag{4}$$

and

$$\Pi_i^{NC/NC} = [1 - q_i - q_j + n(y_i + y_j)]q_i \tag{5}$$

Following recently established literature (e.g., Goering, 2007, 2008; Lambertini & Tampieri, 2010, 2015; Kopel & Brand, 2012; Kopel et al., 2014; Lambertini et al., 2016; Fanti & Buccella, 2017, 2018; Kopel & Lamantia, 2018; Planer-Friedrich & Sahm, 2020),[6] we assume that the social concerns can be interpreted as part of the consumer surplus (CS). This, in turn, implies that the feature of a CSR firm is to be sensitive towards CS. Therefore, each firm aims at maximising a weighted sum of profits (that accrue to its owners) and the consumer surplus (that accrues to its stakeholders). More formally, we define b as the percentage of the consumer surplus captured by the stakeholders. If the stakeholders fully capture the surplus, then $b = 1$. If none of the consumer surplus accrues to the stakeholders, then $b = 0$ and the firm's objective boils down to profit maximisation. The inclusion of a fraction of the welfare of consumers can also be thought of as being part of the firm's social concern or care for consumer outcomes. Then, each firm aims at weighting the consumer surplus with $b \in (0, 1)$, which is the general case. This parameter can be exogenous or endogenous.

[4] For the sake of completeness, we report the demand for the product of network i in the asymmetric scenario in which only one firm, say firm i, credibly commits itself to an announced output level and the rival, say firm j, does not (details are reported in the appendix). This is given by the expression $p_i^{C/NC} = 1 - q_i - q_j + n(q_i + y_j)$. This is done by assuming that consumers are homogeneous and the C-firm neglects the influence on NC-firm's consumers.

[5] The profit functions of the committing firm i and the non-committing firm j in the asymmetric scenario C/NC are respectively given by the following expressions $\Pi_i^{C/NC} = [1 - q_i - q_j + n(q_i + y_j)]q_i$ and $\Pi_j^{C/NC} = [1 - q_i - q_j + n(q_i + y_j)]q_j$.

[6] On the empirical side, see Siegel & Vitaliano (2007) and Fernández-Kranz & Santaló (2010), who provide some empirical findings about CSR.

Both cases have been analysed in the literature (Fanti & Buccella, 2017; Planer-Friedrich & Sahm, 2020) especially in models dealing with CSR decision games that contrast the binary choice CSR versus profit maximisation (PM). However, this is not the main aim of the present article as all the firms are CSR-oriented and must choose to play C or NC (the commitment decision game). In this work, therefore, we assume that parameter b is taken exogenously by each firm.

The CSR-oriented firm i's objective (utility) function U_i may be specified as a simple parameterised combination of profits and consumer surplus, and it is given by:

$$U_i^{C/C} = \Pi_i^{C/C} + bCS^{C/C} \tag{6}$$

if both firms credibly commit themselves to an announced output level, and

$$U_i^{NC/NC} = \Pi_i^{NC/NC} + bCS^{NC/NC} \tag{7}$$

if both firms do not commit themselves to an announced output level, where.[7]

$$CS^{C/C} = \frac{1}{2}(1-n)(q_i+q_j)^2 \tag{8}$$

and

$$CS^{NC/NC} = \frac{1}{2}(q_i+q_j)[q_i+q_j-n(y_i+y_j)] \tag{9}$$

In the first (decision) stage of this non-cooperative game with complete information, each firm chooses to commit or not to commit themselves to an announced output level. This represents a standard binary choice, which can be based either on profits (if the decision is owners-oriented) or on the firm's utility (if the decision is stakeholders-oriented). Both objectives can be justified but follow a different narrative. The rationale to consider profits as the main decision variable in the first decision-making stage implies, following Brand and Grothe (2015), that firms aims at remaining operative in the market over time so that their profits must be non-negative (see also Friedman, 1970, who pointed out that the maximisation of profits to shareholders is the only social responsibility of businesses).

The rationale to consider a broader objective (i.e., the firm's utility) as a strategic variable is more in line with the main goal of a stakeholders-oriented firm and follows, e.g., Buccella et al. (2022b). In this regard, Kopel et al., (2014, p. 397) provide a nice interpretation to justify the strategic use of the firm's utility: a combined objective of profits and weighted consumer surplus aims at capturing "a firm type referred to as syncretic stewardship model…, referring to an organization which embraces economic as well as non-economic goals. Likewise, we can think of a hybrid organizational structure, which enables the firm to pursue profit and non-profit motives".

In the second (market) stage, each firm competes à la Cournot in the product market and then chooses the quantity to maximise the firm's utility U.

2.1 Firms commit themselves to an announced output level

Consider the symmetric sub-game in which both firms commit themselves to an announced output level (C/C). Given the CSR firm's objective and then considering the expressions in

[7] In the asymmetric scenario C/NC, firm i's and firm j's objectives are respectively given by $U_i^{C/NC} = \Pi_i^{C/NC} + bCS^{C/NC}$ and $U_j^{C/NC} = \Pi_j^{C/NC} + bCS^{C/NC}$, where the consumer surplus $CS^{C/NC} = \frac{1}{2}(q_i+q_j)[q_i+q_j-n(q_i+y_j)]$.

(4), (6) and (8), the equilibrium output must satisfy the first-order condition in the second stage of the game:

$$\frac{\partial U_i^{C/C}}{\partial q_i} = 0 \Leftrightarrow 1 - q_i(1-n)(2-b) - q_j(1-n)(1-b) = 0 \tag{10}$$

Equation (6) allows us to obtain the firm i's reaction functions:

$$q_i(q_j) = \frac{1 - q_j(1-n)(1-b)}{(1-n)(2-b)} \tag{11}$$

From (11), the reaction function of firm i is negatively sloped. This implies that products are strategic substitutes (i.e., the network effect and the social concern do not affect the standard slope of the reaction functions). This also holds for the case NC/NC.

Solving the system of output reaction functions composed by (11) and the symmetric counterpart for firm j, the exogenous Nash equilibrium output, profits and firm's utility in the symmetric sub-game C/C are respectively the following:

$$q_i^{*C/C}(n,b) = q_j^{*C/C}(n,b) = q^{*C/C}(n,b) = \frac{1}{(1-n)(3-2b)} \tag{12}$$

$$\Pi_i^{*C/C}(n,b) = \Pi_j^{*C/C}(n,b) = \Pi^{*C/C}(n,b) = \frac{1-2b}{(1-n)(3-2b)^2} \tag{13}$$

and

$$U_i^{*C/C}(n,b) = U_j^{*C/C}(n,b) = U^{*C/C}(n,b) = \frac{1}{(1-n)(3-2b)^2} \tag{14}$$

An increase in the extent of the social concern increases the firm output (as expected) and this, in turn, reduces the market price that consumers are willing to pay. The former has a positive effect on profits. The latter, instead, has a negative effect on profits and always dominates the former. Therefore, an increase in the social concern is always profit-reducing. If b becomes too large, profits are being eroded too much and become negative. Therefore, a technical (feasibility) condition that must be satisfied to guarantee positive profits in the sub-game C/C is $b < b_T^{C/C} := \frac{1}{2}$. In words, as expected, the social concerns should not be too large for ensuring a viable equilibrium industry.

The social welfare is defined as $W = CS + \Pi_i + \Pi_j$. Therefore, by considering the Nash equilibrium values for the sub-game C/C one gets:

$$CS^{*C/C}(n,b) = \frac{2}{(1-n)(3-2b)^2} \tag{15}$$

and

$$W^{*C/C}(n,b) = \frac{4(1-b)}{(1-n)(3-2b)} \tag{16}$$

As b has a positive monotonic effect on the consumer surplus (b increases production and consumer welfare) and a negative monotonic effect on profit (b increases production and reduces the market price, so that profits reduce), it has an a priori ambiguous effect on social welfare. However, the positive consumer-surplus-effect dominates the negative profit-effect so that social welfare monotonically increases with b in the sub-game C/C.

2.2 Firms do not commit themselves to an announced output level

Consider the symmetric sub-game in which both firms do not commit their output to an announced level (NC/NC). Given the expressions in (5), (7) and (9), the equilibrium output must satisfy the first-order condition in the second stage of the game:

$$\frac{\partial U_i^{NC/NC}}{\partial q_i} = 0 \Leftrightarrow 2 - 2(2-b)q_i - 2(1-b)q_j + n(2-b)(y_i + y_j) = 0 \tag{17}$$

Equation (17) allows us to obtain the firm i's reaction functions:

$$q_i(q_j, y_i, y_j) = \frac{2 - 2(1-b)q_j + n(2-b)(y_i + y_j)}{2(2-b)} \tag{18}$$

By imposing the usual "rational expectation condition" such that $y_i = q_i$ and $y_j = q_j$ (Choi & Lim, 2022; Hoernig, 2012; Katz & Shapiro, 1985) and solving the system of output reaction functions composed by (18) and its counterpart for firm j, the exogenous Nash equilibrium output, profits and firm's utility in the symmetric sub-game NC/NC are respectively given by the following expressions:

$$q_i^{*NC/NC}(n,b) = q_j^{*NC/NC}(n,b) = q^{*NC/NC}(n,b) = \frac{1}{1-b+(1-n)(2-b)} \tag{19}$$

$$\Pi_i^{*NC/NC}(n,b) = \Pi_j^{*NC/NC}(n,b) = \Pi^{*NC/NC}(n,b) = \frac{1-b(2-n)}{[1-b+(1-n)(2-b)]^2} \tag{20}$$

and

$$U_i^{*NC/NC}(n,b) = U_j^{*NC/NC}(n,b) = U^{*NC/NC}(n,b) = \frac{1-bn}{[1-b+(1-n)(2-b)]^2} \tag{21}$$

Remarks like those detailed above hold here about the effects on consumer surplus and profits. If b becomes too large profits are being eroded eventually becoming negative. The feasibility condition that must be satisfied to guarantee positive profits in the sub-game NC/NC is $b < b_T^{NC/NC}(n) := \frac{1}{2-n}$. This condition is not binding (compared to $b_T^{C/C}$) if the network externality is positive. It becomes binding if the network externality is negative.

By considering the Nash equilibrium values for the sub-game NC/NC one gets the consumer surplus and the social welfare:

$$CS^{*NC/NC}(n,b) = \frac{2(1-n)}{[1-b+(1-n)(2-b)]^2} \tag{22}$$

and

$$W^{*NC/NC}(n,b) = \frac{2(1-n)(1-b)}{[1-b+(1-n)(2-b)]^2} \tag{23}$$

Given the threshold that must hold to guarantee the positivity of profits, there exists a positive monotonic effect of an increase in b on the consumer surplus and a negative monotonic effect of an increase in b on profits. Like the previous case, the social welfare function increases monotonically with b, so that (by accounting also for threshold $b < b_T^{C/C}$) the positive consumer surplus effect dominates at the societal level.

