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Insurers as Asset Managers and Systemic Risk

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Financial intermediaries often provide guarantees resembling out-of-the-money put options, exposing them to undiversifiable tail risk. We present a model in the context of the U.S. life insurance industry in which the regulatory framework incentivizes value-maximizing insurers to hedge variable annuity (VA) guarantees, though imperfectly, and shifts risks into high-risk and illiquid bonds. We calibrate the model to insurer-level data and identify the VA-induced changes in insurers' risk exposures. In the event of major asset and guarantee shocks and absent regulatory intervention, these shared exposures exacerbate system-wide fire sales to maintain capital ratios, plausibly erasing over half of insurers' equity capital. (*JEL* G11, G12, G14, G18, G22)

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Financial guarantees, which resemble out-of-the money put options, have become a pervasive feature of the financial system, mostly taking the form of "off balance sheet commitments." This paper proposes and studies a new mechanism through which an important example of such guarantees, those embedded in variable annuities (VAs),[1] can induce "reaching-for-yield" behavior through elevated investments in illiquid assets among the life insurers that offer VAs. Given the various risk exposures associated with guarantees and the constraints of capital adequacy regulation, these shared investments in illiquid assets may foster systemic risk through fire-sale externalities.

As the U.S. retirement landscape has moved away from employer-sponsored defined benefit plans, VAs have grown to fill part of this gap and, as a result, life insurers' major product lines have evolved from traditional life insurance to saving and investment products, including VAs and other types of annuities (Figure 1). These VAs often embed various equity-linked investment guarantees (Koijen and Yogo 2017, 2022). While the importance of guarantees is not unique to the insurance industry, U.S. insurance data offer a remarkable level of measurement detail with respect to asset holdings, policy generation, and regulatory constraints. In addition, given their market size and the nature of their return commitments, VAs are attracting attention from policy makers as a potential source of systemic risk, especially because the insurance industry is a large pillar of the financial system with deep connections to other sectors (Billio et al. 2012 and Figure 2).

A critical feature of the guarantees associated with VAs is that they promise minimum returns to policyholders that must be honored by the issuing insurers. Given the put option–like nature of these products, two related problems manifest during a period of sustained financial market stress. First, an individual insurer may become financially distressed as the moneyness of its guarantees sharply increases. Second, the distress is now correlated across insurers as guarantees go in-the money at the same time for all insurers with VAs. It is

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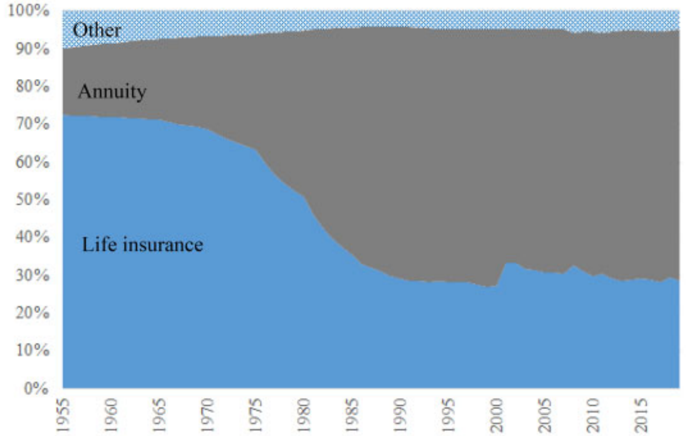
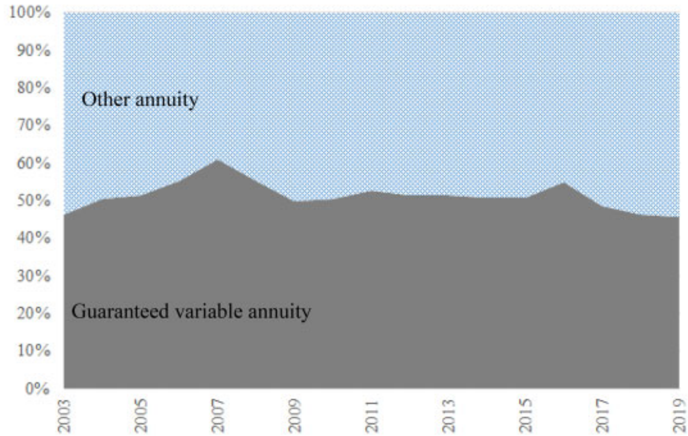
[1] A VA is a life insurance policy generally sold to individuals saving for retirement. We provide an overview of the different types of VAs in Internet Appendix A.



A *Shares of life insurance, annuities, and other products in life insurers' liabilities*



B *Shares of guaranteed VA and other annuities in life insurers' annuity-related liabilities*

Figure 1
Life insurance product shares

Panel A plots product shares of life insurance, annuities, and other products in life insurers' total liabilities over the period 1955–2019. Panel B plots shares of guaranteed variable annuities and other annuities in life insurers' annuity-related liabilities over the period 2003–2019. *Source*: American Council of Life Insurers, 2020 Life Insurers Fact Book, and NAIC 2003–2019 Annual Statutory Reports (as filed by life insurers and obtained through S&P Market Intelligence).

precisely this type of shared risk that has raised significant concerns about financial stability within the insurance sector and, more broadly, across other parts of an interconnected financial system.

We develop a model in which guarantee writing changes an insurer's overall risk exposures in two ways that reinforce each other. First, the guarantees expose the insurer to (undiversifiable) stock market shocks, which can be

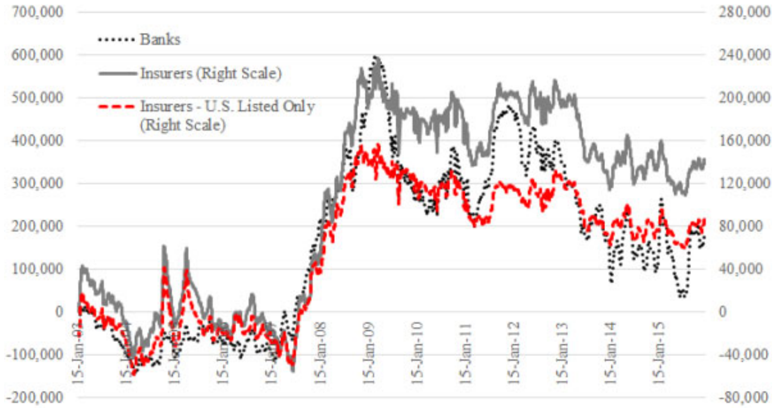


Figure 2
SRISK of largest banks and insurers over time
This figure plots the time series of (sum of) capital shortfall that would be experienced in a potential future financial crisis, or SRISK, ($ million) for (i) 10 largest publicly traded banks in the U.S. (dotted line), (ii) 10 publicly traded insurers with the largest outstanding guaranteed variable annuities in the U.S. (solid line), and (iii) a subset of insurers in (ii) whose stocks are publicly traded in the United States (dashed line). The SRISK data are from NYU Stern Volatility Lab (vlab.stern.nyu.edu/welcome/risk) from January 1, 2003, until the end of December 2015.

mitigated by hedging. Second, the guarantees change the insurer's portfolio choice, especially its investments in illiquid assets, which in turn changes the insurer's exposures to illiquid asset price shocks and its vulnerability to fire-sale feedback effects. Our model considers an insurer with two lines of business: traditional life insurance and VAs with guarantees linked to the stock market. The mix between these two businesses is exogenously given.[2] The insurer maximizes shareholder value by choosing the investments in three broad assets–liquid bonds, illiquid bonds, and common stocks–and the hedge coverage for the guarantees, subject to a risk-based capital (RBC) constraint. The liquid bonds are considered safe while the illiquid bonds and common stocks are risky and thus subject to RBC charges. The insurer has incentives to invest in risky assets, despite their RBC charges, because it perceives higher asset returns relative to the market. One reason, for example, may be that the insurer faces limited liability and thus underprices downside risk. We assume that, consistent with empirical evidence, an insurer targets a specific stock allocation and thus changes asset risk only through the mix of illiquid and liquid bonds.

The RBC constraint dictates how much the insurer can invest in illiquid bonds, and it is precisely through the RBC constraint that VA guarantees affect the insurer's portfolio choice. First, the traditional life insurance and VA businesses may yield different profits, which means different amounts of equity capital to support illiquid bond investments. Second, VAs with guarantees are

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[2] Koijen and Yogo (2022) provide a complementary model in which the insurer optimally chooses the amount and generosity of guarantees to write.

risky and subject to RBC charges. The insurer has incentives to hedge the guarantees, as doing so reduces the RBC charges and thus creates more room for additional illiquid bond investments. Consistent with empirical evidence and the applicable regulation, we focus on dynamic delta hedging, that is, selling stocks and buying bonds to dynamically offset the guarantees' delta, and assume that it is imperfect; its effectiveness and associated RBC relief are limited and decrease as the insurer hedges more.[3] The diminishing hedging benefit, along with a constant hedging cost, determines the optimal hedge coverage.

Together, if the profits from the VA guarantees, net of (after-hedging) RBC charges, are higher than those of the traditional life insurance business, then guarantee writing will relax the RBC constraint and lead to additional investments in illiquid bonds. We refer to these illiquid bond investments as reaching for yield (RFY), given the presence of risk-taking incentives and the fact that the insurer does not internalize externalities caused by any fire sales of these bonds. How much of the guarantee exposures are hedged and whether RFY obtains are ultimately empirical questions, which we address by calibrating the model to the data.

We use the National Association of Business Commissioners (NAIC) (panel) data on guaranteed VAs' account values, gross reserves, portfolio holdings, and derivatives positions for the period 2010–2019. We split assets into three broad groups corresponding to liquid bonds, illiquid bonds, and common stocks in the model, where the term bond is a shorthand for fixed income assets including non bond fixed income assets such as mortgages and loans. We begin by establishing some facts that inform our empirical specifications. First, a relatively small sample of life insurers write VAs, and the ones that do are very large. Second, insurers with higher exposures to VA guarantees have smaller allocations to liquid bonds and common stocks and significantly larger allocations to illiquid bonds, consistent with the predictions of our model, under the parameter space in which VA guarantees relax the RBC constraint.

We calibrate the model by fitting two fundamental relations that govern the profitability and hedging channels, through which VA guarantees affect illiquid bond investments. First, we estimate profitability parameters for VAs with guarantees and traditional lines of insurance by exploiting the variation in gross underwriting profits across insurers with different mixes of the two businesses. We find that the profit from writing guarantees is initially much higher than that from traditional insurance (consistent with the higher risk and RBC charges that we also estimate, as well as the significantly higher broker commissions, as documented by Egan, Ge, and Tang [2021]), but the profit differential decreases

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[3] Regulation caps the maximum relief at 70% of the RBC requirement. As the hedging effectiveness (measured by the correlation between the hedging instruments and the underlying stocks) decreases, the RBC relief also decreases. We focus on delta hedging, essentially selling short equities and investing the proceeds in both the liquid and illiquid bonds. While insurers may also use put options to hedge the guarantees, only about 8.6% of guarantee exposures are hedged with put options, according to the National Association of Insurance Commissioners' Schedule DB data.

rapidly. Second, we calibrate the hedging-related parameters using the binding RBC constraint, under which the allocation to illiquid bonds is a function of the RBC from other assets and liabilities in the balance sheets. We prespecify some of the parameters using historical and regulatory values, and estimate the remaining parameters from the data through an iterative OLS procedure. We assume that the guarantee is an 18-year put option on the stock market, and calculate its reserve value, RBC charge, and RBC relief from hedging using the distribution of maximum capital shortfalls under different simulated paths of the stock market. In addition, given that large insurers write more guarantees and seem to have a preference for illiquid bonds, we allow the insurer's asset size to affect the portfolio choice both indirectly through the RBC relief from hedging and directly through a prespecified preference function.

Using our calibrated parameters, we infer insurers' hedge coverage and calculate their hypothetical portfolios that would obtain if they did not write the guarantees. Our findings suggest that VA guarantees contribute to systemic risk both by raising the insurers' stock market exposures and by inducing RFY. On average, insurers hedge about 71.5% of their stock market exposures (delta) from the guarantees. The remaining exposures, considering varying degrees of hedging effectiveness, amount to about 3.4% of assets on average, effectively doubling the insurers' stock market allocation. In addition, their illiquid bond allocations would be up to 6.4% lower, absent the VA guarantees. Based on observed insurer balance sheets in 2019, these reductions would amount to about $174 billion, almost 17% of actual illiquid bonds held by the insurers with medium to high guarantee exposures.

When viewed from the perspective of the insurance industry, the RFY incentive engenders elevated collective holdings of illiquid bonds among insurers, prompting the question as to the consequent likelihood of *collective* fire sales in times of adverse shocks. For example, upon a large and prolonged decline in stock and bond markets, as happened over the 2008–2009 financial crisis, the insurers' asset values drop while their regulatory reserves spike as the guarantees become closer to or in the money. As a result, insurers need to shore up their capital positions. Since the insurers' liabilities are difficult to adjust in the short run and issuing new equity is likely challenging during a financial crisis, de-risking by selling illiquid bonds (and buying liquid bonds) is a likely outcome. This may cause contagion to other insurers (and, outside of the model, potentially other parts of the financial system), as they (partially) mark their illiquid bond positions to market, thereby facing tightened regulatory constraints and further contributing to fire-sale feedback loops.