2.3 The asymmetric sub-game: one firm commits itself to an announced output level, the rival does not

Let now us consider the sub-game in which firm i can commit itself to an announced output level, given the network size, whereas the rival, firm j, takes as given the consumers' expectations about the size of the network. This implies that the profit functions of firm i and firm j can respectively be written as follows:

$$\Pi_i^{C/NC} = [1 - q_i - q_j + n(q_i + y_j)]q_i \tag{24}$$

and

$$\Pi_j^{C/NC} = [1 - q_j - q_i + n(y_j + q_i)]q_j \tag{25}$$

The consumer surplus, instead, is given by (see also Footnote 6):

$$CS^{C/NC} = \frac{1}{2}(q_i + q_j)[q_i + q_j - n(q_i + y_j)] \tag{26}$$

Therefore, the utility of the stakeholders of firm i and firm j are respectively given by:

$$U_i^{C/NC} = \Pi_i^{C/NC} + bCS^{C/NC} \tag{27}$$

and

$$U_j^{C/NC} = \Pi_j^{C/NC} + bCS^{C/NC} \tag{28}$$

The maximisation of the expressions in (27) and (28) allows us to get the reaction functions of firm i and firm j in the asymmetric sub-game C/NC, that is:

$$q_i(q_j, y_j) = \frac{2 - [2(1-b) + bn]q_j + n(2-b)y_j}{2(1-n)(2-b)} \tag{29}$$

and

$$q_j(q_i, y_j) = \frac{2 - q_i[1 - b - n + (1-n)(1-b)] + n(2-b)y_j}{2(2-b)} \tag{30}$$

By assuming $y_j = q_j$ and solving the system of output reaction functions (29) and (30), one gets the exogenous equilibrium values of the quantity, profits and firm's utility of both the commitment-firm i and no-commitment-firm j, that is

$$q_i^{*C/NC}(n, b) = \frac{2 - bn}{(1-n)[2(1-n) + 4(1-b) + bn]} \tag{31}$$

$$q_j^{*C/NC}(n, b) = \frac{2(1-n) + bn}{(1-n)[2(1-n) + 4(1-b) + bn]} \tag{32}$$

where $q_j^{*C/NC}(n, b) > q_i^{*C/NC}(n, b)$ if $n < 0$ and $q_j^{*C/NC}(n, b) < q_i^{*C/NC}(n, b)$ if $n > 0$, and

$$\Pi_i^{*C/NC}(n, b) = \frac{(2 - bn)[2 - b(4-n)]}{(1-n)[2(1-n) + 4(1-b) + bn]^2} \tag{33}$$

$$\Pi_j^{*C/NC}(n, b) = \frac{[2(1-n) + bn][2 - b(4-n)]}{(1-n)[2(1-n) + 4(1-b) + bn]^2} \tag{34}$$

and

$$U_i^{*C/NC}(n, b) = \frac{4 - bn(4-n)(2-b)}{(1-n)[2(1-n) + 4(1-b) + bn]^2} \tag{35}$$

$$U_j^{*C/NC}(n,b) = \frac{4(1-n) + bn[4(1-b) + bn]}{(1-n)[2(1-n) + 4(1-b) + bn]^2} \tag{36}$$

The feasibility condition that must be satisfied to guarantee positive profits in the sub-game C/NC is $b < b_T^{NC/NC}(n) := \frac{1}{2-n}$. This condition is not binding (compared to $b < b_T^{C/C}$ and $b < b_T^{NC/NC}(n)$) in both cases of positive and negative network externalities. In the former case, $b_T^{C/C}$ is binding. In the latter case, $b_T^{NC/NC}(n)$ is binding. This is of course relevant to have meaningful sub-games when studying the emergence of SPNE of the non-cooperative CSR-CDG.

By considering the Nash equilibrium values for the sub-game C/NC one gets the consumer surplus and the social welfare:

$$CS^{*C/NC}(n,b) = \frac{2(2-n)^2}{(1-n)[2(1-n) + 4(1-b) + bn]^2} \tag{37}$$

and

$$W^{*C/NC}(n,b) = \frac{2(1-b)[8 - n(4-n)(1+b)]}{(1-n)[2(1-n) + 4(1-b) + bn]^2} \tag{38}$$

The welfare effects of an increase in b on social welfare follow the mechanism described so far but (passing through the consumer surplus and profits) but are now more articulated, so that the positive consumer-surplus effect can dominate (be dominated) by the negative profit-effect depending on the size of the network externality. This will clearer later in the article.

Like Katz & Shapiro (1985) (1) there exists a strict ranking for the output levels depending on whether the network is positive or negative (Ranking 1 and Ranking 2); and (2) there exists a strict ranking for the consumer surplus depending on whether the network is positive or negative (Ranking 3 and Ranking 4). These rankings hold by assuming that the feasibility condition $b < b_T^{C/C}$ is fulfilled for any $n > 0$ and the feasibility condition $b < b_T^{NC/NC}(n)$ is fulfilled for any $n < 0$.

Ranking 1. Let n be positive. Then, $q_i^{*C/NC}(n,b) > q^{*C/C}(n,b) > q^{*NC/NC}(n,b) > q_j^{*C/NC}(n,b)$. The positive network externality fosters the production of the C-firm.

Ranking 2. Let n be negative. Then, $q_j^{*C/NC}(n,b) > q^{*NC/NC}(n,b) > q^{*C/C}(n,b) > q_i^{*C/NC}(n,b)$. The negative network externality fosters the production of the NC-firm.

Ranking 3. Let n be positive. Then, $CS^{*C/C}(n,b) > CS^{*C/CN}(n,b) > CS^{*NC/NC}(n,b)$. When the network externality is positive, consumers are better off if firms can commit themselves to an announced output level. This is because the output of the C-firm is larger than the production of the NC-firm.

Ranking 4. Let n be negative. Then, $CS^{*NC/NC}(n,b) > CS^{*C/CN}(n,b) > CS^{*C/C}(n,b)$. When the network externality is negative, consumers are better off if firms cannot commit themselves to an announced output level. This is because the output of the NC-firm is larger than the production of the C-firm.

In addition, there also exists a strict ranking for social welfare depending on whether the network externality is positive or negative (Ranking 5 and Ranking 6), with relevant possible policy implications given the emergence of the different SPNE of the game, as will be shown in Sect. 3.

Ranking 5. Let n be positive. Then, $W^{*C/NC}(n,b) > W^{*C/C}(n,b) > W^{*NC/NC}(n,b)$.

Ranking 6. Let n be negative. Then, $W^{*NC/NC}(n,b) > W^{*C/C}(n,b) > W^{*C/NC}(n,b)$.

3 Equilibrium analysis: the endogenous market configuration

This section aims at studying the firms' incentive to play C or NC in the first decision-making stage by considering two main parameters: the extent of the (positive or negative) network effect and the extent of the social concerns measuring the firm's interest towards the consumer welfare. This is done by considering either profits or the firm's utility as the main decision variable (as was discussed in Sect. 2). In the former case, the decision is owner oriented. In the latter case, the decision is stakeholder oriented (see Kopel et al., 2014; Kopel & Lamantia, 2018).

The feasibility conditions that must hold to have well-defined equilibria in pure strategies for every strategic profile (one for each player) in both cases in which the decision is owner oriented and stakeholder oriented are: $b < b_T^{C/C}$ if the network externality is positive ($n > 0$), and $b < b_T^{NC/NC}(n)$ if the network externality is negative ($n < 0$).

3.1 Profit as the decision variable in the first decision-making stage of the CSR-CDG

This section examines the decision-making stage, in which CSR firms choose whether to commit themselves to an announced output level in a non-cooperative quantity-setting environment à la Cournot with network externalities. The decision is assumed to be owner-oriented and then profits represent the decision variable of each firm. In this regard, Table 1 summarises the payoff matrix with the profit functions in the symmetric and asymmetric sub-games, as was detailed in the previous section (see Eqs. (13), (20), (33) and (34)).

To derive all the possible equilibria of the game one must study the sign of the profit differentials for $i = \{1, 2\}, i \neq j$, that is:

$$\Delta\Pi_A(n,b) := \Pi_i^{*C/NC}(n,b) - \Pi_i^{*NC/NC}(n,b) \tag{39}$$

$$\Delta\Pi_B(n,b) := \Pi_i^{(*NC/NC)}(n,b) - \Pi_i^{(*C/C)}(n,b) \tag{40}$$

and

$$\Delta\Pi_C(n,b) := \Pi_i^{NC/NC}(n,b) - \Pi_i^{C/C}(n,b) \tag{41}$$

Table 1 The CSR-CDG à la Cournot (payoff matrix). Profit

Firm $j \rightarrow$ Firm $i \downarrow$	C	NC
C	$\Pi_i^{*C/C}(n,b), \Pi_j^{*C/C}(n,b)$	$\Pi_i^{*C/NC}(n,b), \Pi_j^{*C/NC}(n,b)$
NC	$\Pi_i^{*NC/C}(n,b)\Pi_j^{*NC/C}(n,b)$	$\Pi_i^{*NC/NC}(n,b), \Pi_j^{*NC/NC}(n,b)$

The first threshold defines the incentive of firm i to deviate from C to NC when its sign is negative (and vice versa when its sign is positive) when the rival, firm j, is playing NC. The second threshold defines the incentive of firm i to deviate from NC to C when its sign is negative (and vice versa when its sign is positive) when the rival, firm j, is playing C. The third threshold determines the Pareto efficiency/inefficiency of a symmetric SPNE.

From (39), the sign of $\Delta\Pi_A(n, b)$ is positive (resp. negative) if $n > n_A(b)$ (resp. $n < n_A(b)$, where $n_A(b)$ is the threshold value of the extent of the network externality (as a function of the extent of the firm's social concern) such that $\Delta\Pi_A(n, b) = 0$. We do not report the value of $n_A(b)$ as it is analytically cumbersome and not informative.

From (40), the sign of $\Delta\Pi_B(n, b)$ is negative (resp. positive) if $n > n_B(b)$ (resp. $n < n_B(b)$, where

$$n_B(b) := \frac{2(3 - 2b)[b(5 - 2b) - 1]}{(2 - b)[b(3 - 2b) + 1]} \tag{42}$$

is the threshold value of the extent of the network externality (as a function of the extent of the firm's social concern) such that $\Delta\Pi_B(n, b) = 0$.

From (41), the sign of $\Delta\Pi_C(n, b)$ is negative (resp. positive) if $n > n_C(b)$ (resp. $n < n_C(b)$, where

$$n_C(b) := \frac{(1 + 2b)(3 - 2b)}{4 - b^2 + b(1 - b)} \tag{43}$$

is the threshold value of the extent of the network externality (as a function of the extent of the firm's social concern) such that $\Delta\Pi_C(n, b) = 0$.

If $b = 0$ results about the prevailing SPNE of the game exactly replicates those of the CDG with profit-maximising firms: if $0 < n < 0.75$, the unique SPNE is the Pareto inefficient (C, C) and the CDG is a prisoner's dilemma, in which C is the dominant strategy and self-interest and mutual benefit of output commitment conflict; if $0.75 < n < 1$, the unique SPNE is the Pareto efficient (C, C) and the CDG is an anti-prisoner's dilemma (deadlock), in which C is the dominant strategy and self-interest and mutual benefit of output commitment do not conflict; if $-1 < n < 0$, the unique SPNE is the Pareto inefficient (NC, NC) and the CDG is a prisoner's dilemma, in which NC is the dominant strategy and self-interest and mutual benefit of output non-commitment conflict. If $b > 0$, the SPNE scenarios of the CSR-CDG sharply change and are summarised in Proposition 1.

Proposition 1 The endogenous market structure of the CSR-CDG when profit is the decision variable is the following.

Let n be positive.

[1] If $0 \leq b < 0.219$ then (1.1) (C, C) is the unique Pareto inefficient SPNE and the CSR-CDG is a prisoner's dilemma for any $0 \leq n < n_C(b)$, in which self-interest and mutual benefit of output commitment conflict; (1.2) (C, C) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $n_C(b) < n < 1$, in which self-interest and mutual benefit of output commitment do not conflict.

[2] If $0.219 < b < 0.313$ then (2.1) (NC, NC) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $0 \leq n < n_B(b)$, in which self-interest and mutual benefit of output non-commitment do not conflict; (2.2) (C, C) and (NC, NC) are two symmetric SPNE (NC payoff dominates C), and the CSR-CDG is a coordination game for any $n_B(b) < n < n_A(b)$; (2.3) (C, C) is the

unique Pareto inefficient SPNE and the CSR-CDG is a prisoner's dilemma for any $n_A(b) < n < n_C(b)$, in which self-interest and mutual benefit of output commitment conflict; (2.4) (C, C) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $n_C(b) < n < 1$, in which self-interest and mutual benefit of output commitment do not conflict.

[3] If $0.313 < b < 0.393$ then (3.1) (NC, NC) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $0 \leq n < n_A(b)$, in which self-interest and mutual benefit of output non-commitment do not conflict; (3.2) (C, NC) and (NC, C) are two asymmetric SPNE, and the CSR-CDG is an anti-coordination game for any $n_A(b) < n < n_B(b)$; (3.3) (C, C) is the unique Pareto inefficient SPNE and the CSR-CDG is a prisoner's dilemma for any $n_B(b) < n < n_C(b)$, in which self-interest and mutual benefit of output commitment conflict; (3.4) (C, C) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $n_C(b) < n < 1$, in which self-interest and mutual benefit of output commitment do not conflict.

[4] If $0.393 < b < 0.4013$ then (4.1) (NC, NC) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $0 \leq n < n_A(b)$, in which self-interest and mutual benefit of output non-commitment do not conflict; (4.2) (C, NC) and (NC, C) are two asymmetric SPNE, and the CSR-CDG is an anti-coordination game for any $n_A(b) < n < n_B(b)$; (4.3) (C, C) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $n_B(b) < n < 1$, in which self-interest and mutual benefit of output commitment do not conflict.

[5] If $0.4013 < b < 0.5$ then (5.1) (NC, NC) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $0 \leq n < n_A(b)$, in which self-interest and mutual benefit of output non-commitment do not conflict; (5.2) (C, NC) and (NC, C) are two asymmetric SPNE, and the CSR-CDG is an anti-coordination game for any $n_A(b) < n < 1$.

Let n be negative.

[6] If $0 \leq b < 0.115$ then (NC, NC) is the unique Pareto inefficient SPNE and the CSR-CDG is a prisoner's dilemma for any $0 > n > -1$, in which self-interest and mutual benefit of output non-commitment conflict.

[7] If $0.115 < b < 0.168$ then (7.1) (NC, NC) is the unique Pareto inefficient SPNE and the CSR-CDG is a prisoner's dilemma for any $0 > n > -n_B(b)$, in which self-interest and mutual benefit of output non-commitment conflict; (7.2) (C, C) and (NC, NC) are two symmetric SPNE (C payoff dominates NC) and the CSR-CDG is a coordination game for any $-n_B(b) > n > -1$.

[8] If $0.168 < b < 0.219$ then (8.1) (NC, NC) is the unique Pareto inefficient SPNE and the CSR-CDG is a prisoner's dilemma for any $0 > n > -n_B(b)$, in which self-interest and mutual benefit of output non-commitment conflict; (8.2) (C, C) and (NC, NC) are two symmetric SPNE (C payoff dominates NC) and the CSR-CDG is a coordination game for any $-n_B(b) > n > -n_A(b)$; (8.3) (C, C) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $-n_A(b) > n > -1$, in which self-interest and mutual benefit of output commitment do not conflict.

[9] If $0.219 < b < 0.333$ then (C, C) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $0 > n > -1$, in which self-interest and mutual benefit of output commitment do not conflict.

[10] If $0.333 < b < b_T^{NC/NC}(n)$ then (C, C) is the unique Pareto efficient SPNE and the CSR-CDG is an anti-prisoner's dilemma (deadlock) for any $0 > n > -\frac{1-2b}{b}$, in which self-interest and mutual benefit of output commitment do not conflict.

Proof. Appendix.

The economic intuition behind the results of Proposition 1 is simple and proceeds following the standard mechanism of price/quantity on the firm's profit. The results are driven by the interaction between the network size and the degree of the firm's social concern (Fig. 1). If firms are profit-maximising entities ($b = 0$), the model replicates the outcomes of Choi & Lim (2022), in which the commitment scenario emerges when the network externality is positive (committing to an announced output level is favoured by the bandwagon effect of the positive network externalities), and the no-commitment scenario emerges when the network externality is negative (not committing to an announced output level is favoured by the snob effect of the negative network externalities).

Unlike this case, if firms are CSR-oriented and the network externality is positive, an increase in the degree of social concern sharply changes the SPNE and then the welfare outcomes (Fig. 2, Panels A-B), working out favouring (i) the no commitment scenario (which is Pareto efficient for firms but not consumers oriented) if the extent of the network externality is relatively small, (ii) the mixed scenario, in which only firm aims at committing to an announced output level, but he rival does not if the extent of the network externality becomes larger. This outcome also leads to the highest social welfare, although it does not represent

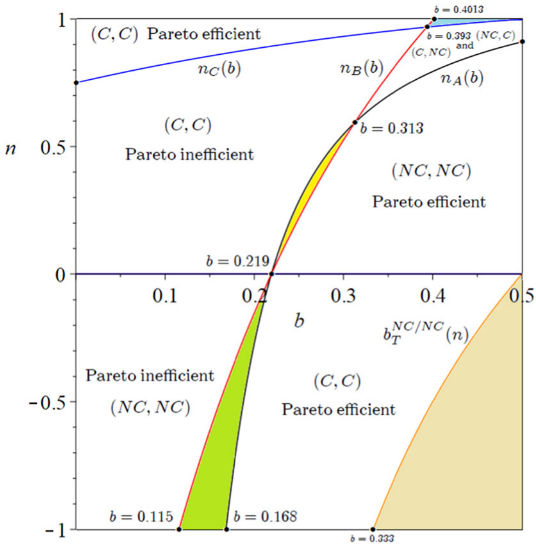

Fig. 1 The CSR-CDG: SPNE when n and b vary and the firm's profit is the decision variable. The sand-coloured region represent the unfeasible parameter space, which is bounded by the threshold $b < \frac{1}{2-n} := b_T^{NC/NC}(n)$. The feasibility condition must be satisfied to guarantee positive profits in the sub-game NC/NC. Yellow region: (C, C) and (NC, NC) are two symmetric SPNE (NC payoff dominates C) and the CSR-CDG is a coordination game. Green region: (C, C) and (NC, NC) are two symmetric SPNE (C payoff dominates NC) and the CSR-CDG is a coordination game. Light-blue region: (C, NC) and (NC, C) are two asymmetric SPNE with the C-firm getting the best outcome and the CSR-CDG is an anti-coordination game. This accords with the region below, in which the CSR-CDG is an anti-coordination game and C-firm continues to get the best outcome

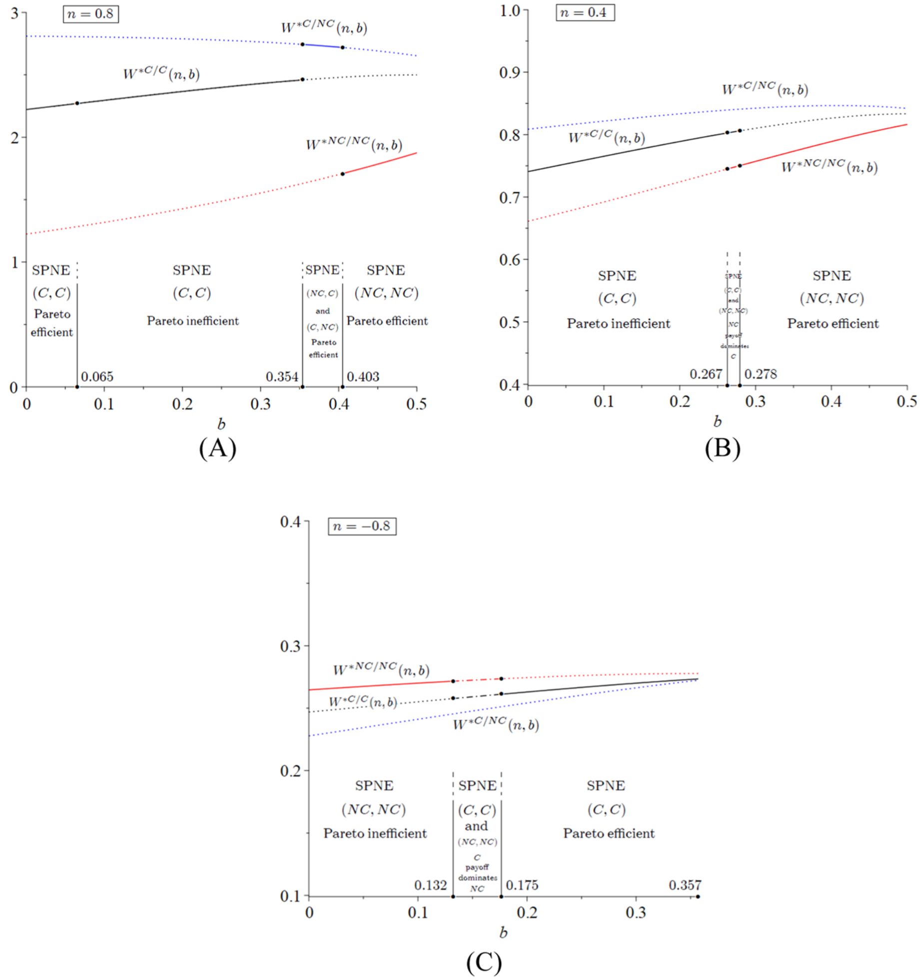

Fig. 2 Equilibrium social welfare as a function of b. The black (resp. red) [resp. blue] curve refers to the case C/C (resp. NC/NC) [resp. C/NC]. The solid lines represent the social welfare prevailing at the SPNE. The dash-dotted lines represents the social welfare level that can emerging when there is indeterminacy, i.e., a multiplicity of SPNE in pure strategies and the CSR-CDG is a coordination game. The dotted lines are fictitious and are drawn only for comparison purposes. Panel A: $n = 0.8$; Panel B: $n = 0.4$; Panel C: $n = -0.8$

a win–win solution from a societal perspective as consumers would be better off by under C/C. The extent of social concern, therefore, favours the emergence of the no-commitment scenario. This is because an increase in b increases the production of the firms in all the strategic profiles, but the percentage increase in the output of the C-firms is larger than the percentage increase in the output of the NC-firms. Therefore, the profits of the C-firm reduce more than the profits of the NC-firm and this, in turn, tends to let NC become the dominant strategy if the network externality is not too large. When the extent of the externality becomes larger the effects of an increase in b are different. First, there is a standard positive effect of production in all the strategic profiles and this, in turn, reduces the corresponding market price. However, the percentage increase in output of the C-firm is larger than the percentage increase in output of the NC-firm. In addition, the reduction in the market price when firms are committing is larger than when firms are not committing. This, in turn, leads to a reduction in profits under C and an increase in profits under NC with the emergence of an anti-coordination game, in which each firm plays the opposite strategy of the rival in equilibrium.