We follow Greenwood, Landier, and Thesmar (2015) and perform a quantitative exercise to assess the amount and costs of fire sales in the face of various market shocks. Further, we use the calibrated model to attribute the fire sales to guarantee writing as well as its two main facets that contribute to systemic risk: exposure to undiversifiable shocks and RFY. We consider three types of shocks–a shock to the stock price, a shock to the illiquid bond price, and a shock to the value of the guarantee–and two levels of magnitude that we label as "adverse" (10th percentile on the left tail) and "severely adverse" (worst in the past 20 years). We take into consideration that life insurers do not have to mark to market many of their bond positions. Collectively, we find that insurers have sufficient capital to withstand the fire-sale costs associated with adverse shocks.[4] However, a *prolonged*, severely adverse but uncorrelated shock can stress insurers' balance sheets, potentially erasing up to 15% of their equity capital. The worst case occurs when these shocks are correlated and accompanied by significant ratings downgrades, as in the 2008–2009 financial crisis. Without any policy intervention, insurers would have to liquidate over 80% of their illiquid bond holdings, leading to the fire-sale costs of $147 billion (57% of the insurers' equity capital), of which 65% are due to VA guarantees. In this scenario, regulatory forbearance, government intervention, or both may be necessary to halt contagion and prevent large-scale insolvency.

Our paper contributes to the systemic risk literature along several dimensions. First, there is scant understanding of how various forms of guarantees written by financial intermediaries affect financial stability. Defined benefit pension funds, for example, provide explicit guarantees to their claimants. Coupled with a realized degree of underfundedness, these guarantees may incentivize RFY behavior in their portfolio investments (Rauh 2006, 2009; Klinger and Sundaresan 2019). Similarly, in securitization deals, banks provide and structure their guarantees to outside investors to minimize capital requirements (Acharya, Schnabl, and Suarez 2013; Landsman, Peasnell, and Shakespeare 2008; Nui and Richardson 2006). While data limitations make it difficult to comprehensively analyze the impact of guarantees on the portfolio choices of pension funds and banks, our analysis makes an important contribution to a broader literature on the risk-taking incentives associated with guarantees and their systemic risk implications.

Second, our findings add to a small but growing body of evidence on hedging and risk management in the insurance industry. Ankirchner, Schneider, and Schweizer (2014), Bauer (2020), and Li, Moenig, and Augustyniak (2021) study the hedging of stock market risk of mutual funds that underlie VAs. Sen (Forthcoming) studies the impact of regulatory incentives on the extent to which life insurers hedge the interest rate and stock market risks of VA guarantees. She finds that life insurers hedge a large fraction of stock market risk but only hedge the interest rate risk of VA products for which the RBC is sensitive to interest rates. While our hedging results accord with Sen (Forthcoming), the shifting of risk to shadow insurers that she documents is noteworthy because it adds another channel through which contagion can spread outside of the formal insurance industry.

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[4] For example, an adverse stock market shock of 19% would result in high-guarantee insurers selling $131 billion of illiquid bonds. The guarantee-induced fire-sale costs would be $3 billion, representing just 1% of the insurers' equity capital.

Third, another strand of the literature addresses the issue of liquidity provision in times of market stress, proposing that some intermediaries, by virtue of their long horizons and balance sheet structure, can take on that vital role. Chodorow-Reich, Ghent, and Haddad (2021) show that life insurers behave like asset insulators during normal times but do not during market meltdowns. Coppola (2021) shows that in crises, investment-grade bonds predominantly held by insurers decrease in value less than those predominantly held by mutual funds. Our focus is instead on fire sales of illiquid bonds, and only a small segment of investment-grade corporate bonds are classified as illiquid. Coppola (2021) also shows that insurers' capital constraints matter, and similarly in our case, VA exposures matter for the extent of fire sales induced by adverse shocks. More broadly, our findings suggest that in a crisis, life insurers may not just stop insulating assets but may even become a source and an amplifier of fire sales, with far reaching implications for the stability of the entire financial system.

Fourth, we theoretically propose and empirically investigate a new mechanism that can potentially cause correlated risk exposures within the life insurance industry, both through the market-dependent guarantee liabilities and the elevated holdings of illiquid assets (Girardi et al. 2021). The existing literature has identified correlated investments as a potential source of systemic risk for banks (Wagner 2010, 2011; Allen, Babus, and Carletti 2012; Greenwood, Landier, and Thesmar 2015). Figure 2 plots a systemic risk measure reported by the NYU Stern Volatility Lab (see, among others, Acharya et al. 2017; Brownlees and Engle 2017), showing that since the global financial crisis, insurers have become significantly systemically riskier. We gauge the ex-ante potential for systemic risk embedded in both insurers' liabilities and their assets, and our model-based approach allows for counterfactual analyses to assess the effects of business and regulatory policies.

1. Institutional Background: VAs and Guarantees

A variable annuity is a policy designed for the accumulation of wealth for retirement. A policyholder contributes funds, which are allocated to subaccounts invested in mutual funds and other investments. An insurer allocates policyholder savings to a separate account, and acts as a delegated asset manager for the policyholder's funds. Absent any guarantees, the separate account is a pass-through account in which the policyholder bears all investment risk. The life insurance component of the VA is the option held by the policyholder to convert the funds to an annuity at retirement. Once the policyholder reaches the contractually specified retirement age, she can make lump sum withdrawals or annuitize the account balance.

As the value of the account fluctuates with the performance of the stock market, the policyholders' savings are exposed to stock market risk. Starting in the 1990s, insurers offered various types of financial guarantees that effectively protect the policy balance in the case of poor stock market performance. To attract funds in a highly competitive market, insurers are incentivized to offer generous guarantees to secure contract origination, but these guarantees may, of course, later prove costly in certain states of the world. As an example of the practical importance of such guarantees, 76% of policyholders purchased a financial guarantee, according to the Life Insurance Marketing and Research Association (LIMRA) (Radu 2017).

A guarantee embedded in VAs is a put option–like instrument. Unlike the mortality risks of traditional life insurance products, which are diversifiable, the risk associated with the VA guarantees is not. Regulators require that insurers set aside reserves to ensure that they can meet the obligations in the event that the guarantee is triggered. To ensure solvency, insurers are also required to hold additional capital to absorb extreme losses that might arise from the guarantees (see Junus and Motiwalla 2009). To determine the reserve and required RBC, insurance regulators supply various scenarios for the joint path of several asset classes. Insurers then simulate the values of their VAs under each supplied scenario to gauge any possible equity deficiency (keeping the highest present value of equity deficiency in each path). The reserve is computed as the conditional mean over the upper 30th percentile of the distribution of equity deficiencies. The RBC is calculated as the conditional mean over the upper 10th percentile minus the reserve.

For traditional insurance risks, insurers' reserves are set to match (a profit margin–adjusted) expected periodic payment to policyholders. Based upon standard asset-liability matching, reserves are usually invested in bonds. An insurer may still face insufficient reserves if it underestimates, say, the average longevity risk of its clients, but any fluctuation in reserves as a result of prior estimation errors would be largely idiosyncratic to the insurer. In contrast, the size of the reserves associated with guarantees, now among the largest liabilities on some insurers' balance sheets, fluctuates with stock market performance and interest rates, and hence is highly correlated across insurers.

To demonstrate this feature, Figure 3 plots the evolution of insurers' reserves for VA guarantees (panel A) in relation to their returns on equity (panel B), a measure of the insurers' financial health, for the period from 2004 to 2019. Each year, life insurers with VAs are divided into three groups by the ratio of gross reserve to capital. The "high," "medium," and "low" groups include life insurers whose ratios of gross reserve to capital are in the top, middle, and bottom terciles, respectively.

As expected, VA reserves spike when two conditions emerge: declining stock markets (and increasing stock volatilities) and declining interest rates, which are the conditions that feature in most recessions. Panel A highlights the effects of these conditions on the VA reserves during the 2008–2009 financial crisis and the 2011–2012 European debt crisis. This is central to the understanding of how the VA business can suffer contagion. As the VA reserves spike, the equity capital is impaired, as manifested by negative returns on equity in panel B, and

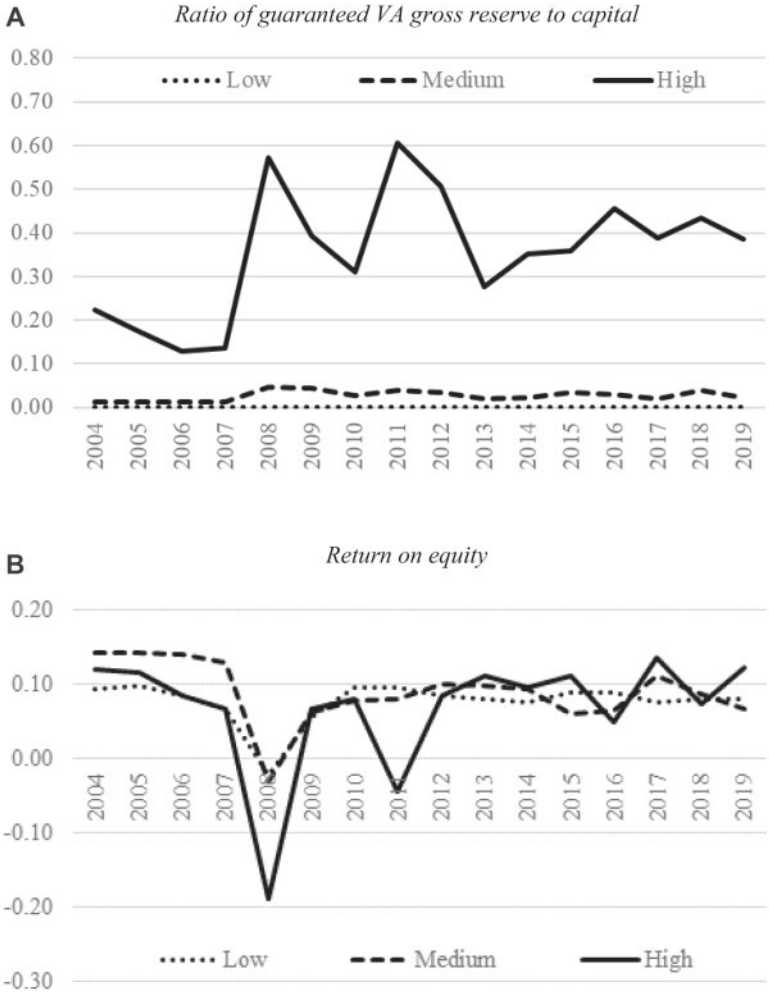

Figure 3
Exposure to guaranteed variable annuities and firm performance
This figure plots the time series of the ratio of gross reserve to capital (panel A), and return on equity (panel B) for life insurers over the period 2004–2019. Each year, life insurers with guaranteed VA liabilities are divided into three groups by the ratio of gross reserve to capital. The "high," "medium," and "low" group include life insurers with ratio of VA gross reserve to capital in the top, middle, and bottom terciles, respectively. For each variable, the annual averages across insurers in each group are plotted. The solid, dashed, and dotted lines represent the high, medium, and low groups, respectively.

RBC ratios rapidly deteriorate. The affected insurers will be under pressure to improve their capital positions by either issuing new equity or reducing risk in their balance sheet by selling risky assets, akin to a de-risking. Given that issuing equity may be difficult during stress periods, selling of risky assets becomes more likely. Any aggressive investments in risky and illiquid securities, which we later show are also a result of writing VAs, may further exacerbate the fire-sale externality.

While not necessarily implying at this stage that VAs cause systemic risk, the time-series patterns that we present highlight how the growing VA business may affect insurers' financial health in the event of sustained market stress. The impact that such undiversifiable risk poses to life insurers is borne out when one considers the experiences of some prominent insurers, such as Hartford.[5]

2. A Model of Guarantee Hedging and Portfolio Choice

We examine an insurer's asset portfolio choice along with its hedging of VA guarantees, and take as given the liabilities from all insurance contracts including the guarantees themselves. The insurer maximizes the market value of equity, and has incentives to take risk in its asset portfolio because it perceives higher returns on risky assets relative to those reflected in traded asset prices. The wedge between the insurer's and market's perceptions of asset returns may arise, for example, from the insurer's underpricing of downside risk, possibly due to its limited liabilities, or from the insurer's comparative advantage in trading certain types of risky assets, possibly due to the long-term nature of its liabilities (Chodorow-Reich, Ghent, and Haddad 2021). The insurer faces a financial friction in the form of a regulatory capital constraint that limits its investment in risky assets. The insurer has incentives to hedge risks arising from the guarantees because doing so relaxes the constraint and allows it to invest more in risky assets.

Guarantee writing may increase the systemic risk of the insurance industry not only because it exposes insurers to stock market return and volatility shocks, which may not be perfectly hedged, but also because it may permit additional investments in riskier, illiquid bonds, which amplify the effects of shocks through fire sales. In our model, the latter obtains because of the following three assumptions, which we will justify in Section 2.5. First, VA guarantees are more profitable for the insurer than traditional insurance contracts, and hence profits from the guarantees help relax the regulatory capital constraint. Second, as discussed, hedging the guarantees provides regulatory capital relief, further relaxing the constraint, but the effectiveness of hedging and its associated capital relief decrease as the insurer hedges more. This ensures that there is a limit to hedging and allows for the optimal hedge coverage to reflect an interior solution. Third, the insurer has a fixed stock-bond allocation, and hence any changes in risk taking occur within the bond exposure through elevated investments in riskier, illiquid bonds.

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[5] AIG, Hartford Financial Services Group, and Lincoln National were among those that aggressively wrote investment-oriented life policies that had minimum return guarantees attached to them. Besides the well-known case of AIG, Hartford Financial was also bailed out by the Troubled Asset Relief Program in 2009, and the reason was precisely the significant losses arising from the VA business unit. Hartford eventually sold its VA business in 2013.

Our characterization of the insurer's balance sheet builds on Koijen and Yogo (2015, 2016). We focus on the asset side of the balance sheet, taking the VA guarantees and other liabilities as given.[6] Koijen and Yogo (2022) present a complementary model that focuses on the product or liability side, featuring financial frictions (also related to the costs of regulatory capital constraint), insurers' market power due to differentiated product demand, and their impacts on the equilibrium contract terms and quantities of VAs with guarantees. In addition, we primarily consider stock market risk and insurers' incentives to hedge such risk, abstracting from other sources such as interest rate risk (Sen 2022) and social inflation risk (Oh 2021).