Things change under negative network externalities (Figs. 1 and 2, Panel C). In this case, an increase in the extent of the social concern works out in the direction of increasing the importance of the snob effect, so that an increase in the (absolute value of) strength of the network externality tends to shift inwards the market demand at the highest intensity. This has negative effects on profits in every strategic profile. However, the reduction in output experienced under C is larger than under NC letting the no-commitment scenario become the dominant strategy. However, as firms under NC produce more (and sell at a lower price) than under C there is a joint incentive to deviate towards the commitment scenario, but no one has the unilateral incentive to comply with the (non-cooperative) agreement, so each firm is entrapped in the poverty trap to do not commit to an announced output level. As the NC-firm produces more than the C-firm when n is negative, an increase in the extent of b has exactly the opposite effects as those described above so that the reduction in profits of the NC-firm is larger than the reduction in profits of the C-firm and C becomes the dominant strategy for larger values of b. As consumers are now always better off under NC there are no win–win solutions also in this case.

The policy recipes are perhaps quite articulated (there are no win–win situations, in which both consumers and firms are better off) and counterintuitive. If the network externality is positive, increasing the importance of the social concern favours the emergence of no-commitment scenario, which is detrimental to consumers and social welfare. Therefore, if the extent of the network externality is relatively high, the public authority should not favour the emergence of a credible commitment for all the firms as the best outcome at the societal level is a situation in which one firm commits to an announced output level, but the rival does not (though consumers would be better off under the full commitment scenario), and the extent of the social concern should be set an intermediate level; if the extent of the network externality is relatively low, the public authority should favour the emergence of a credible commitment for all the firms (though the society would be better off under the mixed scenario), but increasing the importance of the social concern is detrimental to firms. If the network externality is negative, increasing the importance of the social concern favours the emergence of the commitment scenario, which is detrimental to consumers and social welfare as society would be better off under no commitment. Therefore, the extent of the social concern should be set at an intermediate level.

3.2 Stakeholders' utility as the decision variable in the first decision-making stage of the CSR-CDG

Unlike Sect. 3.1, this section examines the first stage of the game by assuming that the choice to commit or not commit is stakeholder-oriented and then the firm's utility represents the decision variable. In this regard, Table 2 summarises the payoff matrix with the firm's utility functions in the symmetric and asymmetric sub-games (see Eqs. (14), (21), (35) and (36)).

To derive all the possible equilibria of the game one must study the sign of the firm's utility differentials for $i = \{1, 2\}$, $i \neq j$, that is:

$$\Delta U_A(n, b) := U_i^{*C/NC}(n, b) - U_i^{*NC/NC}(n, b) \tag{44}$$

$$\Delta U_B(n, b) := U_i^{*NC/C}(n, b) - U_i^{*C/C}(n, b) \tag{45}$$

Table 2 The CSR-CDG à la Cournot (payoff matrix). Firm's utility

Firm $j \rightarrow$ Firm $i \downarrow$	C	NC
C	$U_i^{*C/C}(n,b), U_j^{*C/C}(n,b)$	$U_i^{*C/NC}(n,b), U_j^{*C/NC}(n,b)$
NC	$U_i^{*NC/C}(n,b), U_j^{*NC/C}(n,b)$	$U_i^{*NC/NC}(n,b), U_j^{*NC/NC}(n,b)$

and

$$\Delta U_C(n,b) := U_i^{*NC/NC}(n,b) - U_i^{*C/C}(n,b) \tag{46}$$

From (44), the sign of $\Delta U_A(n,b)$ is positive (resp. negative) if $n > n_A^U(b)$ (resp. $n < n_A^U(b)$, where $n_A^U(b)$ is the threshold value of the extent of the network externality (as a function of the extent of the firm's social concern) such that $\Delta U_A(n,b) = 0$. We do not report the value of $n_A^U(b)$ as it is analytically cumbersome and not informative.

From (45), the sign of $\Delta U_B(n,b)$ is negative (resp. positive) if $n > n_B^U(b)$ (resp. $n < n_B^U(b)$, where

$$n_B^U(b) := \frac{(3-2b)(1-2b)}{b^2-b-1} \tag{47}$$

is the threshold value of the extent of the network externality (as a function of the extent of the firm's social concern) such that $\Delta U_B(n,b) = 0$.

From (46), the sign of $\Delta U_C(n,b)$ is negative (resp. positive) if $n > n_C^U(b)$ (resp. $n < n_C^U(b)$, where

$$n_C^U(b) := \frac{(1+2b)(3-2b)}{4-b(9-4b)} \tag{48}$$

is the threshold value of the extent of the network externality (as a function of the extent of the firm's social concern) such that $\Delta U_C(n,b) = 0$.

To avoid lengthening the article further we do not write an explicit proposition to detail the SPNE outcomes and the endogenous market structure of the CSR-CDG emerging when the firm's utility is the decision variable. However, without loss of generality, results are reported Fig. 3, which resembles Fig. 1 above. It is interesting to observe that, as expected, when the network externality is positive and the decision is stakeholder-oriented in the first stage of the CSR-CDG, production weights more in the firm's objective than when profit is the decision variable, and this favour the emergence of the commitment scenario irrespective of the level of the social concern, in sharp contrast with the previous case. This "opposite outcome" is also observed when the network externality is negative. In this case, in fact, the no-commitment scenario emerges for a wider parameter configuration than when profit is the decision variable. This is because, in this case, production is higher under no commitment than under commitment.

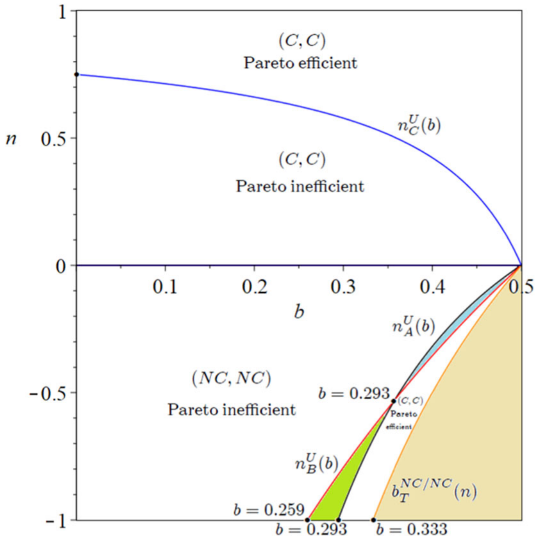

Fig. 3 The CSR-CDG: SPNE when n and b vary and the firm's utility is the decision variable. The sand-coloured region represent the unfeasible parameter space, which is bounded by the threshold $b < \frac{1}{2-n} := b_T^{NC/NC}(n)$. The feasibility condition must be satisfied to guarantee positive profits in the sub-game NC/NC. Green region: (C,C) and (NC, NC) are two symmetric SPNE (C payoff dominates NC) and the CSR-CDG is a coordination game. Light-blue region: (C, NC) and (NC, C) are two asymmetric SPNE with the NC-firm getting the best outcome and the CSR-CDG is an anti-coordination game

4 Conclusions

The increasingly rapid spread of network goods, which is making consumption externalities a major issue, and Corporate Social Responsibility, which is making socially oriented firms more and more important in the market, is becoming a stylised fact especially in strategic competitive contexts.

Under network consumption externalities, consumers must either form (rational) expectations or know the information disclosed by the firms about the size of competing networks. This article aimed at directly extending Katz & Shapiro (1985) and Choi & Lim (2022) by considering CSR instead of profit-maximising firms. In doing this, it endogeneised the choice to credibly commit to an announced output level, given the network size, before consumers make their purchasing decisions. The article then considers output commitment as a strategic device that depends on the extent of the network size and the degree of firms' social concern capturing the importance of consumer welfare in production. Unlike Choi & Lim (2022), who study the non-cooperative, multi-stage, commitment decision game with profit-maximising firms, the present work considers the non-cooperative, multi-stage, CSR commitment decision game.

Our non-cooperative game allows us to observe several equilibrium scenarios, in sharp contrast to Choi & Lim (2022), ranging from the prisoner's dilemma to the anti-prisoner's dilemma (deadlock), passing through the anti-coordination game and offers some testable implications to econometricians about the market configuration that can have relevant welfare effects, which are driven by the extent of the firm's social concern.

The present work belongs to an emerging industrial organization literature (e.g., Griva & Vettas, 2011; Hagiu & Halaburda, 2014; Leppänen, 2020; Nakamura, 2021; Suleymanova & Wey, 2012; Toshimitsu, 2019, 2021) on the consumers' expectations and network externalities and may be extended in some directions, for instance by adding R&D investments, managerial delegation, unions and so on.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Appendix

The microeconomic foundations of the market demand when both firms do not commit themselves to an announced output level

This section briefly presents the microeconomic foundations of the market demand reported in Eq. (2) in the main text. It tackles this issue by considering firms that do not commit themselves to an announced output level before consumers make their purchase decisions. This follows the mechanism detailed by Katz & Shapiro (1985) in the model presented in the main body of the text of their contribution with consumers having rational expectations.

On the supply side, the economy is bi-sectorial with a competitive sector producing the numeraire good m and a duopolistic industry in which firm i and firm j ($i = \{1, 2\}$; $i \neq j$) produce network goods of variety (network) i and variety (network) j, respectively. These goods are perceived as homogeneous by customers.

On the demand side, there are identical consumers with preferences described by the utility function $V = Z + m$, which is linear in the numeraire good m. The utility V is maximised subject to the budget constraint $p_i q_i + p_j q_j + m = R$, where Z is a twice continuously differentiable function, q_i and q_j are the control variables of the problem, p_i and p_j represent the price of (i.e., the marginal willingness to pay of the representative consumer for) product of variety i and variety j, respectively, and R is the consumer's exogenous nominal income. This income is high enough to avoid the existence of income effects on the demand of q_i and

q_j (i.e., the goods enter non-linearly in V). In this regard, in fact, the utility function V is quasi-linear in m so that all the related properties about the demand of m and that of q_i and q_j hold (Amir et al., 2017; Choné & Linnemer, 2020).

Different from the traditional IO literature, we assume the existence of network externalities in consumption, i.e., one person's demand also depends on the demand of other customers. In this regard, y_i represents an external effect denoting the consumers' expectations about firm i's equilibrium total sales.

The simple mechanism of network effects described so far follows Katz & Shapiro (1985) and resembles several recent articles belonging to the IO literature framed in strategic competitive markets using with linear market demand (Buccella et al., 2022a, 2022b; Chirco & Scrimitore, 2013; Choi & Lim, 2022; Hoernig, 2012; Naskar & Pal, 2020; Shrivastav, 2021; Song & Wang, 2017).

The function Z follows the specification of a quadratic utility[8]:

$$Z = q_i + q_j - \frac{1}{2}(q_i^2 + q_j^2 + 2q_i q_j) + n[q_i(y_i + y_j) + q_j(y_j + y_i)] - \frac{n}{2}(y_i^2 + y_j^2 + 2y_i y_j) \tag{A.1}$$

where $-1 < n < 1$ is the strength of the network effect ($n = 0$ represents the standard case of non-network goods). Positive (resp. negative) values of n reflect a positive (resp. negative) consumption externality. Though several network industries exhibit positive externalities (mobile communications, software, internet-related activities, online social networks, fashion, etc.), as a greater number of users contribute to increase the value of the product to each consumer (there exists a positive feedback loop if the network becomes more valuable), one may think about real-world industries that exhibit both kinds of externalities. In this regard, the automobile industry is worth to be mentioned. This industry might also show negative consumption externalities as the more cars are sold, the greater the traffic congestion, parking difficulties and other related issues are. Therefore, a negative network externality implies that an increasing number of users reduces the value of the goods for each consumer (for instance, traffic congestion or network congestion over limited bandwidth). The term $y_i + y_j$ represents the expected effective network size of firm i's consumers and of firm j's consumers.

The solution of the maximisation problem of the surplus by the representative consumer following Amir et al. (2017) and Choné & Linnemer (2020) gives the following linear inverse demand for product of network i:

$$p_i^{NC/NC} = 1 - q_i - q_j + n(y_i + y_j) \tag{A2}$$

8 The network externality effect in the utility function Eq. (A.1) can more generally be written as $n[q_i(y_i + k_i y_j) + q_j(y_j + k_j y_i)] - \frac{n}{2}[y_i^2 + y_j^2 + (k_i + k_j)y_i y_j]$, where the parameter $k_i \in [0, 1]$ measures the degree of compatibility of the network of product j towards the network of product i. Pairwise, the parameter $k_j \in [0, 1]$ measures the degree of compatibility of the network of product i with the network of product j. Considering the case of common compatibility $k_i = k_j = k$, the network externality term becomes $n[q_i(y_i + ky_j) + q_j(y_j + ky_i)] - \frac{n}{2}(y_i^2 + y_j^2 + 2ky_i y_j)$, which can be further simplified by assuming full compatibility between the products of the two networks, i.e., $k_i = k_j = 1$, so that $n[q_i(y_i + y_j) + q_j(y_j + y_i)] - \frac{n}{2}(y_i^2 + y_j^2 + 2y_i y_j)$. If the degree of compatibility between the products of the two networks was zero, i.e., $k_i = k_j = 0$, we would have $n(q_i y_i + q_j y_i) - \frac{n}{2}(y_i^2 + y_j^2)$.

The microeconomic foundations of the market demand when both firms commit themselves to an announced output level

This section considers the opposite situation in which both firms credibly commit themselves to an announced output level before consumers make their purchase decision. This follows the mechanism detailed by Katz & Shapiro (1985) in an ad hoc appendix of their contribution. In this situation, consumers know the quantity that will be produced by that both firms (the consumers' expectations about the production of the two committing firms are realised immediately, i.e., at the same logical time as the announcement without the need to know the prevailing equilibrium). As consumers consider the committed output of both firms, substituting out $y_i = q_i$ and $y_j = q_j$ into Eq. (A.2) gives the linear inverse demand for product of network i under C/C, that is:

$$p_i^{C/C} = 1 - q_i - q_j + n(q_i + q_j) \tag{A3}$$

The microeconomic foundations of the market demand when one firm credibly commits itself to an announced output level and the rival does not

In the asymmetric scenario in which only firm, say firm i, can credibly commit itself to an announced output level, consumers know the quantity that will be produced by that firm (the consumers' expectations about the production of the committing firm are realised immediately, i.e., at the same logical time as the announcement without the need to know the prevailing equilibrium), whereas they have expectations about the network size of the non-committing firm, say firm j. In this case, consumers' expectations about the quantity produced by firm j become binding, and they are realised in equilibrium. Consumers are rational so that they can know in advance (in the logical timing of the events) the prevailing equilibrium value of the output. Knowing that consumers consider the committed output only of firm i, substituting out $y_i = q_i$ into Eq. (A.2) gives the linear inverse demand for product of network i under C/NC, that is:

$$p_i^{C/NC} = 1 - q_i - q_j + n(q_i + y_j) \tag{A4}$$

Proof of Proposition 1

Let n be positive. If $0 \leq b < 0.219$ then $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) > 0$ for any $0 \leq n < n_C(b)$; $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) < 0$ for any $n_C(b) < n < 1$ so that Point [1] holds. If $0.219 < b < 0.313$ then $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) > 0$ for any $0 \leq n < n_B(b)$; $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) > 0$ for any $n_B(b) < n < n_A(b)$; $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) > 0$ for any $n_A(b) < n < n_C(b)$; $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) < 0$ for any $n_C(b) < n < 1$ so that Point [2] holds. If $0.313 < b < 0.393$ then $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) > 0$ for any $0 \leq n < n_A(b)$; $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) > 0$ for any $n_A(b) < n < n_B(b)$; $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) > 0$ for any $n_B(b) < n < n_C(b)$; $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) < 0$ for any $n_C(b) < n < 1$ so that Point

[3] holds. If $0.393 < b < 0.4013$ then $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) > 0$ for any $0 \leq n < n_A(b)$; $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) > 0$ or $\Delta\Pi_C(n,b) < 0$ for any $n_A(b) < n < n_B(b)$; $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) < 0$ for any $n_B(b) < n < 1$ so that Point [4] holds. If $0.4013 < b < 0.5$ then $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) > 0$ for any $0 \leq n < n_A(b)$; $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) > 0$ or $\Delta\Pi_C(n,b) < 0$ for any $n_A(b) < n < 1$ so that Point [5] holds. Let n be negative. If $0 \leq b < 0.115$ then $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) < 0$ for any $0 > n > -1$. If $0 \leq b < 0.115$ then $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) < 0$ for any $0 > n > -1$ so that Point [6] holds. If $0.115 < b < 0.168$ then $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) < 0$ for any $0 > n > -n_B(b)$; $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) < 0$ for any $-n_B(b) > n > -1$ so that Point [7] holds. If $0.168 < b < 0.219$ then $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) > 0$ and $\Delta\Pi_C(n,b) < 0$ for any $0 > n > -n_B(b)$; $\Delta\Pi_A(n,b) < 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) < 0$ for any $-n_B(b) > n > -n_A(b)$; $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) < 0$ for any $-n_A(b) > n > -1$ so that Point [8] holds. If $0.219 < b < 0.333$ then $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) < 0$ for any $0 > n > -1$ so that Point [9] holds. If $0.333 < b < b_T^{NC/NC}(n)$ then $\Delta\Pi_A(n,b) > 0$, $\Delta\Pi_B(n,b) < 0$ and $\Delta\Pi_C(n,b) < 0$ for any $0 > n > -\frac{1-2b}{b}$ so that Point [10] holds. **Q.E.D.**

The CSR-CDG in a Stackelberg environment

The CSR-CDG is a multi-stage non-cooperative game with complete information in which, in the first decision-making stage, each firm chooses to credibly commit or not to commit to an announced output level. This is a binary choice that happens *before* consumers make their purchase decisions and then affects their expectations about the prevailing network size. Consumers are rational and have rational expectations. If at least one of the two firms is committing, the consumers' expectations about the production of the committing firm(s) are realised immediately, i.e., at the same logical time as the announcement without the need to know the prevailing equilibrium. If at least one of the two firms is not committing, consumers' expectations become binding, and they are realised in equilibrium as consumers are rational so that they can know in advance (in the logical timing of the events) the prevailing equilibrium value of the output. This hold in an intermediate stage between the decision-making stage and the market stage. In the market stage, competition between firms can occur simultaneously (Cournot) or sequentially (Stackelberg). In the former case, any decision of each firm can only be expected by the rival because the logical timing of events is such that no player can observe each other's decisions regardless of whether a firm credibly commits itself to an announced output level. This case has been analysed in the main text of the article. In the latter case, one firm (the leader) chooses before its rival, who is therefore forced to become a follower choosing its production after having observed the production – and the decision to credibly commit or not to commit – by the leader. This section deals with this case and develops the CSR-CDG in which at the market stage quantity competition occurs sequentially according to the Stackelberg rules.

Let us assume that firm i is the leader and firm j is the follower. The game is solved by using the backward induction logic and considers profits as the main variable at the decision-making stage. The structure of the four sub-games is the following: (i) both firms credibly commit themselves to an announced output level (C/C), the leader commits itself before the follower does; (ii) both firms do not (or cannot) commit themselves to an announced output level (NC/NC), the leader does not commit before the follower does; (iii) the leader credibly commits itself to an announced output level before the follower produces through the non-commitment strategy (C/NC); (iv) the leader does not commit itself to an announced output level before the follower produces through the commitment strategy (NC/C).

The follower first maximises its utility (given by a weighted sum of its profits and the consumer surplus) as in the Cournot model. Unlike the Cournot setting, the leader considers the follower's reaction function and chooses its output based on it by maximising its utility. The equilibrium values of output and profits emerging in each sub-game are reported below (the superscript S stands for Stackelberg).

Sub-game C/C.

$$q_i^{S,*C/C}(n,b) = \frac{2(1-b)+b^2}{(1-n)(4-3b)} \tag{A.5}$$

$$q_j^{S,*C/C}(n,b) = \frac{1+b(1-b)}{(1-n)(4-3b)} \tag{A.6}$$

and

$$\Pi_i^{S,*C/C}(n,b) = \frac{(1-2b)[2(1-b)+b^2]}{(1-n)(4-3b)^2} \tag{A.7}$$

$$\Pi_i^{S,*C/C}(n,b) = \frac{(1-2b)[1+b(1-b)]}{(1-n)(4-3b)^2} \tag{A.8}$$

where $b < \frac{1}{2} := b_T^{C/C}$ must hold to guarantee that profits are positive in this sub-game.

Sub-game NC/NC.

$$q_i^{S,*NC/NC}(n,b) = \frac{2+(1-b)[2(1-b)+nb]}{2[1-b+(1-n)(3-2b)]} \tag{A.9}$$

$$q_j^{S,*NC/NC}(n,b) = \frac{2+b(1-b)(2-n)}{2[1-b+(1-n)(3-2b)]} \tag{A.10}$$

and

$$\Pi_i^{S,*NC/NC}(n,b) = \frac{[1-b(2-n)]\{2+(1-b)[2(1-b)+nb]\}}{2[1-b+(1-n)(3-2b)]^2} \tag{A.11}$$

$$\Pi_i^{S,*NC/NC}(n,b) = \frac{[1-b(2-n)][2+b(1-b)(2-n)]}{2[1-b+(1-n)(3-2b)]^2} \tag{A.12}$$

where $b < \frac{1}{2-n} := b_T^{NC/NC}(n)$ must hold to guarantee that profits are positive in this sub-game.