2.1 Set-Up

We consider one insurer in the context of a three-period model with the following main ingredients. For ease of exposition, we drop the time subscript for variables at date 0.

Initial Balance Sheet. At $t = 0$, an insurer is endowed with the general account liabilities that consist of traditional insurance contracts and guaranteed VAs. The statutory values of the two liabilities, or the statutory reserves, equal T and G, respectively, and so the insurer's total liabilities are $L = T + G$. The insurer collects the premia for assuming the liabilities, which, after subtracting the statutory reserves, provides the insurer with the initial statutory equity capital $K = \Pi(T, G)$. Thus, the insurer's total assets at date 0 are $A = L + K = L + \Pi(T, G)$. The insurer allocates A to three groups of assets: liquid bonds, illiquid bonds, and common stocks, with the portfolio weights α_F, α_I, and α_S, respectively; $\alpha_F + \alpha_I + \alpha_S = 1$. Liquid bonds are risk-free and generate a *gross* return at date 2 equal to r_f. Illiquid bonds and stocks are risky and generate *excess* returns equal to R_I and R_S, respectively. Further, we assume that liquid bonds and stocks can be traded without cost,[7] but illiquid bonds may suffer a fire-sale discount if sold quickly and in large quantities.

Capital Constraint. The insurer faces a financial friction in the form of a regulatory risk-based capital constraint, which is given by

$$\frac{K}{\widehat{K}} \geq \rho \Leftrightarrow K - \rho \widehat{K} \geq 0. \tag{1}$$

It states that the ratio of statutory capital K to combined risk-based capital $\widehat{K}$ must remain above a threshold ρ, which reflects the tightness of the

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[6] We argue that our setting is realistic because most life insurance products are long-term, and hence life insurers' liabilities, as reported in their balance sheets, have been accumulated over several years.

[7] Insurers are small players in the U.S. stock market. Based on the U.S. Financial Account Table L.223 for Corporate Equities, life insurers hold collectively only about 1% of the overall U.S. equity market capitalization over our sample. The notional amount of stock futures and options used to hedge VA guarantees is even smaller, typically less than 20% of the insurers' (cash) stock holdings.

capital regulation and plausibly the sensitivity of insurance product demand to the insurer's financial strength and reputation (Hortacsu and Syverson 2004; Doherty and Schlesinger 1990).[8] The combined RBC is a weighted sum of the components of the insurer's assets and liabilities, with the weight, or RBC charge $\gamma_i, i = F, I, S, T, G$, reflecting the regulator's perceived risk of each item. The RBC charge for liquid bonds is set to zero, $\gamma_F = 0$, indicating that they are safe. All the other RBC charges are positive.

Risk of VA Guarantees and Hedging. Traditional insurance and guarantee liabilities differ in their exposure to stock market risk. The value of traditional insurance liabilities (e.g., life insurance contracts) T is not affected by stock market shocks. By contrast, the VA guarantees are essentially put options on the stock market, and hence their value G rises during stock market downturns. We denote the sensitivity of guarantee reserve value to a small change in the stock price with $\delta < 0$ so that a one unit decline in the stock price increases the guarantee reserve by $|\delta|G$. The level of the guarantee delta, $|\delta|$, reflects the generosity of the guarantees as well as other contract characteristics.

The insurer may reduce the exposure to stock market risk by delta hedging, that is, selling short a fraction $h \in [0,1]$ of the amount $|\delta|G$ of common stocks and investing the proceeds in bonds. Doing so helps to reduce the RBC charge on the guarantees γ_G. However, hedging is typically imperfect, meaning that some basis risk remains (see Section 2.5). As noted earlier, we assume that the hedging effectiveness as well as the associated RBC reduction is capped at $\kappa < 1$ per unit of hedging, and, as the insurer hedges more, the hedging effectiveness diminishes. The rate of diminishing effectiveness depends on the insurer's risk management skills and sophistication η, with $0 < \eta < 1$. Given these assumptions, the RBC charge on the guarantees, net of hedging, equals $(1 - h\kappa(1 - \eta h))\gamma_G$.

Stock Allocation. As noted, we assume that on the asset side, the insurer targets a fixed stock allocation $\overline{\alpha}_S$, which may reflect its risk preferences and/or risk management skills. This is consistent with the evidence that insurers keep their stock allocations fairly stable over time and adjust their asset portfolios by instead changing the mix of liquid and illiquid bonds (see Section 2.5). Since the insurer may hedge the guarantees by selling stocks short, we cannot directly observe $\overline{\alpha}_S$. Rather, we only observe the insurer's stock allocation, net

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[8] The market typically demands capital above the regulatory minimum, as capital positions form an important ingredient in the insurer's credit and strength ratings that critically determines its ability to (a) generate/maintain business and (b) raise funds in the marketplace (Ellul, Jotikasthira, and Lundblad 2011 and Ellul, Jotikasthira, Lundblad, and Wang 2015). Hence, consistent with the data, insurers target an RBC ratio that is higher than the regulatory minimum, and seem to retain this target as if it were a binding regulatory constraint.

of guarantee hedging, which is given by[9]

$$\alpha_S = \overline{\alpha}_S - h|\delta|\frac{G}{A}. \tag{2}$$

Future Dates. At $t=1$, the insurer may be forced to de-risk its portfolio. This occurs if the insurer is hit by random asset or liability shocks that prompt a violation of its capital constraint (1). We model the insurer's portfolio de-risking explicitly in Section 4. Here, to simplify the analysis, we take a reduced-form approach by assuming that the insurer must liquidate a fraction $\tau \in [0,1]$ of its illiquid bonds and invest the proceeds in liquid bonds. We assume that illiquid bonds can only be sold at a fire-sale discount $c \geq 0$ (Ellul, Jotikasthira, and Lundblad 2011). Both the fraction of illiquid bonds sold τ and the fire-sale discount c are variables whose magnitudes depend on the severity of the shocks hitting the insurer as well as the magnitude of "economy-wide" fire sales. Together, de-risking of the insurer's portfolio induces losses equal to $c\tau\alpha_I A$.

At the final date, $t=2$, all assets pay out their returns, the liabilities are due, and the insurer uses the assets to settle its liabilities. The market and statutory values are the same for all balance sheet items. The value of assets equals the buy-and-hold returns on the initial portfolio minus the date-1 fire-sale costs,

$$A_2 = (\alpha_I R_I + \alpha_S R_S + r_f)A - c\tau\alpha_I A. \tag{3}$$

By including guarantee hedging as part of the assets, as in (2), we can treat the value of liabilities as given or outside the insurer's control. The insurer's decisions at $t=0$ only affect the value of its equity at $t=2$, K_2, through the value of assets A_2 as given by (3).

2.2 Optimization problem

At $t=0$, the insurer chooses its portfolio allocations, α's, and hedge coverage, h, to maximize the market value of equity, subject to the capital constraint (1). In the absence of arbitrage, there exists a strictly positive stochastic discount factor M (Chapter 4 of Cochrane 2005) so that the market value of the insurer's equity is given by

$$V = \mathbf{E}[MK_2], \tag{4}$$

where $\mathbf{E}[.]$ denotes the expectation operator at date 0. Since the insurer's decisions have no bearing on the value of liabilities at $t=2$, we can write the value of its date-2 equity within the maximization problem as a function of just the assets: $K_2 = \Phi(A_2)$. Specifically, we assume that date-2 equity (weakly) increases in date-2 assets, that is, $\Phi'(A_2) \geq 0$, but the rate of increase is not necessarily one-to-one, for example, due to the limited liability of shareholders.

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[9] In the estimation, we index insurers by an insurer-specific stock allocation $\overline{\alpha}_S$, which is anchored by their past behavior and allowed to vary with sophistication η. Doing so helps capture the heterogeneity across insurers that impacts asset allocations irrespective of the guarantees and hedging.

2.3 Optimal hedge ratio and illiquid bond investment

Given the insurer's net stock allocation defined by (2) and the identity $\alpha_F + \alpha_S + \alpha_I = 1$, we can express the optimization problem (4) as a function of two choice variables: the illiquid bond allocation α_I and the hedge ratio h, subject to the capital constraint (1). The Lagrangian is

$$\Lambda(\alpha_I, h) = \mathbf{E}[M\Phi(A_2)] + \lambda[K - \rho\widehat{K}],$$

where $\lambda \geq 0$ is the Lagrange multiplier on the capital constraint and the combined RBC, $\widehat{K}$, is given by

$$\widehat{K} = \gamma_T T + (1 - h\kappa(1 - \eta h))\gamma_G G + \gamma_I \alpha_I A + \gamma_S \overline{\alpha}_S A. \tag{5}$$

The first-order conditions for α_I and h are as follows:

$$\mathbf{E}[M\Phi'(A_2)R_I] - \mathbf{E}[M\Phi'(A_2)c\tau] - \lambda\rho\gamma_I = 0, \tag{6}$$

$$-|\delta|\mathbf{E}[M\Phi'(A_2)R_S] + \lambda\rho\kappa(1 - 2\eta h)\gamma_G = 0. \tag{7}$$

The law of one price implies that, in equilibrium, the excess returns of assets discounted at the stochastic discount factor is zero. Denoting the equilibrium excess returns of stocks and illiquid bonds obtained by the marginal investors by R_I^* and R_S^* (which may or may not be the same as the returns earned by the insurer), we have $\mathbf{E}[MR_I^*] = \mathbf{E}[MR_S^*] = 0$. Hence, the first order conditions can be rewritten as

$$\mathbf{E}[M((\Phi'(A_2) - 1)R_I + R_I - R_I^*)] - \mathbf{E}[M\Phi'(A_2)c\tau] - \lambda\rho\gamma_I = 0, \tag{8}$$

$$-|\delta|\mathbf{E}[M((\Phi'(A_2) - 1)R_S + R_S - R_S^*)] + \lambda\rho\kappa(1 - 2\eta h)\gamma_G = 0. \tag{9}$$

Equation (8) depicts the trade-off that the insurer faces in investing in illiquid bonds. The term $\mathbf{E}[M((\Phi'(A_2) - 1)R_I + R_I - R_I^*)]$ is the discounted buy-and-hold return on illiquid bonds as perceived by the insurer in excess of that implied by market prices. A positive excess return means that the insurer has an incentive to invest in illiquid bonds. One plausible reason is that changes in asset value A_2 may not always translate one-to-one into changes in equity value, $\Phi'(A_2) \neq 1$. Notably, limited liability and state guarantee funds partially protect insurers in bad states of the world, which implies that $\mathbf{E}[M(\Phi'(A_2) - 1)R_I] > 0$,[10] reflecting a classic risk-shifting incentive. Another source of the excess return for illiquid bonds is a possibility that the insurer may earn a higher-than-market average return on its bond portfolio, $R_I \neq R_I^*$ and $\mathbf{E}[R_I - R_I^*] > 0$. For instance, Chodorow-Reich, Ghent, and Haddad (2021) provide evidence that due to the nature of their liabilities, insurers have a comparative advantage in holding and trading illiquid bonds.

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[10] Consider the case of limited liability. We have that $\Phi'(A_2) = 0$ in default states, and $\Phi'(A_2) = 1$ otherwise. Since, in default states, we are likely to have $R_I < 0$, it follows that $\mathbf{E}[M(\Phi'(A_2) - 1)R_I] > 0$ even if $\mathbf{E}[MR_I] = 0$.

The insurer also faces the disincentives to invest in illiquid bonds, which arise from the expected (future) fire-sale costs $\mathbf{E}[M\Phi'(A_2)c\tau]$ and the shadow costs of regulatory capital $\lambda\rho\gamma_I$. By taking c and τ as given, the insurer does not consider potential equilibrium effects of its investment in illiquid bonds, leading to overinvestment. This is because the insurer ignores the fact that by investing more in illiquid bonds at $t=0$, there will be (marginally) more fire sales in the economy at $t=1$ (if adverse shocks hit), hurting all insurers through even lower liquidation prices (fire-sale externality, as in Stein [2012] among others).

Equation (9) describes the insurer's hedging incentives. The benefit of hedging is that it releases regulatory capital, though at a declining rate, and hence reduces the shadow costs by $\lambda\rho\kappa(1-2\eta h)\gamma_G$. The cost of hedging arises from the fact that it reduces the net stock allocation α_S and hence deprives the insurer of the excess return of the stocks $|\delta|\mathbf{E}[M((\Phi'(A_2)-1)R_S+R_S-R_S^*)]$.

Combining Equations (8) and (9), we obtain the optimal hedge ratio:

$$h^* = \frac{1}{2}\frac{1}{\eta}\left(1-\frac{\psi_S}{\psi_I}\right), \tag{10}$$

where

$$\psi_I = \frac{\mathbf{E}[M((\Phi'(A_2)-1)R_I+(R_I-R_I^*))]-\mathbf{E}[M\Phi'(A_2)c\tau]}{\gamma_I}, \tag{11}$$

$$\psi_S = \frac{|\delta|\mathbf{E}[M((\Phi'(A_2)-1)R_S+(R_S-R_S^*))]}{\kappa\gamma_G}. \tag{12}$$

The term ψ_I captures the "benefit" of hedging, which equals the discounted excess return per unit of RBC for illiquid bonds. The term ψ_S captures the "cost" of hedging, which equals the discounted excess return on stocks per unit of RBC relief (from the first dollar of guarantee hedging). Equation (10) shows that guarantee hedging is optimal, that is, $h^*>0$, only if $\psi_I>\psi_S$.

Substituting h^* into the capital constraint (1), we obtain the optimal investment in illiquid bonds:

$$\alpha_I^* = \frac{1}{\gamma_I A}\left(\frac{K}{\rho}-\gamma_S\overline{\alpha}_S A-\gamma_T T-(1-h^*\kappa(1-\eta h^*))\gamma_G G\right). \tag{13}$$

Optimal allocations to stocks α_S^* and liquid bonds α_F^* can then be derived from condition (2) and the identity $\alpha_F+\alpha_S+\alpha_I=1$.