Sub-game C/NC.

$$q_i^{S,*C/NC}(n,b) = \frac{8(1-b)+2b^2(1-n)+nb[2(1-n)+nb]}{2\{(2-n-b)[2(1-b)+nb]+4(1-n)(1-b)\}} \tag{A.13}$$

$$q_j^{S,*C/NC}(n,b) = \frac{4(1-n)+4b(1-b)+nb[2b+n(2-b)]}{2\{(2-n-b)[2(1-b)+nb]+4(1-n)(1-b)\}} \tag{A.14}$$

and

$$\Pi_i^{S,*C/NC}(n,b) = \frac{(1-n)[2-b(4-n)]\{8(1-b)+2b^2(1-n)+nb[2(1-n)+nb]\}}{2\{(2-n-b)[2(1-b)+nb]+4(1-n)(1-b)\}^2} \tag{A.15}$$

$$\Pi_i^{S,*C/NC}(n,b) = \frac{(1-n)[2-b(4-n)]\{4(1-n)+4b(1-b)+nb[2b+n(2-b)]\}}{2\{(2-n-b)[2(1-b)+nb]+4(1-n)(1-b)\}^2} \tag{A.16}$$

where $b < \frac{1}{4-n} := b_T^{NC/NC}(n)$ must hold to guarantee that profits are positive in this sub-game.

Sub-game NC/C.

$$q_i^{S,*NC/C}(n,b) = \frac{2[1+(1-b)^2]}{4(1-b)+2(1-n)(2-b)+nb^2} \tag{A.17}$$

$$q_j^{S,*NC/C}(n,b) = \frac{2-nb+2b(1-b)(1-n)}{(1-n)[4(1-b)+2(1-n)(2-b)+nb^2]} \tag{A.18}$$

and

$$\Pi_i^{S,*NC/C}(n,b) = \frac{2\left[1+\left(1-b\right)^2\right][2-b(4-n)+nb^2]}{[4(1-b)+2(1-n)(2-b)+nb^2]^2} \tag{A.19}$$

$$\Pi_i^{S,*NC/C}(n,b) = \frac{[2-nb+2b(1-b)(1-n)][2-b(4-n)+nb^2]}{(1-n)[4(1-b)+2(1-n)(2-b)+nb^2]^2} \tag{A.20}$$

where $b < \frac{4-n-\sqrt{(4-n)^2-8n}}{2n} := b_T^{NC/C}(n)$ must hold to guarantee that profits are positive in this sub-game.

To examine the SPNE emerging in CSR-CDG à la Stackelberg, we consider that firms are owner-oriented and then profits represent the decision variable. Table A.1 summarises the payoff matrix with the profit functions prevailing in the four sub-games. To derive all the possible SPNE one must study the sign of three profit differentials for firm i and three profit differential form firm j. This is because in an environment à la Stackelberg the game is basically asymmetric. These differentials are given by the following equations:

$$\Delta\Pi_{i,A}(n,b) := \Pi_i^{S,*C/NC}(n,b) - \Pi_i^{S,*NC/NC}(n,b) \tag{A.21}$$

$$\Delta\Pi_{i,B}(n,b) := \Pi_i^{S,*NC/C}(n,b) - \Pi_i^{S,*C/C}(n,b) \tag{A.22}$$

$$\Delta\Pi_{i,C}(n,b) := \Pi_i^{S,*NC/NC}(n,b) - \Pi_i^{S,*C/C}(n,b) \tag{A.23}$$

and

$$\Delta\Pi_{j,A}(n,b) := \Pi_j^{S,*NC/C}(n,b) - \Pi_j^{S,*NC/NC}(n,b) \tag{A.24}$$

$$\Delta\Pi_{j,B}(n,b) := \Pi_j^{S,*C/NC}(n,b) - \Pi_j^{S,*C/C}(n,b) \tag{A.25}$$

$$\Delta\Pi_{j,C}(n,b) := \Pi_j^{S,*NC/NC}(n,b) - \Pi_j^{S,*C/C}(n,b) \tag{A.26}$$

The first threshold defines the incentive of firm i to deviate from C to NC when its sign is negative (and vice versa when its sign is positive) when the rival, firm j, is playing NC. The second threshold defines the incentive of firm i to deviate from NC to C when its sign is negative (and vice versa when its sign is positive) when the rival, firm j, is playing C. The third threshold determines the Pareto efficiency/inefficiency of a symmetric SPNE for firm i. The fourth threshold defines the incentive of firm j to deviate from C to NC when its sign is positive (and vice versa when its sign is negative) when the rival, firm i, is playing NC. The fifth threshold defines the incentive of firm j to deviate from NC to C when its sign is positive (and vice versa when its sign is negative) when the rival, firm i, is playing C. The sixth threshold determines the Pareto efficiency/inefficiency of a symmetric SPNE for firm j.

The SPNE of the CSR-CDG à la Stackelberg are summarised in Figure A.1 and the main results prevailing in the space (n, b) are reported below. The figure reports only the loci of points such that $\Delta\Pi_{i,A}(n,b) = 0$ and $\Delta\Pi_{i,B}(n,b) = 0$ for firm i (the leader), i.e., $n_{i,A}(b)$ and $n_{i,B}(b)$, respectively, and the loci of points such that $\Delta\Pi_{j,A}(n,b) = 0$ and $\Delta\Pi_{j,B}(n,b) = 0$ for firm j (the follower), i.e., $n_{j,A}(b)$ and $n_{j,B}(b)$, respectively. Results are qualitatively like those obtained in a basically symmetric, Cournot-like, environment. However, there are differences that must be pinpointed. These differences regard especially the emergence of a unique asymmetric SPNE in which either the leader or the follower commits itself to an announced output level and the rival does not.

Positive network effects

- Area A: (C, C) is the unique SPNE ($\Delta\Pi_{i,A}(n,b) > 0$, $\Delta\Pi_{i,B}(n,b) < 0$, $\Delta\Pi_{j,A}(n,b) > 0$ and $\Delta\Pi_{j,B}(n,b) < 0$), which is Pareto efficient for the leader and Pareto inefficient for the follower.
- Area B: (C, C) is the unique SPNE ($\Delta\Pi_{i,A}(n,b) < 0$, $\Delta\Pi_{i,B}(n,b) < 0$, $\Delta\Pi_{j,A}(n,b) > 0$ and $\Delta\Pi_{j,B}(n,b) < 0$), which is Pareto inefficient for both firms.
- Area C: (NC, C) is the unique asymmetric SPNE ($\Delta\Pi_{i,A}(n,b) < 0$, $\Delta\Pi_{i,B}(n,b) > 0$, $\Delta\Pi_{j,A}(n,b) > 0$ and $\Delta\Pi_{j,B}(n,b) < 0$), which is Pareto inefficient for the leader and Pareto efficient for the follower. The follower is the only player that efficiently chooses to credibly commit itself to an announced output level.
- Area D: (C, NC) is the unique asymmetric SPNE ($\Delta\Pi_{i,A}(n,b) > 0$, $\Delta\Pi_{i,B}(n,b) < 0$, $\Delta\Pi_{j,A}(n,b) > 0$ and $\Delta\Pi_{j,B}(n,b) > 0$), which is Pareto efficient for the leader and Pareto inefficient for the follower. The leader is the only player that efficiently chooses to credibly commit itself to an announced output level.
- Area E: (NC, C) is the unique asymmetric SPNE ($\Delta\Pi_{i,A}(n,b) < 0$, $\Delta\Pi_{i,B}(n,b) > 0$, $\Delta\Pi_{j,A}(n,b) > 0$ and $\Delta\Pi_{j,B}(n,b) > 0$), which is Pareto inefficient for the leader and Pareto efficient for the follower. The follower is the only player that efficiently chooses to credibly commit itself to an announced output level.
- Area F: (NC, NC) is the unique SPNE ($\Delta\Pi_{i,A}(n,b) < 0$, $\Delta\Pi_{i,B}(n,b) > 0$, $\Delta\Pi_{j,A}(n,b) < 0$ and $\Delta\Pi_{j,B}(n,b) > 0$), which is Pareto efficient for both firms.
- Area G: (C, C) is the unique SPNE ($\Delta\Pi_{i,A}(n,b) < 0$, $\Delta\Pi_{i,B}(n,b) < 0$, $\Delta\Pi_{j,A}(n,b) > 0$ and $\Delta\Pi_{j,B}(n,b) > 0$), which is Pareto inefficient for both firms.
- Area H: (NC, C) and (C, NC) are two asymmetric SPNE ($\Delta\Pi_{i,A}(n,b) < 0$, $\Delta\Pi_{i,B}(n,b) > 0$, $\Delta\Pi_{j,A}(n,b) > 0$ and $\Delta\Pi_{j,B}(n,b) > 0$), which are Pareto efficient for both firms.

Negative network effects

Table A.1 The CSR-CDG à la Stackelberg (payoff matrix). Profit

Firm $j \rightarrow$ Firm $i \downarrow$	C	NC
C	$\Pi_i^{*C/C}(n,b)$, $\Pi_j^{*C/C}(n,b)$	$\Pi_i^{*C/NC}(n,b)$, $\Pi_j^{*C/NC}(n,b)$
NC	$\Pi_i^{*NC/C}(n,b)$, $\Pi_j^{*NC/C}(n,b)$	$\Pi_i^{*NC/NC}(n,b)$, $\Pi_j^{*NC/NC}(n,b)$

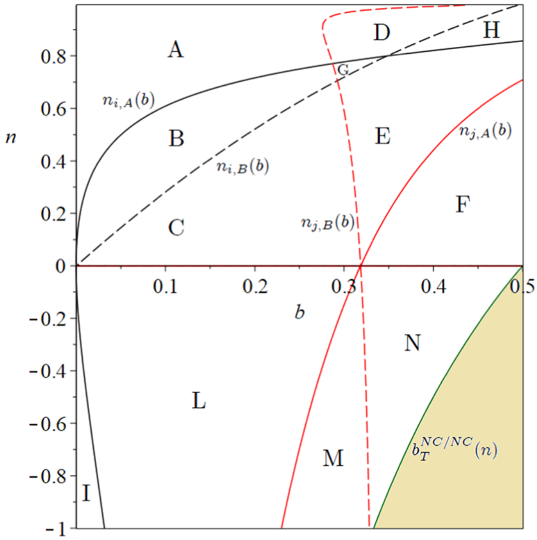

Fig. A.1 The CSR-CDG à la Stackelberg: SPNE when n and b vary and the firm's profit is the decision variable. The sand-coloured region represent the unfeasible parameter space, which is bounded by the threshold $b < \frac{1}{2-n} := b_T^{NC/NC}(n)$. The feasibility condition must be satisfied to guarantee positive profits in the subgame NC/NC

- Area I: (NC, NC) is the unique SPNE ($\Delta\Pi_{i,A}(n,b) < 0$, $\Delta\Pi_{i,B}(n,b) < 0$, $\Delta\Pi_{j,A}(n,b) < 0$ and $\Delta\Pi_{j,B}(n,b) > 0$), which is Pareto inefficient for both firms.
- Area L: (C, NC) is the unique asymmetric SPNE ($\Delta\Pi_{i,A}(n,b) > 0$, $\Delta\Pi_{i,B}(n,b) < 0$, $\Delta\Pi_{j,A}(n,b) < 0$ and $\Delta\Pi_{j,B}(n,b) > 0$), which is Pareto inefficient for the leader and Pareto efficient for the follower. The leader is the only player that inefficiently chooses to credibly commit itself to an announced output level.
- Area M: (C, NC) is the unique asymmetric SPNE ($\Delta\Pi_{i,A}(n,b) > 0$, $\Delta\Pi_{i,B}(n,b) < 0$, $\Delta\Pi_{j,A}(n,b) > 0$ and $\Delta\Pi_{j,B}(n,b) > 0$), which is Pareto efficient for both firms.
- Area N: (C, C) is the unique SPNE ($\Delta\Pi_{i,A}(n,b) > 0$, $\Delta\Pi_{i,B}(n,b) < 0$, $\Delta\Pi_{j,A}(n,b) > 0$ and $\Delta\Pi_{j,B}(n,b) < 0$), which is Pareto efficient for both firms.

An alternative scenario: the CSR-CDG in a Cournot-Stackelberg environment

The analysis so far considered the endogenous choice of commitment device to consumers' expectations with network effects, following the early intuition of Katz & Shapiro (1985), later augmented by Choi & Lim (2022), who build on the commitment decision game with profit maximising firms: (i) the "commitment" (C) strategy implies that firms first compete in quantities (or, in other words, they make a binding/credible announcement on the chosen quantity produced or that will be produced). This holds according to the Cournot (simultaneous moves) rules (see the analysis in the main text) or the Stackelberg (sequential moves) rules (see the previous analysis in this appendix). Then, consumers form expectations about the network size (or, in other words, rational consumers expect exactly the production chosen by the firms; this alternatively means that expectations in this case are not binding) and finally consumers make optimal purchasing decisions; (ii) the "no commitment" (NC) strategy implies that consumers first form expectations about network sizes (expectations are binding in this case), then firms compete in quantities (or, in other words, the production is chosen by considering the exogenous consumer expectation about the network size) – according to the Cournot (simultaneous moves) rules or the Stackelberg (sequential moves) rules – and, finally, consumers (which rationally expect a network size corresponding to the production that hold in equilibrium) make optimal purchasing decisions.

Therefore, the logical timing of the events of the non-cooperative CSR-CDG is the following: in the first (decision-making) stage, firms make the binary decision to commit or not to commit to a given output; in the second (market) stage, firms compete on quantities following the Cournot rivalry (main text) or Stackelberg rivalry (appendix) setting.

To sum up, the timing of the game developed in the main text is the following: the expectations in the case of non-commitment imply that firms first compete in quantities, then consumers form expectations about the network size and consumers eventually optimise the utility and make optimal purchasing decisions. The expectations in the case of commitment imply that consumers first form expectations about the network size, firms then compete in quantities, and consumers eventually optimise and make optimal purchasing decisions. This timing does not mean that the commitment strategy forces, at the market stage of the game, the firm that is unilaterally committing to becoming the market leader.

Following the suggestion of an anonymous reviewer, this section considers an alternative scenario, which is logically consistent with the one adopted by Choi & Lim (2022) and was previously considered in the present paper in the Cournot and Stackelberg scenarios, that modifies the asymmetric sub-games C/NC and NC/C. In this case, the timing in the formation of consumers' expectations above described should force Stackelberg competition, in the market stage of the game, under asymmetric consumers' expectations formation. Therefore, another interesting scenario is possible.

The new Cournot-Stackelberg environment implies that the C-firm, say firm i, credibly commits to its quantity choice ex-ante when it selects strategy C. This observable quantity level q_i is selected to maximize the firm's objective given the anticipated reaction of the rival firm, say firm j, and the firm's consumers. This choice cannot be altered in the production/market stage as the firm is committed to it. The timing of the game, therefore, is: 1) C-firm credibly commits itself to quantity q_i, 2) consumers form expectations y_j about q_j (i.e., about the production of the NC-firm) and consumers know $q_i = y_i$ so the expected market size is $q_i + y_j$, 3) NC-firm determines q_j, firms produce, consumers make purchases. Beliefs are required to be consistent. Therefore, C-firm is the (Stackelberg) leader and NC-firm is the (Stackelberg) follower. This holds in the sub-game C/NC. As C-firm is always the leader, the timing of the events in the asymmetric sub-game NC/C leads to symmetric results than those discussed above.

Therefore, the logical timing of the events of the non-cooperative CSR-CDG is now the following: in the first (decision-making) stage, firms make the binary decision to commit or not to commit to a given output; in the second (market) stage, firms compete on quantities following the Cournot simultaneous moves rules in the symmetric sub-games C/C and NC/NC and the Stackelberg sequential moves rules in the asymmetric sub-games C/NC and NC/C by considering that C-firm is always the leader.

The equilibrium output and profits under the symmetric sub-games C/C and NC/NC are exactly those reported in Sects. 2.1 and 2.2 in the main text (see Eqs. (12) and (13) for the case C/C, and Eqs. (19) and (20) for the case NC/NC). The symmetric equilibrium output and profits under the asymmetric sub-games C/NC and NC/C are those reported in the previous section of this appendix, and they are given by Eqs. (A.13), (A.14) and (A.15), (A.16) for the caseC/NC, and Eqs. (A.14), (A.13) and (A.16), (A.15) for the caseNC/C.

It could be instructive to detail here the logical and mathematical processes leading to the equilibrium output in the Stackelberg sub-game C/NC, in which the C-firm is the leader and the NC-firm is the follower.

If the C-firm publicly commits to a quantity level, then $y_i = q_i$ and at the same time the rival NC-firm can also observe this announcement quantity. Hence, if q_i^e is the firm j's prediction (or expectations) on firm i's quantity, we have that $q_i^e = q_i$. Applying backward induction, the follower, firm j, maximises $U_j = \Pi_j + bCS_j$, where $\Pi_j = [1 - q_j - q_i^e + n(y_j + q_i^e)]q_j$ and $CS_j = \frac{1}{2}(q_i^e + q_j)[q_i^e + q_j - n(q_i^e + y_j)]$, subject to the constraint $q_i^e = q_i$. This yields the reaction function $q_j(q_i, y_j) = \frac{-2+[2+b(-2+n)-2n]q_i+(-2+b)ny_j}{2(-2+b)}$. The C-firm anticipates the reaction of the NC-firm (assuming it neglects the influence on firm j's consumers) and maximises $U_i = \Pi_i + bCS_i$, where $\Pi_i = [1 - q_i - q_j^e + n(q_i + y_j)]q_i$ and $CS_j = \frac{1}{2}(q_i + q_j^e)[q_i + q_j^e - n(q_i + y_j)]$, subject to the constraint $q_j^e = q_j(q_i, y_j)$, which represents the firm i's prediction (or expectations) on firm j's quantity. This process yields $q_i(y_j) = \frac{8+4(-2+b)b-(-2+b)^2 n(-2+bn)y_j}{[2+(-2+b)n]\{8+b[-6+(-2+b)n]\}}$. Imposing consistency of beliefs, we have $q_j = \frac{-2+[2+b(-2+n)-2n]q_i+(-2+b)ny_j}{2(-2+b)}$, $q_i = \frac{8+4(-2+b)b-(-2+b)^2 n(-2+bn)y_j}{[2+(-2+b)n]\{8+b[-6+(-2+b)n]\}}$ and $y_j = q_j$. Then, solving for equilibrium gives Eqs. (A.13) and (A.14). The other equilibrium values follow accordingly. These solutions, therefore, coincide with those reported in the sub-game C/NC in the Stackelberg environment studied in the previous section of this appendix. Therefore, the logical and mathematical procedures considered here coincide with those used to build on the sub-game C/NC à la Stackelberg. We pinpoint that the difference between the approach used throughout the paper (in both Cournot and Stackelberg environments) and the one adopted in the present section is about the logical timing of the events and the transmission of information. In the Cournot model developed in the main text as well as in the Stackelberg model in which firm i is the leader irrespective of whether it is credibly committing to an announced output level, each firm chooses to credibly commit or not to commit to an announced output level in the first decision-making stage. This holds before consumers make their purchase decision. Competition in the market stage between the two firms can hold simultaneously or sequentially. In the former case, each firm has a prediction on the quantity produced (alternatively, credibly committed and that will be produced) by the rival, so that each firm takes the rival's quantity as given when it maximises its objective function. This is because the events hold at the same logical timing for both firms, so no one can observe and immediately internalise the effective production of the rival. In the latter stage, the leader moves first and considers the quantity produced (or will be produced) by the rival when it maximises its objective function; this holds irrespective of whether the follower is committing or not committing to an announced output level. Unlike this, according to the anonymous reviewer's suggestion in the Stackelberg model in which the committing firm is always the leader, choosing to commit to an announced output level implies that: (i) only a firm's consumers are affected by this announcement and not the rival's consumers, and (ii) the non-committing firm becomes a Stackelberg follower so that the C-firm anticipates the reaction of the NC-firm (by assuming that C-firm neglects the influence on NC-firm's consumers), i.e., the credible public announcement of output commitment by the C-firm occurs ex-ante, i.e., before the rival takes any action when it selects strategy C. *In the Cournot model studied in the main text of the present article we have indeed assumed that the credible public announcement of output commitment by the C-firm occurs at the same logical timing of the events as the rival's action, irrespective of whether the rival is selecting* *strategy Cor strategy NC. In addition, we have also assumed, in the sub-game $NC/C$of the Stackelberg model presented in the previous section of this appendix, in which the leader is not committing, that, the credible public announcement of output commitment by the by the C-firm occurs ex-post, i.e., after the rival selected strategy NC.*

Therefore, according to the reviewer's point, the symmetric sub-games can be played according to the Cournot rules but in the asymmetric sub-games the C-firm (either firm i or firm j) always becomes a Stackelberg leader. This implies that the equilibrium results of the asymmetric sub-game C/NC obtained by considering a Stackelberg scenario in which firm i is the leader are the same as those emerging in the model of the present section. This implies that CSR-CDG à la Stackelberg is an asymmetric game and the CSR-CDG à la Cournot-Stackelberg is a symmetric game.

To sum up, in the Cournot-Stackelberg CSR-CDG both C-firms in the symmetric sub-game C/C public announce, for some reason, their output commitment in the same logical timing of the events, whereas the C-firm in the asymmetric sub-games C/NC and NC/C public announces, for some reason, its output commitment before the rival's takes any actions and then the C-firm becomes a Stackelberg leader. Of course, this modifies the logical timing of the events occurring (simultaneously) in the symmetric sub-games from those occurring (sequentially) in the asymmetric sub-games.