2.4 Impact of VA guarantees on insurer's risk exposures

The characterization of an insurer's optimal hedge coverage and asset allocation allows us to assess the impact of guarantee writing on the insurer's risk exposures. First, the guarantees expose insurers to stock market shocks. Second, writing the guarantees may help relax the RBC constraint and increase illiquid bond investments, possibly amplifying the impact of shocks through fire-sale externality. We refer to the additional illiquid bond investments, if positive, as reaching for yield (RFY), as they are motivated by discounted excess returns of illiquid bonds, per unit of RBC, being positive (thus higher than those of liquid bonds, for which we normalize the return and RBC to zero) and potentially higher than those of common stocks. In addition, assuming that the traditional insurance and VA guarantees have the same exposures to longevity and interest rate risks, another important incremental risk is volatility risk, which we do not formally analyze here but do so in reduced form in our fire-sale simulation. Below, we present some comparative statics to help guide our subsequent empirical exercise, in which we evaluate whether and to what extent VA guarantees increase the insurer's stock market exposure and engender RFY.

To identify the impact of guarantees on stock market exposure, we consider an insurer with liabilities (G,T), and compare its actual total stock market exposure (from both assets and liabilities) $\alpha^*_{S,Total}(G,T)$ to a counterfactual exposure $\alpha^*_{S,Total}(0,L)$. The latter is the total stock market exposure if the insurer hypothetically maintained an equal-size liability portfolio, $L=G+T$, but replaced the guarantee by traditional insurance liabilities, that is, $G=0$ and $T=L$. Since the target stock allocation $\overline{\alpha}_S$ is a common component in both $\alpha^*_{S,Total}(G,T)$ and $\alpha^*_{S,Total}(0,L)$, the guarantees increase the insurer's total stock market exposure, net of hedging, by

$$\triangle\alpha^*_{S,Total}=\alpha^*_{S,Total}(G,T)-\alpha^*_{S,Total}(0,L)=\frac{(1-h^*\kappa(1-\eta h^*))|\delta|G}{A}, \tag{14}$$

assuming that the RBC relief from hedging accurately reflects the hedging effectiveness.

By the same calculation, the RFY tendency arising from the guarantees is captured by

$$\begin{aligned}\triangle\alpha^*_I &= \alpha^*_I(G,T)-\alpha^*_I(0,L) \qquad (15)\\ &= \frac{1}{\rho\gamma_I A}\big[(\Pi(G,T)-\Pi(0,L))-\rho((1-h^*\kappa(1-\eta h^*))\gamma_G-\gamma_T)G\big].\end{aligned}$$

Equation (15) shows that guarantee writing engenders RFY ($\triangle\alpha^*_I>0$) if the additional equity capital generated by the guarantees $(\Pi(G,T)-\Pi(0,L))$ exceeds the additional required capital $\rho((1-h^*\kappa(1-\eta h^*))\gamma_G-\gamma_T)G$.[11]

Finally, it is important to note that the two sources of risk arising from the guarantees are linked by guarantee hedging. As the insurer hedges more, the stock exposure decreases, resulting in a decrease in RBC and hence a larger capacity to invest in illiquid bonds. Equations (10), (14), and (15) also show that the transformation of stock into illiquid bond exposures increases in the insurer's sophistication $\frac{1}{\eta}$ and hedging effectiveness κ. The two parameters

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[11] Note that $\Delta\alpha^*_I$ is measured as a *fraction* of assets. Since guarantee writing increases the assets if $\Pi(G,T)>\Pi(0,L)$, the condition that illiquid bond allocation increases in *absolute* terms is weaker than $\Delta\alpha^*_I>0$.

work both directly and through the optimal hedge coverage. For the same hedge coverage, higher $\frac{1}{\eta}$ and κ mean hedging is more effective in reducing stock market risk, which means a larger reduction in stock exposures and a larger RBC relief. In addition, more effective hedging naturally leads to a higher optimal hedge coverage, which further decreases stock exposures and increases RFY.

2.5 Discussion of key assumptions

The mechanism through which guarantee writing leads to RFY, $\triangle\alpha_I^* > 0$, rests on three key assumptions, as outlined earlier. Below, we briefly discuss supporting empirical evidence.

Guarantee writing is more profitable than traditional business.[12] Grouping VA-writing insurers into terciles based on the ratio of guarantee reserve to capital, we find that profitability monotonically increases as we go from the lowest to highest terciles. The average ratios of net premium to total reserves over the period 2010–2019 are 22%, 23%, and 26% for the low, medium, and high reserve to capital ratio groups, respectively. The corresponding medians are 16%, 18%, and 22%.[13]

Hedging the guarantees provides RBC relief but the effectiveness of hedging and its associated RBC relief are limited and decrease as the insurer hedges more. Guarantees are essentially put options on various mutual funds, ranging from pure index to actively managed funds. Insurers hedge their guarantees by dynamically selling the guarantee deltas (e.g., selling futures and (short) selling ETFs) or buying put options on relevant stock indices. Ankirchner, Schneider, and Schweizer (2014), Bauer (2020), and Li, Moenig, and Augustyniak (2020) study guarantee hedging and find that hedging effectiveness varies and significant basis risk remains in most cases. Regulators recognize the variation in hedging effectiveness, and set the RBC relief accordingly, with the maximum being 70% for the most effectively hedged guarantees.

In Schedule DB, where insurers report their derivatives positions, derivatives used for hedging are grouped into “effective” and “others” with effective hedging defined as the correlation between the returns on the hedged item and the hedging instruments being 0.9 or higher. The NAIC’s Capital Markets Special Report (2021) documents that just about 10% of life insurers’ hedging using derivatives is considered effective. With a gradation of hedging effectiveness for various written guarantees, insurers should optimally hedge the guarantees for which hedging is most effective first, as doing so results in the largest RBC relief and is potentially least costly (e.g., due to less frequent

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[12] Our analysis does not, per se, require that guaranteed VAs be more profitable than traditional business. However, as shown in equation (15), one of the two sources of capital that supports RFY is the business profit.

[13] In our estimation, we specify and estimate an insurer’s profit function, allowing each insurer’s profits to depend on the mix of guaranteed VA and traditional insurance businesses. We do not impose any restrictions that force VAs to be more profitable.

rebalancing). As the insurers hedge more, they cover increasingly difficult-to-hedge guarantees, and hedging effectiveness declines. We estimate that insurers in the top tercile of guarantee reserve to capital ratio hedge, on average, about 80% of their guarantee exposures (Section 3). Among the derivatives reported as being used for hedging VA guarantees, about 4% are classified as effective. In contrast, insurers in the bottom tercile of guarantee reserve to capital ratio hedge about 60% of their guarantee exposures, and classify 10% of the derivatives used for that purpose as effective.

Insurers have a fixed stock-bond preference and therefore adjust risk taking primarily by changing the mix of liquid and illiquid bonds. Insurers invest mainly in two groups of risky assets, illiquid fixed income assets and common stocks. On average, VA-writing insurers invest about 4% in (cash) common stocks and 36-42% in the fixed income assets that we classify as illiquid bonds (Table 1). We find that insurers maintain a fairly stable stock allocation. Internet Appendix Table B.2 shows that insurer-level fixed effects explain about 82% of the variation of (cash) stock holdings, and even the stock holdings in 1996 alone explain almost 30% of the variation of stock holdings up to 23 years later. In contrast, Internet Appendix Table B.3 shows that insurer-level fixed effects explain about 43-67% of the variation for most illiquid bond types, and the holdings in 1996 explain only 6-15% of the variation. The exceptions are mortgages and loans, with holdings that tend to be stickier, perhaps due to their extreme illiquidity. In addition, looking forward to the results, a case study of insurers' trading during the global financial crisis (in Panel A of Table 7) shows that insurers de-risk primarily by selling illiquid bonds to buy liquid bonds.

3. Data and Model Calibration

3.1 Data

To calibrate the model, we use the National Association of Insurance Commissioners data, obtained through S&P Global Market Intelligence, on guaranteed VAs' account values and gross reserves (from the General Interrogatories [2003–2016] and Variable Annuities Supplement, Parts 1 and 2 [2017–2019]), portfolio holdings (from Schedules A, B, BA, and D), and derivatives positions (from Schedule DB). The data frequency is annual and the unit of observation is firm-year, where each firm refers to a stand-alone life insurer or a consolidated balance sheet of all life insurers in the same group. While our data start from 2003, we only use the sample spanning 2010–2019 for the calibration; in 2009, the NAIC changed the method for calculating the reserve and RBC for VA guarantees. The reserves and RBC ratios, pre- and post-2009, are not directly comparable.

Figure 4 plots the total guaranteed VAs' account value (i.e., value of policyholders' accounts as determined by the underlying separate account assets and the guarantees), summed across all insurers, and the associated gross reserve over time. The account value increased significantly from about

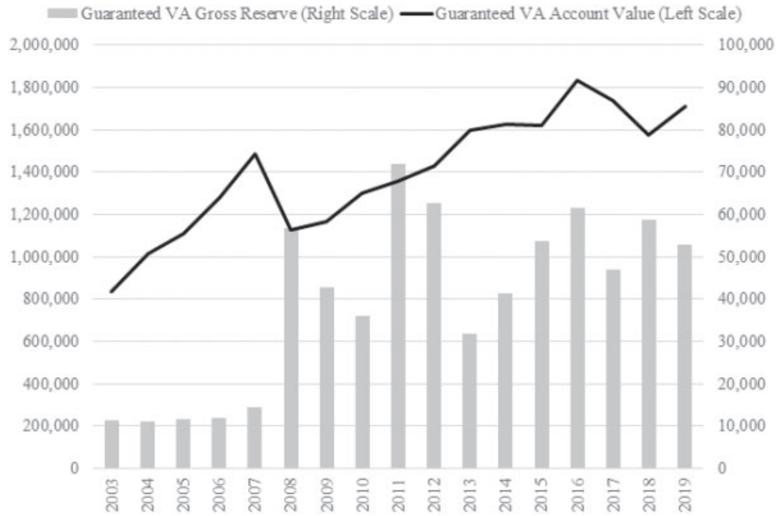

Figure 4
Guaranteed VA account value and gross reserve over time
This figure plots the time series of account value (line) and gross reserve (bar) ($ million) for guaranteed VA, as reported annually by life insurers in the NAIC's general interrogatories form (up to 2016) and variable annuities supplement form (2017–2019). The sample period is from 2003 to 2019. The account value and gross reserve are summed over individual life insurers with outstanding guaranteed VA.

$840 billion in 2003 to almost $1.5 trillion in 2007, due in part to the rise in the stock market. Over the same period, the gross reserve remained relatively low, because the guarantees were deep out-of-the-money. In 2008, as the stock market collapsed, the aggregate account value dipped, and the gross reserve spiked from about $10 billion to almost $60 billion. Since then, the account value recovered and eventually surpassed the previous peak in 2007. Despite the recovery, the gross reserve remained relatively high and volatile (partly due to the new method in calculating the reserve, as discussed in Section 1), ranging between $32 and $72 billion.

Only a fraction of life insurers write VAs with guarantees, and an even smaller number do so for a significant amount. These insurers tend to be large and sophisticated, and are not directly comparable to those that do not write VAs (see Internet Appendix Table B.1). We thus focus on the sample of 90 insurers that write VAs with guarantees at some point, and identify the effects of interest using their cross-sectional differences. Our sample covers about 20% (by number) and 88% (by assets) of the life insurance industry in 2019. To observe firm attributes and asset allocations that may be associated with guarantee exposures, we divide the insurers each year into three groups—high, medium, and low - by the ratio of VA gross reserve to capital. Table 1 presents summary statistics on several relevant variables. The statistics are pooled across firm-years in each group.

Table 1
Summary statistics of life insurers' characteristics and asset allocations

	[1] High			[2] Medium			[3] Low			[1] − [2]	[1] − [3]
	Mean	Std. dev.	Median	Mean	Std. dev.	Median	Mean	Std. dev.	Median	Mean	Mean
A. Firm characteristics											
Gross reserve to capital (%)	38.175	47.768	17.994	2.666	2.196	1.944	0.052	0.089	0.019	35.508***	38.123***
Gross reserve to acct. val. (%)	10.695	22.902	3.453	4.214	16.896	0.761	0.780	1.711	0.181	6.482**	9.915***
Assets ($ Million)	76,672	69,453	58,623	53,122	70,706	29,110	48,563	73,765	18,053	23,549	28,109
Capital and surplus ($ Million)	7,635	7,147	6,095	5,506	6,650	3,236	5,372	8,307	1,893	2,129	2,263
RBC ratio	9.288	2.468	9.488	10.867	5.939	9.868	11.083	4.774	9.786	−1.578	−1.795*
Return on equity	0.093	0.154	0.096	0.086	0.098	0.080	0.087	0.068	0.073	0.007	0.007
Stock return	0.135	0.273	0.105	0.148	0.327	0.119	0.137	0.251	0.099	−0.013	−0.002
B. Asset allocation											
Liquid bonds	0.578	0.096	0.568	0.589	0.109	0.584	0.600	0.113	0.589	−0.012	−0.023
Cash	0.034	0.030	0.030	0.026	0.036	0.017	0.022	0.022	0.015	0.008	0.012**
Synthetic cash	0.049	0.060	0.025	0.010	0.040	0.000	0.001	0.003	0.000	0.039***	0.048***
Pub. bonds in NAIC 1	0.235	0.076	0.239	0.253	0.084	0.244	0.263	0.115	0.238	−0.019	−0.028
Pub. bonds in NAIC 2	0.212	0.051	0.212	0.229	0.063	0.230	0.225	0.060	0.213	−0.017	−0.013
Agency ABS in NAIC 1	0.046	0.040	0.039	0.070	0.053	0.051	0.090	0.063	0.081	−0.024*	−0.044***
Agency ABS in NAIC 2	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.000

(*Continued*)

Table 1
Continued

	[1] High			[2] Medium			[3] Low			[1] − [2]	[1] − [3]
	Mean	Std. dev.	Median	Mean	Std. dev.	Median	Mean	Std. dev.	Median	Mean	Mean
Illiquid bonds	0.416	0.061	0.420	0.370	0.096	0.386	0.350	0.102	0.363	0.046**	0.066***
Long-term assets	0.035	0.029	0.030	0.031	0.028	0.026	0.029	0.026	0.024	0.005	0.006
Priv. bonds in NAIC 1	0.028	0.011	0.023	0.028	0.011	0.027	0.030	0.017	0.027	0.000	−0.002
Priv. bonds in NAIC 2	0.027	0.011	0.024	0.027	0.011	0.027	0.028	0.013	0.027	0.000	−0.001
Bonds in NAIC 3–6	0.035	0.019	0.030	0.032	0.017	0.032	0.030	0.018	0.026	0.003	0.004
Agency ABS in NAIC 3–6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Priv.-label ABS in NAIC 1	0.100	0.046	0.099	0.094	0.050	0.094	0.100	0.080	0.093	0.006	0.000
Priv.-label ABS in NAIC 2	0.015	0.014	0.010	0.011	0.012	0.008	0.013	0.017	0.008	0.004	0.002
Priv.-label ABS in NAIC 3–6	0.009	0.008	0.006	0.006	0.006	0.005	0.006	0.006	0.004	0.003**	0.003**
Mortgages	0.110	0.041	0.112	0.091	0.055	0.102	0.081	0.061	0.087	0.019**	0.029**
Loans	0.042	0.036	0.029	0.042	0.042	0.029	0.027	0.026	0.020	−0.001	0.015*
Derivatives for income gen.	0.017	0.021	0.012	0.008	0.011	0.005	0.006	0.012	0.001	0.009**	0.012***
Common stock exposures	−0.011	0.074	0.006	0.029	0.046	0.017	0.038	0.037	0.024	−0.040***	−0.049***
Common stocks	0.038	0.039	0.030	0.039	0.032	0.033	0.039	0.037	0.025	−0.001	−0.001
Synthetic common stocks	−0.049	0.059	−0.024	−0.010	0.041	−0.001	−0.001	0.003	0.000	−0.039***	−0.048***
Other assets	0.015	0.018	0.008	0.011	0.010	0.009	0.011	0.011	0.008	0.004	0.004
Preferred stocks	0.003	0.005	0.002	0.004	0.006	0.001	0.003	0.004	0.001	−0.001	0.000
Real estates	0.007	0.017	0.002	0.003	0.003	0.002	0.005	0.008	0.002	0.006	0.003
Securities lending	0.006	0.011	0.001	0.005	0.007	0.001	0.003	0.007	0.000	0.001	0.003

This table presents summary statistics of general firm characteristics (panel A) and asset allocations (panel B). The data are from the NAIC, obtained through S&P Global Market Intelligence. The sample period is from 2010 to 2019 (to match the sample used for parameter estimation), and the observations are insurer-year. The sample includes only (NAIC group-level) insurers that write guaranteed variable annuities. A total of 90 unique insurers have guaranteed VA at some point during the sample period. Each year, insurers are divided into three groups by the ratio of guaranteed VA gross reserve to capital. Insurers in groups 1, 2, and 3 have the ratios of guaranteed VA gross reserve to capital that are in the top, middle, and bottom terciles, respectively. For each insurer, assets include all invested assets at book values in the general account, and capital and surplus are assets minus liabilities. The risk-based capital ratio is the (adjusted) statutory capital divided by the required risk-based capital. Return on equity is net income divided by common equity, as reported under GAAP. Stock return includes both percentage price change and dividend, and is available only for public insurer groups. Assets are divided into NAIC-defined categories, which are then grouped into liquid bonds, illiquid bonds, common stocks, and others. Asset allocation is the value of assets in each category or broad group divided by the value of all invested assets. Tests of difference in mean are conducted using pooled panel regressions with standard errors clustered by insurer. $^{*}p<.1$; $^{**}p<.05$; $^{***}p<.01$.

Panel A shows that the guarantee exposures of the life insurance industry are concentrated in the high-exposure group, with the average gross reserve to capital ratio of over 38%. The corresponding averages are only 2.67% and 0.05% for the medium and low groups. Insurers with higher guarantee exposures also write more generous guarantees, as reflected by the ratios of gross reserve to account value. The average Spearman's rank correlation between the reserve to capital ratio and the reserve to account value ratio is 0.94, suggesting that the sorting of insurers by either metric produces similar results. The statistics also show that insurers with high guarantee exposures are generally larger than the others, both in terms of assets (in the general account) and capital and surplus. The insurers in the high group also have slightly lower RBC ratios than those in the other two groups; the differences, however, are not economically significant. The returns on equity and stock returns are about the same for all three groups.

In panel B, we present the portfolio allocations. We split assets into three broad groups corresponding to liquid bonds, illiquid bonds, and common stocks in the model.[14] We use the term bond as a shorthand for fixed income assets, which encompasses non-bond fixed income assets such as mortgages, loans, and derivatives. The composition of each group is listed underneath the group's heading. The insurers with high guarantee exposures have a slightly lower liquid bond allocation than those in the other two groups (not significant at conventional levels). Considering different components of the liquid bonds, we find that the high-exposure insurers hold significantly less agency asset-backed securities (ABS) in NAIC class 1 than the others, but the differences are more or less offset by the synthetic cash from selling stock futures.

High-VA insurers have a significantly higher allocation to illiquid bonds than those in the medium and low groups (42% vs. 35–37%). The differences are generally in the same direction for all types of illiquid bonds, although the magnitudes are most economically and statistically significant among private-label ABS (collectively across all NAIC classes), mortgages, loans, and derivatives. In Internet Appendix Table B.3, we run separate, reduced-form OLS regressions of the allocation to each type of illiquid bond on its own lag in 1996 (several years before the start of our sample), the ratio of the gross reserve to capital, and other time-varying attributes. Our goal is to absorb the unobserved time-invariant insurer effects and potential time-varying confounders. Interpreting the coefficient of reserve to capital ratio as capturing a within-insurer effect, we find that the same insurers increase allocations to most types of illiquid bonds as their guarantee exposures increase. As the reserve to capital ratio increases from 0 to 38%, equivalent to an increase from the average of insurers with low guarantees to the average of insurers with high guarantees, the combined allocation to illiquid bonds increases by approximately 3%.

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[14] We focus on these three groups, lumping together all others, which account for a very small fraction of portfolios, as *other assets*.

For common stocks, we consider both cash stocks and equity derivatives, and find that insurers with high guarantee exposures have slightly smaller allocations to cash stocks, sell significantly more stock futures and other derivatives (e.g., total return swaps), and hence have significantly lower net asset-side stock exposures (-1.1% vs. 2.9–3.8%) (these include the guarantee hedging, which is reported as part of the assets, but excludes the guarantees' gross stock exposures, which we will estimate in the next section). In column (5) of Internet Appendix Table B.2, we run reduced-form OLS regressions of the (cash) stock allocation on its own lag in 1996, the reserve to capital ratio, and other time-varying controls. We find that insurers decrease the (cash) stock allocation by less than 0.5% as their reserve to capital ratio increases from 0 to 38%. In column (6), we replace the (cash) stock allocation by the net asset-side stock exposure, and find that it decreases by about 3% for the same 38% increase in the reserve to capital ratio.

Together, the summary statistics for the asset allocations and the results from reduced-form multivariate analyses are consistent with our model's predictions for the parameter space in which guarantee writing increases investments in illiquid bonds (i.e., $\triangle\alpha_I^*$ in Equation (15) is positive).[15]

3.2 Model calibration

To formally isolate the effects of VA guarantees on insurers' asset allocations, we calibrate the parameters of our model using two main relations. First, we use Equation (13) and exploit the variation in illiquid bond holdings and other observed quantities on the right-hand side to measure the model parameters that govern insurers' asset allocation and guarantee hedging. Second, the guaranteed VA and traditional businesses have different profitabilities, which implies that the gross underwriting profits, $\Pi(G,T)$, vary across insurers with different mixes of the two businesses. Below, we summarize our calibration procedure, and Internet Appendix C.1 provides full details.

We parameterize different components of Equation (13), and rearrange to obtain the following regression model:

$$\frac{\frac{K}{\rho}-\gamma_I\alpha_I^*A-\gamma_G G}{A}=b_0+b_1\widehat{\ln(1+A)}+\sum_{i\geq 2}b_i x_i+c_1\frac{T_{Life}}{A}+c_2\frac{T_{Annuity}}{A}$$

$$+c_3\left(-\frac{\kappa\gamma_G}{4}\cdot\frac{G}{A}\cdot\frac{1}{\widehat{\ln(1+A)}}\right)$$

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[15] In Internet Appendix Table B.4, we try to isolate the hedging channel by including the amount of linear derivatives used to hedge the guarantees in the regression of illiquid bond investments. We continue to use the reserve to asset ratio to capture the profitability channel. Our results show that both channels are significant. However, it is important to exercise some caution as insurers can hedge the guarantees simply by reducing the amount of (cash) common stocks held, and hence hedging can only be inferred once the target holding of common stocks is known. This is precisely why we estimate the hedge ratios for different insurers by imposing our model on the data.

$$+c_4\left(\frac{\gamma_I^2|\delta|^2}{4\kappa\gamma_G}\cdot\frac{G}{A}\cdot\frac{1}{\widehat{\ln(1+A)}}\right)+\epsilon, \tag{16}$$

where b_0 absorbs the average bias in estimating the RBC, which we assume to be orthogonal to all of the terms on the right-hand side, $b_1=\gamma_S\bar{\alpha}_{S1}\eta_0$, $b_i=\gamma_S\bar{\alpha}_{Si}$ for $i\geq 2$, $c_1=\gamma_{T-Life}$, $c_2=\gamma_{T-Annuity}$, $c_3=1/\eta_0$, $c_4=\Theta^2/\eta_0$, and ϵ is the random error. In estimating these coefficients, we allow b_0 to vary over time by including year fixed effects. The left-hand side of (16) is the residual RBC, defined as the total RBC minus the RBC from illiquid bonds and VA guarantees (pre-hedging) as a fraction of assets, while the right-hand side captures the RBC associated with other sources that depend on the model parameters. These sources include the fixed stock allocation, the traditional business, and the guarantee hedging credit.

We define the fixed stock allocation $\overline{\alpha}_S=\overline{\alpha}_{S1}\eta+\sum_{i\geq 2}\overline{\alpha}_{Si}x_i$, and the inverse sophistication index $\eta=\eta_0\widehat{\ln(1+A)}$ with $\widehat{\ln(1+A)}$ equal to four minus the normalized logged total assets.[16] The parameterization implies that larger insurers are more sophisticated, and allows the sophistication to directly influence the preference for bonds versus stocks (in addition to its indirect effect on stock exposures through guarantee hedging). Plausibly, larger insurers can invest more to improve their risk management capabilities and capture an economy of scale in managing risk. In addition, we assume that the traditional business consists of life and annuity businesses, and $\gamma_T T=\gamma_{T-life}T_{Life}+\gamma_{T-Annuity}T_{Annuity}$. Finally, we use Θ to represent the ratio of insurers' discounted excess return on common stocks to discounted excess return on illiquid bonds (again, both over the returns implied by the market prices):

$$\Theta=\frac{\mathbf{E}[M((\Phi'(A_2)-1)R_S+(R_S-R_S^*))]}{\mathbf{E}[M((\Phi'(A_2)-1)R_I+(R_I-R_I^*)]-\mathbf{E}[M\Phi'(A_2)c\tau]}, \tag{17}$$

which affects the RBC relief from guarantee hedging and the optimal hedge ratio (Equation (10)).

With the exception of $|\delta|$ and γ_G, we obtain the variables for the regression from insurers' annual filings, and pre-specify the parameters, other than those to be estimated, using regulatory values. Specifically, we use the general account assets as A, the VA gross reserve as G, the reserve for life contracts as T_{Life}, the reserve for annuities as $T_{Annuity}$, the adjusted capital as K, the RBC ratio as ρ, and the illiquid bond allocation as α_I^*. In addition, we assume the maximum RBC relief $\kappa=0.7$ and the RBC charge for stocks $\gamma_S=0.3$, and calculate γ_I for each insurer as the weighted average RBC charge for all types of fixed income assets that make up the illiquid bonds.

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[16] In our model, the inverse sophistication index η must be positive and, by definition, decreasing in sophistication. We use A to capture sophistication; thus, for our empirical implementation, we need $\widehat{\ln(1+A)}$ to be positive and decreasing in A. Normalized logged assets has a mean of zero and standard deviation of one, and lies within a range from -3 to 3. By multiplying it by -1 and adding 4, we ensure that $\widehat{\ln(1+A)}$ is positive and decreasing in A.

We include the insurer's stock allocation in 1996 and the ratio of capital and surplus to assets as x_2 and x_3, as these are additional determinants of the stock allocation that would have prevailed in the absence of guarantees. The results in columns (3)–(5) of Internet Appendix Table B.2 show that the stock allocation in 1996 captures well the time-invariant firm-specific preferences (in the spirit of Lemmon, Roberts, and Zender [2008]). It explains almost 30% of the variation in insurer's common stock investments, and its combination with $\widehat{\ln(1+A)}$ and the capital to assets ratio allows further flexibility to equate the means of predicted and observed stock allocations.