The payoff matrix (profits) that firms face in the first decision-making stage of the CSR-CDG played in a Cournot-Stackelberg environment is composed of Eqs. (13), (20), (A.15) and (A.16). The SPNE of the game are obtained by computing the usual profit differentials that we do not report here to save space. The main results are summarised in Fig. A2 , which rigorously reports the loci in which the profit differentials are zero and the feasibility condition that must be satisfied to guarantee positive profits in the sub-game NC/NC. The figure reports the loci of points such that $\Delta\Pi_A^{CS}(n,b)=0$, $\Delta\Pi_B^{CS}(n,b)=0$ and $\Delta\Pi_C^{CS}(n,b)=0$, i.e., $n_A^{CS}(b)$, $n_B^{CS}(b)$ and $n_C^{CS}(b)$ respectively, where the super-script CS stands for "Cournot-Stackelberg". Results are qualitatively like those obtained in the Cournot model of the main text and the Stackelberg model of the previous section of the appendix.

Positive network effects

- Area A: (C,C) is the unique SPNE ($\Delta\Pi_A^{CS}(n,b)>0$, $\Delta\Pi_B^{CS}(n,b)<0$ and $\Delta\Pi_C^{CS}(n,b)>0$), which is Pareto inefficient. The CSR-CDG à la Cournot-Stackelberg is a prisoner's dilemma with committing firms.
- Area B: (C,C) is the unique SPNE ($\Delta\Pi_A^{CS}(n,b)>0$, $\Delta\Pi_B^{CS}(n,b)<0$ and $\Delta\Pi_C^{CS}(n,b)<0$), which is Pareto efficient. The CSR-CDG à la Cournot-Stackelberg is an anti-prisoner's dilemma with committing firms.
- Area C: (C,C) and (NC,NC) are two symmetric SPNE ($\Delta\Pi_A^{CS}(n,b)<0$, $\Delta\Pi_B^{CS}(n,b)<0$ and $\Delta\Pi_C^{CS}(n,b)>0$), but NC payoff dominates C. The CSR-CDG à la Cournot-Stackelberg is a coordination game.
- Area D: (NC,NC) is the unique SPNE ($\Delta\Pi_A^{CS}(n,b)<0$, $\Delta\Pi_B^{CS}(n,b)>0$ and $\Delta\Pi_C^{CS}(n,b)>0$), which is Pareto efficient. The CSR-CDG à la Cournot-Stackelberg is an anti-prisoner's dilemma with non-committing firms.
- Area E and Area F: (C,NC) and (NC,C) are two asymmetric SPNE ($\Delta\Pi_A^{CS}(n,b)>0$, $\Delta\Pi_B^{CS}(n,b)>0$), which are Pareto efficient, and the C-firm gets the best outcome.

Negative network effects

- Area G: (C,C) is the unique SPNE ($\Delta\Pi_A^{CS}(n,b)>0$, $\Delta\Pi_B^{CS}(n,b)<0$ and $\Delta\Pi_C^{CS}(n,b)<0$), which is Pareto efficient. The CSR-CDG à la Cournot-Stackelberg is an anti-prisoner's dilemma with committing firms.

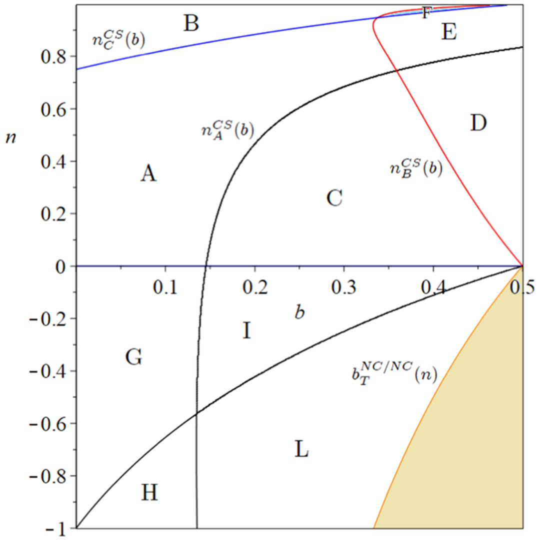

Fig. A.2 The CSR-CDG à la Cournot-Stackelberg: SPNE when n and b vary and the firm's profit is the decision variable. The sand-coloured region represent the unfeasible parameter space, which is bounded by the threshold $b < \frac{1}{2-n} := b_T^{NC/NC}(n)$. The feasibility condition must be satisfied to guarantee positive profits in the sub-game NC/NC. Light-blue region (F): (C, NC) and (NC, C) are two asymmetric SPNE with the C-firm getting the best outcome and the CSR-CDG à la Cournot-Stackelberg is an anti-coordination game. This accords with the region below (E), in which the CSR-CDG à la Cournot-Stackelberg is an anti-coordination game and C-firm continues to get the best outcome

- Area H: (C, C) and (NC, NC) are two symmetric SPNE ($\Delta\Pi_A^{CS}(n, b) < 0$, $\Delta\Pi_B^{CS}(n, b) < 0$ and $\Delta\Pi_C^{CS}(n, b) < 0$), but C payoff dominates NC. The CSR-CDG à la Cournot-Stackelberg is a coordination game.
- Area I: (C, C) and (NC, NC) are two symmetric SPNE ($\Delta\Pi_A^{CS}(n, b) < 0$, $\Delta\Pi_B^{CS}(n, b) < 0$ and $\Delta\Pi_C^{CS}(n, b) < 0$), but C payoff dominates NC. The CSR-CDG à la Cournot-Stackelberg is a coordination game.
- Area L: (C, C) is the unique SPNE ($\Delta\Pi_A^{CS}(n, b) > 0$, $\Delta\Pi_B^{CS}(n, b) < 0$ and $\Delta\Pi_C^{CS}(n, b) < 0$), which is Pareto efficient. The CSR-CDG à la Cournot-Stackelberg is an anti-prisoner's dilemma with committing firms.

Cournot versus Stackelberg

This section discusses some possible advancements of the modelling setup used in the present paper by specifically concentrating on the comparison between the simultaneous Cournot setting and the sequential Stackelberg setting. The discussion begins by considering the delegation literature framed in strategic competitive markets developed by Fershtman and Judd (1987), Sklivas (1987) and Vickers (1985), FJVS henceforth.

The managerial delegation game à la FJSV represents a classic example in which the choices of the players are always simultaneous in all sub-games, i.e., the scenario is the Cournot-Nash one. This implies that in the asymmetric sub-game where, for example, the owner of firm i is delegating the output to a manager and firm j is profit maximizing, the

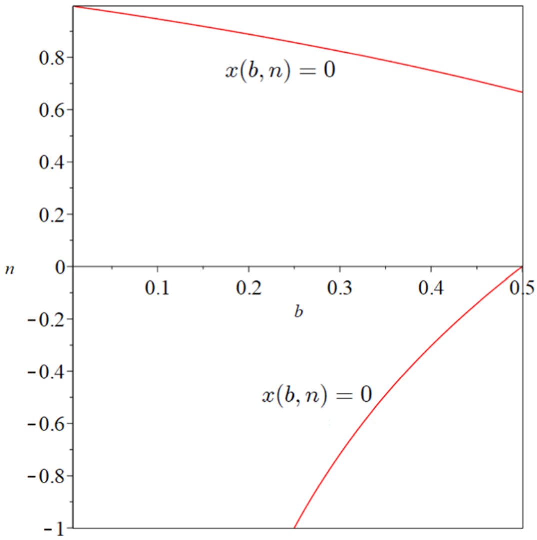

Fig. A.3 The CSR-CDG: Cournot versus Stackelberg. The red line is the curve defining, in the space (b, n), the indifference to play the CSR-CDG according to the Cournot-Nash (simultaneous) rules or according to the Stackelberg (sequential) rules with positive and negative values of n

same payoffs (profits) are obtained as in the case where the delegating firm i is the leader and the profit-maximising firm j is the follower in the product market like in a Stackelberg scenario. This, in turn, implies that profits under Cournot-Nash or Stackelberg coincide (see, for example, the payoff matrix in Fanti et al., 2017, Table 1, p. 498, where this result is clearly illustrated).

However, in a game à la FJSV, the SPNE is always the one in which both firms choose the Cournot-Nash delegation strategy (so the asymmetric case in which one firm is delegating and the rival is profit-maximising does not emerge as a possible SPNE of this kind of endogenous strategic delegation games). Therefore, the overlapping of profits between Cournot and Stackelberg is not strategically interesting in that case. Indeed, it is also well known that adequate changes to the FJSV model may imply, in the asymmetric sub-game in which one firm is delegating and the rival is profit-maximising, that if firms choose according to the Stackelberg rules (sequentially) and not according to the Cournot-Nash rule (simultaneously) then under appropriate parametric conditions the strategic situation of mixed choices (in which it is assumed competition à la Stackelberg) may emerge as the SPNE of the game: this is the case in Basu's (1995) contribution. For the sake of precision, we note that Basu's (1995) model is very specific in the strategic delegation literature à la FJSV, as results require the presence of fixed hiring managerial costs, which are commonly neglected.

The analysis below aims to clarify the following question. Is it possible that the quantities chosen simultaneously in the product market generate the overlapping of the corresponding Cournot-Nash payoffs (profits) with the payoffs (profits) obtained in a sequential Stackelberg scenario in which the committing C-firm is the leader and the non-committing NC-firm is the follower? The answer is positive, but this applies to a specific parametric set. Indeed, there exists a curve defined by the equation $x(b, n) = 0$ such that the profits following the simultaneous Cournot-Nash scenario in which firm i is playing C and firm j is playing NC coincide with the profits following the sequential Stackelberg scenario in which the C-firm i

is the leader and the NC-firm j is the follower. This function is computed as the difference between the profits of firm i (resp. firm j) in the asymmetric sub-game of the Cournot-Nash scenario, as given by Eq. (33) (resp. Equation (34)) in the main text and the profits of firm i (resp. firm j) in the asymmetric sub-game of the Stackelberg scenario in which the C-firm is the leader and the NC-firm is the follower, as given by Eq. (A.15) (resp. Eq. (A.16)). The curve $x(b, n) = 0$, as depicted in Fig. A3 (in which the solutions in the space (b, n) of the profit differentials of firm i and firm j discussed so far just overlap), is defined only in an implicit form as it is not possible to explicit n as a function of b or vice versa in a neat analytical form. Thus, only for the parametric set in the space (b, n) defined by $x(b, n)$ does the Cournot-Nash simultaneous choice hold as if it were a Stackelberg sequential set-up at the market stage. This result for our commitment decision game exactly resembles that of Basu (1995) for his managerial delegation game.

This also means that a parametric region (for instance, above the curve defined by the red line $x(b, n) = 0$ in Fig. A3 for positive values of n) does exist in which the Stackelberg outcome arises endogenously through the commitment strategy within the commitment decision game like the delegation strategy emerges in the strategic delegation game of Basu (1995).

This basic analysis raises questions that can be of importance about the mode of competition (endogenous order of moves) that can emerge as a Nash outcome (SPNE) of an endogenous timing game à la Hamilton and Slutsky (1990) with CSR committing and non-committing firms in a network industry. If the events happen simultaneously, the competition in the sub-game holds à la Cournot. If the events happen sequentially, the competition in the sub-game holds à la Stackelberg. The endogenous SPNE of the game can be either the simultaneous move or the sequential move. This improvement is left for future research.

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