The primary challenge is to determine $|\delta|$, the sensitivity of the guarantee reserve (as a percentage of the reserve G) to a small change in the stock market. We do not directly observe $|\delta|$, but do observe the reserve and the underlying account value. To infer $|\delta|$, we rely on the idea that the more generous is the guarantee, the higher is the value of reserve per unit of the underlying account. We make a simplifying assumption that the guarantee is an 18-year put option and the underlying asset is the S&P 500 index, and calculate the reserve by simulating the path of the index from the end of each year over the next 18 years. Following practice, we calculate the present value of the maximum loss under each simulated path and average it across the worst 30% scenarios to find the reserve. We calibrate the strike price of the put option such that the ratio of the reserve to the underlying account value (net of the hedging credit) matches the observed ratio for each insurer in each year. The effective delta is the sensitivity of the gross reserve to a small change in the S&P 500 index at the calibrated strike price. In addition, for each insurer, we calculate the RBC requirement for the guarantee γ_G as the average of the worst 10% scenarios minus the reserve (normalized by the reserve G) at the same calibrated strike price.

We estimate Equation (16) by OLS using panel data from 2010 to 2019. Column (1) of Table 2 reports the coefficient estimates (panel A) and the implied model parameters (panel B). We use White's heteroskedasticity robust standard errors for the coefficient estimates, and calculate the standard errors of the inferred model parameters by Monte Carlo simulation. The implied model parameters have the predicted signs and reasonable magnitudes. First, η_0 is 0.102 (statistically significant at 1%), suggesting that insurers that are larger tend to be more effective at hedging. Second, the insurer-specific stock allocation loads positively on the inverse sophistication index, the allocation to common stocks in 1996, and the ratio of capital and surplus to assets. The magnitudes of the coefficients are similar to those of the reduced-form estimates in Internet Appendix Table B.2. The parameter $\overline{\alpha}_{S1}$ of 0.039 implies that larger, and by our definition, more sophisticated firms invest less in common stocks and potentially more in illiquid bonds, even in the absence of VA guarantees. As we go from the most sophisticated insurer with an η of 2.189 to the least sophisticated insurer with an η of 6.288, the fixed stock allocation $\overline{\alpha}_S$ increases by about 1.6%.

Third, our estimate of Θ is 3.315 (statistically significant at 1%), indicating that the discounted excess return on common stocks is about three times that of illiquid bonds. The RBC charge for stocks γ_S is 0.3, while the weighted average RBC charge for illiquid bonds held by insurers γ_I (given the composition of different bond types in 2019) is about 0.027, less than 10% of the RBC charge for stocks. Thus, our estimate of Θ implies that insurers subject to the RBC constraint would prefer to take risk in illiquid bonds rather than stocks. In addition, given the average $|\delta|$ of 2.276, average γ_G of 1.223, κ of 0.7, and γ_I of 0.058, the ratio of cost to benefit of hedging, ψ_S/ψ_I, is $0.238 < 1$, indicating that the average insurer faces incentives to hedge the guarantees and use the capital relief from hedging to support illiquid bond investments.

Fourth, our estimates of γ_{T-Life} and $\gamma_{T-Annuity}$ are 0.020 and 0.018, respectively, resulting in the weighted average RBC charge for traditional business γ_T of about 0.019 (statistically significant at 5%). Since the RBC charges for life and annuity-related products do not differ significantly, our estimate of γ_T does not vary much across firms and over time. It is important to note that we do not explicitly model hedging and other forms of risk management for the traditional business and therefore, unlike γ_G, the parameter γ_T should be viewed as net of hedging and reinsurance credits.

Table 2
Estimates of model parameters

A. OLS estimates of parametrized model relation

Dependent variable:	Residual RBC/Assets (1)	Net premium to reserves (2)
$\widehat{\ln(1+A)}$	0.001***	−0.022***
	(0.000)	(0.008)
$\bar{\alpha}_{S,1996}$	0.107***	
	(0.028)	
$1-Leverage$	0.227***	0.443**
	(0.027)	(0.190)
T_{Life}/A	0.020***	
	(0.006)	
$T_{Annuity\ (excl.G)}/A$	0.018*	
	(0.010)	
$-\frac{\kappa\gamma_G}{4}\cdot\frac{G}{A}\cdot\frac{1}{\widehat{\ln(1+A)}}$	9.7696***	
	(0.946)	
$\frac{\gamma_I^2\lvert\delta\rvert^2}{4\kappa\gamma_G}\cdot\frac{G}{A}\cdot\frac{1}{\widehat{\ln(1+A)}}$	108.359***	
	(17.688)	
G/L		1.509***
		(0.310)
$(G/L)^2$		−4.674***
		(1.273)
$(\Pi(G,T)/L)_{1996}$		0.189***
		(0.038)
Year fixed effects	YES	YES
Observations	534	534
R-squared	0.489	0.146

(*Continued*)

Table 2
Continued

B. Implied model parameters

	From residual RBC/Assets eqn. (1)	From net premium to reserves eqn. (2)
η_0	0.102***	
	(0.010)	
Mean(η)	0.339***	
	(0.033)	
$\bar{\alpha}_{S1}$	0.039***	
	(0.009)	
$\bar{\alpha}_{S2}$	0.356***	
	(0.093)	
$\bar{\alpha}_{S3}$	0.757***	
	(0.090)	
Mean($\bar{\alpha}_S$)	0.039***	
	(0.003)	
Θ	3.315***	
	(0.157)	
Mean(γ_T)	0.019**	
	(0.009)	
e_T		0.198***
		(0.006)
e_G		1.725***
		(0.294)
f		4.721***
		(1.199)

This table reports (i) OLS estimates for parameterized relation between guarantee writing/hedging and ratio of residual risk-based capital to assets (panel A, column (1)) and between guarantee writing and overall underwriting profits (panel A, column (2)), and (ii) the implied model parameters (panel B). The sample period is 2010–2019. Only insurers that have guaranteed VA at some point during the sample period are included. Observations are insurer (NAIC insurance group)-year. In column (1) of panel A, the dependent variable is the ratio of residual RBC to assets where the residual RBC equals the total RBC minus the RBC on all bond and other nonstock investments minus the RBC on gross (prehedging) guarantee exposures. The explanatory variables are grouped into three groups: those that explain RBC on stocks investments, those that explain RBC on traditional business, and those that explain RBC credit for hedging the guarantees. The regression is as follows:

$$\frac{\mathit{Residual\ RBC}}{A} = \overbrace{b_0+b_1\ln(\widehat{1+A})+b_2\bar{\alpha}_{S,1996}+b_3(1-\mathit{Leverage})}^{\mathit{RBC\ on\ Stocks}}+\overbrace{c_1\frac{T_{\mathit{Life}}}{A}+c_2\frac{T_{\mathit{Annuity(excl.G)}}}{A}}^{\mathit{RBC\ on\ Traditional\ Business}}$$

$$\underbrace{+d_1\left(-\frac{\kappa\gamma_G}{4}\cdot\frac{G}{A}\cdot\frac{1}{\ln(\widehat{1+A})}\right)+d_2\left(\frac{\gamma_I^2|\delta|^2}{4\kappa\gamma_G}\cdot\frac{G}{A}\cdot\frac{1}{\ln(\widehat{1+A})}\right)}_{\mathit{RBC\ Credit\ for\ Guarantee\ Hedging}}$$

where A denotes total assets (\$ million); $\ln(\widehat{1+A})$ equals four minus normalized logged total assets (to ensure positivity and decreasing relation); G denotes VA gross reserves; $\bar{\alpha}_{S,1996}$ denotes preferred gross common stock allocation as a fraction of assets in year 1996; $(1-\mathit{Leverage})$ is the ratio of capital and surplus to assets; $|\delta|$ denotes sensitivity of the reserves to change in underlying stock price, normalized by the reserves; γ_G and γ_I are, respectively, the RBCs per unit of guarantee reserves and illiquid bond investments; and κ is the maximum credit for hedging. $|\delta|$ is calculated by assuming that the guarantee is an 18-year put option and inferring the strike by iteratively matching the average of 30% worst outcomes to the reported reserve, net of inferred hedging credit, for each insurer-year observation. γ_G is the average of 10% worst outcomes in excess of the reserve, also calculated from the simulation of the put option value over its life. For each insurer, γ_I is the weighted average of RBC charge across all assets that together compose the illiquid bonds. The RBC charges for individual assets and ratings are given by the regulation. κ is 0.7, also given by the regulation. The coefficient estimates, b's, c's, and d's, imply the model parameters in column (1) of panel B, as well as the averages of inverse sophistication, η; stock preference, $\bar{\alpha}_S$; and RBC for traditional business, γ_T. In column (2) of panel A, the dependent variable is the net premium per unit of total liabilities, $\Pi(GT)/L$, which proxies for the underwriting profit, and the explanatory variables are the fraction of reserves that is attributable to the guarantee, G/L, and its square, as in the equation below:

$$\frac{\Pi(G,T)}{L}=\overbrace{a_0+a_1\frac{G}{L}+a_2\left(\frac{G}{L}\right)^2}^{\mathit{Avg.\ Profit\ from\ Underwriting}}\ \overbrace{+a_3\ln(\widehat{1+A})+a_4\left(\frac{\Pi(G,T)}{L}\right)_{1996}+a_5(1-\mathit{Leverage})}^{\mathit{Controls\ for\ Other\ Influences}}.$$

The coefficient estimates, a's, imply the model parameters in column (2) of panel B. In panel A, both regression models include year fixed effects, and White's heteroskedasticity robust standard errors are in parentheses. In panel B, standard errors in parentheses are calculated by Monte Carlo simulation, using the point estimates and the covariance matrices from the regressions in panel A. $*p<.1$; $**p<.05$; $***p<.01$.

We next turn to the profitability of the guarantee relative to that of traditional business. Our model does not specify the functional form for the total profit, $\Pi(G,T)$, which we need in order to determine how much the guarantees contribute to the insurer's capital and support illiquid bond investments. We assume that the traditional business's profits exhibit a constant return to scale at the rate e_T (per unit of reserve), but the grantee's profits exhibit a diminishing return to scale, starting at the rate e_G and decreasing at the rate $2f$. The average profit for an insurer with the reserve of G therefore equals $e_G - fG$, and the total profit $\Pi(G,T) = e_T T + (e_G - fG)G$. Substituting $T = L - G$, dividing by L, and adding some controls, we obtain the following regression model:

$$\frac{\Pi(G,T)}{L} = a_0 + a_1 \frac{G}{L} + a_2 \left(\frac{G}{L}\right)^2 + controls + \xi, \tag{18}$$

where the regression coefficients $a_0 = e_T$, $a_1 = e_G - e_T$, $a_2 = -f$, and ξ is the random error. Variables G, L, and $\Pi(G,T)$ vary across insurers and over time. We include as controls $\widehat{\ln(1+A)}$ to capture the effects of sophistication, $\Pi(G,T)/L$ in 1996 to capture time-invariant firm-specific effects, and the ratio of capital and surplus to assets to absorb the effects of leverage. We also allow the intercept a_0 to vary over time by including year fixed effects, and report the implied parameter e_T by averaging over the time-specific intercepts.

We estimate (18) using the same panel data as above. While G/L (the guarantee over total reserves) can be taken directly from the data, the total profit or equity $\Pi(G,T)$ does not correspond to the statutory capital and surplus, as those are affected by past dividends, equity issuance, as well as accumulated investment returns, and not just the insurer's business mix. We therefore use the gross underwriting profit in place of capital, and regress the profit to total reserve ratio on the guarantee over total reserves and its square.[17]

Column (2) of Table 2 reports the regression coefficients (panel A), with White's heteroskedasticity robust standard errors in parentheses, and the implied profitability parameter estimates (panel B), with standard errors in parentheses calculated by Monte Carlo simulation. Our estimate for e_T is 0.198 (each dollar of traditional business reserve comes with about 19 cents of profit or equity), and our estimate for e_G is 1.725, which implies that the initial profit from VAs with guarantees is much higher than that of traditional business. Our estimate of f is 4.721, suggesting that the guarantee will become less profitable than the traditional business after the guarantee over total reserves reaches about

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[17] We do not explicitly capitalize the gross profits in the profit regression. We do not know the appropriate discount rate, but to the extent that it is the same across insurers and over time, using it to discount the gross profits (assuming a certain growth rate) is equivalent to scaling them by a constant. Using the discounted gross profits as the dependent variable, in place of the gross profits themselves, would then result in all of the coefficient estimates being scaled by the same constant. This implies the same relative profits between the VA guarantees and traditional business as in our estimation (without capitalizing the profits), and would have no impact on our results. We only use the profit parameters to calculate the counterfactual portfolios, and in doing so, we take the observed capital as given, and use the relative profits to partition it into two parts, one coming from the VA guarantees and the other from traditional business.

14%. All but two insurers in our sample have the guarantee over total reserves less than the implied profit maximizing level.

3.3 Counterfactual portfolios

Our goal is to estimate the contribution of VA guarantees to systemic risk, which requires that we determine the insurers' counterfactual risk exposures and asset portfolios absent the guarantees. That is, what if $G=0$ and $T=L$? In Section 2, we answer the question theoretically by deriving Equation (14) for the stock market exposures and Equation (15) for the RFY tendency. Below, we use these equations along with our parameter estimates from Section 3.2 for our empirical assessment.

First, VA guarantees expose insurers to additional stock market risk, which is partially offset by hedging. In panel A of Table 3, we report the summary statistics for guarantee hedging and net total stock market exposures. The first two rows report the statistics for comprehensive (i.e., using put options) hedge coverages. Insurers use equity options relatively sparsely, covering on average about 8.6% of the written guarantees, with 3.4% classified as "effective" hedging and 5.2% "other" hedging. The median insurer does not use any options. Our model parameters imply that insurers delta-hedge much of the remaining guarantee exposures. As reported in the third row, the average hedge ratio is 71.5%, with the 5th and 95th percentiles being 44.0% and 96.7%, respectively. Equation (10) dictates that insurers that write more generous guarantees, holding sophistication constant, hedge less,[18] while those that are more sophisticated hedge more. Insurers with higher guarantee exposures tend to write more generous guarantees and are more sophisticated; so, in theory, it is not clear whether they hedge more or less. Our results show that the sophistication effect dominates; the average hedge ratios are 79.6%, 72.2%, and 59.5%, for the high, medium, and low guarantee exposure groups, respectively.

After hedging, the guarantees increase the insurers' total stock market exposures, as a fraction of assets, by an average of 3.4%. Given that the average fixed stock allocation in the absence of guarantees is 3.9%, the guarantees almost double the insurers' stock market exposures. The increased exposures are positively skewed, with many insurers writing only a small amount of guarantee and, as a result, experiencing a negligible increase in stock market exposure. Besides the guarantee amount, Equation (14) shows that the increased exposures are also a function of the generosity $|\delta|$, hedge coverage h^*, and hedging effectiveness, as determined by κ and η. While insurers with high guarantee exposures hedge more and their hedging is generally more effective, their guarantee-induced stock market exposures, averaged at 8.4% of assets, are

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[18] Generosity increases both $|\delta|$ and γ_G, which generate two opposing effects (given the amount of written guarantees). An increase in $|\delta|$ decreases the optimal hedge ratio while an increase in γ_G increases the optimal hedge ratio. In our calculation assuming the guarantee is an 18-year put option, the first effect slightly prevails.

Table 3
Calibration results and counterfactual portfolios

A. Estimated VA hedge coverages

	Data			Estimation		
	Mean	PCT5	PCT95	Mean	PCT5	PCT95
Comprehensive hedging	0.086	0.000	0.296	—	—	—
Dynamic hedge coverage						
All	Unobserved			0.715***	0.440***	0.967***
				(0.056)	(0.069)	(0.108)
High VA exposures	Unobserved			0.796***	0.457***	0.987***
				(0.065)	(0.081)	(0.118)
Medium VA exposures	Unobserved			0.722***	0.439***	0.976***
				(0.058)	(0.079)	(0.112)
Low VA exposures	Unobserved			0.595***	0.390***	0.905***
				(0.059)	(0.110)	(0.109)
Net guarantee-induced stock market exposures						
All	Unobserved			0.034***	0.000	0.174***
				(0.002)	(0.000)	(0.008)
High VA exposures	Unobserved			0.085***	0.013***	0.288***
				(0.006)	(0.002)	(0.024)
Medium VA exposures	Unobserved			0.017***	0.001**	0.061***
				(0.001)	(0.000)	(0.005)
Low VA exposures	Unobserved			0.002***	0.000	0.007***
				(0.000)	(0.000)	(0.001)

B. Counterfactual portfolios for insurers with high VA exposures

	Portfolio C1: No VA			Portfolio C2: Same VA exposures and hedge ratio; no RFY		
	Mean	Mean − Actual	Mean − Low group	Mean	Mean − Actual	Mean − Low group
Liquid bonds	0.598***	0.020	−0.003	0.618***	0.041*	0.018
	(0.020)	(0.021)	(0.022)	(0.020)	(0.021)	(0.022)
Illiquid bonds	0.352***	−0.064***	0.002	0.371***	−0.045**	0.021
	(0.020)	(0.021)	(0.022)	(0.020)	(0.021)	(0.022)
Common stock exposures	0.033***	0.044***	−0.005	−0.007**	0.004	−0.045***
	(0.003)	(0.006)	(0.004)	(0.003)	(0.006)	(0.004)
Other assets	0.017	0.000	0.006	0.017	0.002	0.006
	—	—	(0.005)	—	—	(0.005)

(*Continued*)

Table 3
Continued

C. Counterfactual portfolios for insurers with medium VA exposures

	Portfolio C1: No VA			Portfolio C2: Same VA exposures and hedge ratio; no RFY		
	Mean	Mean − Actual	Mean − Low group	Mean	Mean − Actual	Mean − Low group
Liquid bonds	0.601***	0.014	0.001	0.606***	0.018*	0.005
	(0.006)	(0.010)	(0.010)	(0.006)	(0.010)	(0.010)
Illiquid bonds	0.351***	−0.020**	0.001	0.353***	−0.017**	0.003
	(0.004)	(0.008)	(0.008)	(0.004)	(0.008)	(0.008)
Common stock exposures	0.035***	0.007*	−0.003	0.029***	0.000	−0.009
	(0.003)	(0.004)	(0.004)	(0.003)	(0.004)	(0.004)
Other assets	0.012	0.000	0.001	0.012	0.001	0.001
	—	—	(0.003)	—	—	(0.003)

This table reports summary statistics on implied delta hedge coverage for VA guarantees and guarantee-induced stock market exposure, net of hedging (panel A), and counterfactual asset allocations for insurers with high (panel B) and medium (panel C) VA exposures. In panel A, total hedge coverage is divided into comprehensive and delta hedge coverages. Comprehensive hedging includes hedging using options with negative delta, for example, buying put options. Only derivatives that have common stocks underlying and are earmarked for hedging VA are included. Delta hedge coverage is estimated using the parameter estimates in column (1) of Table 2, panel B, and the following optimal hedging formula:

$$h^* = \frac{1}{2\eta}\left(1 - \frac{|\delta|\gamma_I}{\kappa\gamma_G} \bullet \Theta\right),$$

where η is insurer (inverse) sophistication; $|\delta|$ denotes sensitivity of the reserves to change in underlying stock price, normalized by the reserves; γ_G and γ_I are, respectively, the RBCs per unit of guarantee reserves and illiquid bond investments; κ is the maximum credit for hedging; and Θ is the ratio of discounted excess return on common stocks to discounted excess return on illiquid bonds. Net stock market exposure from the guarantees is calculated using the above delta hedge coverage, assuming that hedging effectiveness is highest at κ for the first dollar of hedging and decreases at the rate equal to η, as given by:

$$\Delta\alpha^*_{S,Total} = \frac{(1 - h^*\kappa(1 - \eta h^*))|\delta|G}{A}$$

Summary statistics for the hedge coverage and guarantee-induced net stock market exposure are separately reported for insurers with high, medium, and low VA exposures. In panels B and C, counterfactual Portfolio C1 is determined by assuming that the insurer does not write the VA but instead writes traditional insurance products, holding constant the mix between annuity-related and other life insurance products, as well as the total amount of reserves. The estimated $\bar{\alpha}_S$ from Table 2 determines the insurer's stock-bond allocation, and the actual RBC ratio determines the mix between liquid and illiquid bonds.

Counterfactual Portfolio C2 is calculated by keeping the estimated delta hedge coverage the same as actual but without tilting the allocation towards illiquid bonds, that is, keeping the ratio of liquid to illiquid bonds as if the insurer did not underwrite the VA (i.e., the same as the ratio in Portfolio C1). Allocation to other assets is assumed to equal the actual allocation. The pooled averages of Portfolios C1's and C2's allocations are reported, along with their differences from the actual allocations and the allocations of insurers with low VA exposures. Standard errors, calculated by Monte Carlo simulations using the point estimates of model parameters and their covariance matrix from Table 2, are in parentheses. $^*p < .1$; $^{**}p < .05$; $^{***}p < .01$.

still significantly higher than those of insurers with medium and low guarantee exposures.

Next, we turn to the impact of guarantees on asset allocation. We begin by asking what the asset allocation would look like if the same insurers did not write the guarantees? The profitability parameters from Table 2 allow us to estimate the profits in that counterfactual world, which can then be used to infer insurers' capacity to invest in risky assets. Since our estimates of e_T, e_G, and f are based on annual profits but the amount of capital that supports risk taking is the cumulative profits over the past several years, we scale the total profits $e_T T + (e_G - fG)G$ to match the capital for each firm and maintain the proportions of capital that come from the guarantees and traditional business at the same level as the proportions of profits from the two sources. This amounts to scaling all the parameters by the insurer-specific ratio of capital to gross profits.

The hypothetical amount of capital supports both the liabilities and the investments in risky assets, including stocks and illiquid bonds. The estimate of γ_T determines the amount that is tied up with the traditional business, and the estimate of $\overline{\alpha}_S$ tells us the hypothetical stock allocation. To obtain the hypothetical illiquid bond allocation, we use Equation (13) and the assumption that the RBC ratio ρ is the same as the actual, which together pin down the maximum investment in illiquid bonds allowed under the RBC constraint.

We refer to the hypothetical asset allocation in the absence of guarantees as Portfolio C1, and report its averages for insurers with high and medium guarantee exposures in the first column of Table 3, panels B and C, respectively. We do not report the counterfactuals for insurers in the low group, since they are very close to actual (due to their negligible VA business). In the second column of both panels, we report the average differences between Portfolios C1 and the actual portfolios, which show that the illiquid bond allocations would be 6.4% and 2.0% lower, respectively, for insurers with high and medium guarantee exposures. These reductions, amounting to about $174 billion or 17% of illiquid bonds held by these insurers, are made up for by increases in the net asset-side stock exposures (including both cash and synthetic stocks) and allocations to liquid bonds. In the third column, we report the average differences between Portfolios C1 and the actual portfolios of insurers with low (effectively negligible) guarantee exposures, which show that they do not significantly differ. This indicates that our estimation procedure works as intended, using the (conditional) cross-sectional differences in guarantee exposures to identify the effects of guarantees on illiquid bond investments, and that insurer characteristics, other than the guarantee exposures, have small impacts on insurers' asset allocation.

To further separate the effects of (net of hedging) guarantee exposures from those of RFY, we create another hypothetical portfolio, Portfolio C2, representing another counterfactual in which an insurer writes the actual amount of guarantees and applies the actual hedge coverage but does not engage in additional RFY. We assume that, while delta hedging decreases the stock exposure and increases the overall allocation to bonds, the mix between liquid and illiquid bonds in Portfolio C2 (with VA guarantees but no RFY) is the same as that observed in Portfolio C1 (without VA guarantees). The differences between Portfolios C2 and C1 (the fourth minus the first columns of Table 3, panels B and C) result only from the guarantees and their hedging while the differences between the actual portfolio and Portfolio C2 (negative of the fifth column) are due to RFY. We find that without the RFY tendency, the guarantees would have a smaller impact on insurers' illiquid bond allocations. For example, of the average 6.4% increase in the illiquid bond allocation for the high exposure group, only 1.9% comes directly from the guarantees, in isolation, with the remaining 4.5% coming from RFY. However, it is important to note that, while VA guarantees, by themselves, only induce a small increase in illiquid bonds, they still expose insurers to undiversifiable risk, which can trigger fire sales even among existing illiquid bond holdings.

4. Fire Sales and Systemic Risk

VA guarantees are put options whose values can increase significantly during market downturns. Such an increase erodes insurers' equity, violates their RBC constraints, and may force them to de-risk their asset portfolios. Below, we formally derive an individual insurer's portfolio de-risking and its implications for aggregate fire sales. In Section 5, we look at the actual trading of insurers with different guarantee exposures during the global financial crisis and the COVID-19 pandemic.

4.1 Valuation shocks

We consider a combination of two shocks at date 1: (i) a shock that reduces the value of assets, ε_A, and (ii) a shock that increases the (after-hedging) value of the guarantee reserve, ε_G. Both shocks lower an insurer's equity, and hence result in a violation of the capital constraint (1). Restoring it, in theory, can be achieved by reducing the liabilities L_1, raising external equity (which effectively increases K_1), or reducing the combined RBC $\widehat{K}_1$. In practice, insurers' long-term liabilities are essentially fixed in the short run,[19] and raising new external equity during a market downturn can be challenging.[20] As a

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[19] The asset portfolio de-risking is akin to the mechanism of *de-leveraging* by banks (Adrian and Shin 2010, 2014; Capponi and Larsson 2015; Greenwood, Landier, and Thesmar 2015). A distinctive feature of insurers' balance sheets is that their liabilities, which are long-term insurance contracts, remain fixed, and thus the insurer's leverage cannot be adjusted by reducing the balance sheet size.

[20] Berry-Stolzle, Nini, and Wende 2014 document capital raising in 2008–2009 by life insurers from sources outside the insurance industry. However, Niehaus (2018) shows that a substantial amount of the capital raised during the financial crisis actually comes from other non-life insurance entities in the same group. Koijen and Yogo (2015) show that during the financial crisis, insurers sold annuity policies at substantial discounts relative to their actuarial values because doing so results in higher reported capital under statutory accounting rules.

result, the insurer is likely to restore its capital constraint by *de-risking* its asset portfolio. In the context of our model, the insurer does so by selling common stocks and/or illiquid bonds and using the proceeds to buy liquid bonds, which helps to reduce the combined RBC.

As outlined in Section 2, while stocks can be sold at their full value, illiquid bonds potentially suffer from fire-sale discounts. We assume that when an *aggregate* amount N of illiquid bonds is sold in the economy, the bonds will trade at a discount $c = \varphi N$ ($\varphi > 0$). The discount will lead to an additional deterioration in the capital positions of other insurers holding the same illiquid bonds, which will require further liquidations. This is a fire-sale externality (e.g., [Stein, 2012]) at work. Below, we sketch a brief derivation of the aggregate fire-sale amount, given the insurers' asset and liability positions and the shocks ε_A and ε_G. Internet Appendix C.2 provides the full details.

Given the fire-sale discount, a proportional shock to asset value ε_A changes the insurer's asset value from dates 0 to 1 by

$$\triangle A = \left(-\varepsilon_A - \varphi N \alpha_{I,0}\right) A_0, \tag{19}$$

where A_0 is the book value of assets at $t = 0$ (we now use the time subscript for variables at date 0 to keep track of values that change from dates 0 to 1). The first term, $-\varepsilon_A A_0$, is the direct reduction in asset value from the shock. The second term, $-\varphi N \alpha_{I,0} A_0$, represents the losses arising from illiquid bonds being sold and marked-to-market (MTM) at fire-sale prices on the balance sheet. To keep the algebra simple, we assume here that the insurer marks to market all of its illiquid bonds. In our simulations, we take into account partial MTM to be consistent with regulation (Ellul et al. 2015). Partial MTM alleviates the fire-sale externality, and hence our fire-sale amount and costs derived below should be thought of as the upper bound.

We define the proportional shock to guarantee reserve ε_G as a shock that only increases the reserve but does not affect the asset value. Hence, the change in equity $\triangle K$, following both the asset and guarantee shocks together, can be written as:

$$\triangle K = -\varepsilon_G G_0 - (\varepsilon_A + \varphi N \alpha_{I,0}) A_0, \tag{20}$$

where G_0 is the guarantee reserve at $t = 0$.

Under our assumption of a fixed stock allocation, the insurer satisfies its capital constraint by selling illiquid bonds. To derive the sale amount for each insurer, we maintain the pre-shock RBC ratio, ρ, and the optimal hedge ratio at date 1, h_1^*, which is consistent with Equation (10) under the new market circumstances following the shocks. We then solve for the total amount of fire sales in the market using the fact that it must equal the illiquid bond sales of all insurers after accounting fully for the fire-sale externality, $\varphi N \alpha_I A$. The total amount of fire sales in the market is given by:

$$N=\frac{\sum_i \frac{A_0^i}{\gamma_I^i}\left(\frac{\varepsilon_A^i+\varepsilon_G^i\frac{G_0^i}{A_0^i}}{\rho^i}-\gamma_S\overline{\alpha}_S^i\varepsilon_A^i+\left((1+\varepsilon_G^i)(1-\widetilde{h}(\delta_1^i))\gamma(\delta_1^i)-(1-\widetilde{h}(\delta_0^i))\gamma(\delta_0^i)\right)\frac{G_0^i}{A_0^i}\right)}{1-\varphi\sum_i \frac{A_0^i}{\gamma_I^i}\left(\frac{1}{\rho^i}-\gamma_S\overline{\alpha}_S^i\right)\alpha_{I,0}^i}, \tag{21}$$

where $\widetilde{h}_t = h_t^*\kappa(1-\eta h_t^*)$ is the RBC relief from optimally hedging a fraction h_t^* of the guarantee, and $\gamma_{G,t}$ is the RBC charge per unit of the guarantee. We write $\widetilde{h}_t$ and $\gamma_{G,t}$ as (time-invariant) functions of the guarantee's delta on date t to signify the fact that both quantities change through time to reflect the guarantee's evolving moneyness and sensitivity to the stock market. Given our assumption of linear price impact, the total fire-sale cost, which can be interpreted as a measure of systemic risk in the economy, is given by $C = N\cdot\varphi N = \varphi N^2$.[21]

Equation (21) clearly highlights the factors that drive the fire sales of illiquid bonds. The first effect, corresponding to the first term in the brackets, arises because the valuation shocks and the losses in values of illiquid bonds (due to fire sales) reduce equity. The second effect is a reduction in RBC associated with stock investments. As the asset base decreases, the amount invested in stocks also decreases in absolute terms (as the insurer maintains a constant fraction $\overline{\alpha}_S$ in stocks). The third effect arises because the amount of capital needed to support the guarantee changes. This is, first, because the guarantee reserve is now higher (by factor ε_G), but also because the optimal hedge ratio, as well as the RBC charge, changes to the extent that the shocks change the risk profile of the guarantee. For example, a negative stock market shock would push the guarantee closer to the money, increasing the delta and applicable RBC charge and hence changing the optimal hedge ratio, as per Equation (10).

4.2 Valuation shocks plus rating downgrades

In reality, large adverse shocks are fundamental in nature and often accompanied by significant rating downgrades. For example, according to "Moody's Credit Policy," dated March 2009, 35% of structured finance products and almost 20% of corporate finance products were downgraded in 2008, with the average number of downgrade notches being 8.3 and 1.6, respectively. Similarly, S&P's commentary "COVID-19 and Oil Price-Related Public Rating Actions on Corporations, Sovereigns, and Project Finance to Date" on June 5, 2020, shows that about 20% of corporate and sovereign bonds were downgraded by at least one notch in the first 5 months of 2020.

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[21] Systemic risk may lead to welfare losses as it can cause instability in the financial system, for example, due to a failure of insurers in fulfilling their commitments. However, in the absence of any instability issues, fire sales could be viewed as a pure zero-sum event, benefiting the buyers of illiquid bonds at the cost of the sellers.

In our framework, rating downgrades potentially affect fire sales and systemic risk in two ways: (i) increasing the RBC charges for illiquid bonds, from $\gamma_{I,0}$ to $\gamma_{I,1}$, and (ii) increasing the amount of illiquid bonds being subject to fire-sale prices, from α_I to $\alpha_I + \triangle\alpha_I$. For example, if a publicly traded corporate bond in NAIC 2 is downgraded to NAIC 3, its RBC charge will increase from 1.3% to 4.6%, and it will be reclassified as an illiquid bond. Following the same derivation steps as in Section 4.1, we obtain the following expression for the total amount of fire sales of in the market:

$$N = \frac{\sum_i \frac{A_0^i}{\gamma_{I,1}^i} \left(\begin{array}{c} \frac{\varepsilon_A^i + \varepsilon_G^i \frac{G_0^i}{A_0^i}}{\rho^i} - \gamma_S \overline{\alpha}_S^i \varepsilon_A^i + \left((1+\varepsilon_G^i)(1-\widetilde{h}(\delta_1^i))\gamma(\delta_1^i) - (1-\widetilde{h}(\delta_0^i))\gamma(\delta_0^i)\right) \frac{G_0^i}{A_0^i} \\ + \left((\alpha_{I,0}^i + \triangle\alpha_I^i)\gamma_{I,1}^i - \alpha_{I,0}^i \gamma_{I,0}^i\right) \end{array} \right)}{1 - \varphi \sum_i \frac{A_0^i}{\gamma_{I,1}^i} \left(\frac{1}{\rho^i} - \gamma_S \overline{\alpha}_S^i\right)(\alpha_{I,0}^i + \triangle\alpha_I^i)}. \tag{22}$$

4.3 Simulating fire sales

In our simulations, we consider three fundamental shocks. First, we explore a shock that reduces the value of the stock market by ε_S, which, from the perspective of an affected insurer, is effectively a combination of an asset shock of $\varepsilon_A = \overline{\alpha}_S \varepsilon_S$ and a guarantee shock of $\varepsilon_G = (1-\widetilde{h}(\delta_0))|\delta_0|\varepsilon_S$. The latter assumes that the RBC rule correctly reflects the hedging effectiveness, and hence $\widetilde{h}$ is also the fraction of the guarantee that is *effectively* hedged and protected from the shock. Second, we explore a shock that proportionally reduces the value of illiquid bonds by ε_I, which, to an affected insurer, is just an asset shock of magnitude $\varepsilon_A = \alpha_I \varepsilon_I$. Third, we explore a shock to the value of the guarantee ε_G, which we assume, for the purpose of quantifying post-shock guarantee-related variables, arises from an increase in stock market volatility.

Following the procedure for banks' stress tests, we investigate two levels of magnitude: adverse and severely adverse.[22] For each variable (except the guarantee reserve), the adverse shock represents the 10th percentile of a (rolling) 1-year change during the period from 7/1998 to 7/2018, while the severely adverse shock is the worst scenario. We use the S&P 500 index to represent common stocks, and a mixture of 85% Bloomberg Barclays U.S. Credit and Mortgage Index and 15% Bloomberg Barclays High-Yield Corporate Bond Index to represent illiquid bonds. For the guarantee reserve, the adverse shock comes from the change from 2014 to 2015, a period with a very small change in the stock market but a relatively large increase in the reserve across all insurers

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[22] We do not study smaller shocks for two reasons. First, following smaller shocks, insurers may allow their RBC ratios to be temporarily below target and hence do not engage in costly portfolio de-risking, contradicting our assumption. Second, the effects of smaller shocks can be easily confounded by and hence difficult to isolate from forces outside the model, for example, shocks to insurers' traditional insurance liabilities.

(suggesting that the change in reserves is driven by other determinants). The severely adverse shock to the guarantee reserve comes from years 2010 to 2011, when long-term interest rates significantly declined and stock market volatility spiked. To isolate the shock to reserve from the stock market shock (which also affects the guarantee reserve), we adjust the observed change in reserve to take out the effect of the underlying stock price during that time. Finally, we recognize that these shocks may not occur in isolation and therefore use the 2008 financial crisis as the worst case scenario, in which each shock may not be at its worst but all three shocks hit with large magnitudes simultaneously.

We focus on the insurers that, at some point, are in the high-exposure group, and use their balance sheets and RBC ratios in 2019 to perform the fire-sale exercise. As of 2019, these insurers hold over \$2.5 trillion of assets, representing about 54% of the life insurance industry. About 42% of their assets are in the illiquid fixed income assets that we classify in Table 1 as illiquid bonds. We assume that the price impact φ is 18.6 basis points per \$10 billion of sales, following the NSFR estimate for non-agency mortgage-backed securities by Duarte and Eisenbach (2021). Given the size of the private-label ABS/MBS market, our assumption translates to a fire-sale discount of about 0.22% for a sale of 1% of outstanding market capitalization.

Life insurers do not mark to market the vast majority of bonds on their balance sheets.[23] However, they have to write down the book value if the decrease in the bond's price is deemed other than temporary (i.e., recognize other-than-temporary-impairments [OTTI]). Severely adverse shocks are often fundamental in nature, and the decreases in illiquid bond prices that follow, while induced by fire sales, may appear non-temporary, at least in part. By comparing insurers' reported book to fair values, Ellul et al. (2015) estimate that 71–79% of downgraded MBS during the financial crisis are still not subject to MTM. Using data on intrinsic prices provided by PIMCO and BlackRock, Becker et al. (2021) show that, after the regulatory reform in 2009–2010, over 67% of life insurers' MBS positions with non-zero expected credit losses are carried at written-down book prices that are within 0.85% of their intrinsic prices and hence classified as NAIC 1. For our simulation, we take both estimates into account and err on the conservative side, assuming that insurers write down their unsold illiquid bond positions by an average of 25% of the fire-sale discounts. A lower degree of MTM yields smaller fire-sale feedback effects, as a smaller part of fire-sale discounts affect other insurers' equity and induce additional fire sales (Plantin, Sapra, and Shin 2008).

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[23] MTM only applies to sufficiently distressed assets. For bonds, life insurers are required to use MTM if the bonds are in NAIC 6 (in or near default). For MBS, after the change in statutory accounting rules in 2009–2010, life insurers are required to use MTM if the intrinsic price, as provided by PIMCO or BlackRock, is below the amortized costs by 26.5% or more.

In our discussion of the results below, we decompose the fire-sale amount and costs attributable to VA guarantees into the guarantee exposure, hedging, and RFY components using the counterfactual portfolios that we introduced in Section 3.3. However, these components, which add up to the total effects, should be viewed as just indicative, given that these effects actually amplify each other (due to the fire-sale externality). We can clearly see the multiplicative nature of these effects in Equation (21), where the effects of the guarantee exposures to various shocks and hedging policies are reflected in ε_G (scaled by G/A) and $\widetilde{h}(\delta)$ (in the numerator) and the effects of RFY are captured by an increase in $\alpha_{I,0}$ (in the denominator).

Table 4 reports the aggregate fire-sale amounts, N, (top panel) and fire-sale costs, C, (bottom panel) as a result of adverse shocks. The results show that the adverse shocks, or shocks that could occur once every 10 years, do not engender significant systemic risk. Insurers seem to have sufficient capital to easily absorb them. For example, an adverse stock market shock of -19% would result in insurers selling about \$131 billion of illiquid bonds, of which \$85 billion is attributable to VA guarantees. The corresponding fire-sale costs are \$3 billion, which represents just 1% of the insurers' equity. Our decomposition shows that about 90% of guarantee-related fire-sale costs are associated with the net guarantee exposures to the stock market, and the other 10% with RFY. Interestingly, while hedging reduces the amount of fire sales, it does so in a manner less than proportional to the hedge coverage because hedging is not 100% effective ($\widetilde{h} < h^*$). In addition, hedging also slightly increases illiquid bond investments, even without RFY, further blunting its role in alleviating fire sales.

In Table 5, we turn to the severely adverse shocks, the worst in each category over the past 20 years. In the absence of any policy intervention, a stock market shock of -48% would result in \$336 billion of fire sales, of which \$221 billion, the majority, is attributable to VA guarantees. The associated fire-sale costs are \$21 billion, and \$19 billion are due to the VA guarantees. The guarantee-related losses represent 7% of insurers' equity, with the net-of-hedging guarantee exposures and RFY accounting for, as in the case of a smaller stock market shock, about 90% and 10% of this amount, respectively. A severely adverse shock to illiquid bonds of -8% would generate a slightly lower amount of fire sales, compared to the stock market shock. The guarantee-related fire-sale costs would be about \$8 billion, or 3% of the insurers' equity, almost all of which is due to RFY.

A severely adverse increase in the guarantee reserve of 80% would lead to fire sales of over \$452 billion of illiquid bonds, almost half of insurers' holdings (about \$1.1 trillion). The associated fire-sale costs would be about \$38 billion, erasing almost 15% of insurers' equity. By construction, all of the fire-sale costs are attributable to VA guarantees, with about 92% being the direct result of the net guarantee exposures and 8% coming from RFY.

Table 4
Aggregate fire-sale amount and costs in adverse scenarios

		Fire-sale amount ($ million)			Decomposition ($ million)		
Type of shock	Magnitude of shock	Actual	No VA	Net increase from VA	Gross VA exposure	Hedging	RFY
Stock	19%	130,775	45,654	85,121	114,318	−34,238	5,041
Illiquid bond	5%	172,809	116,481	56,328	0	8,220	48,108
Guarantee	30%	208,518	0	208,518	250,139	−49,659	8,037
		Fire-sale cost ($ million)			Decomposition ($ million)		
Type of shock	Magnitude of shock	Actual	No VA	Net increase from VA	Gross VA exposure	Hedging	RFY
Stock	19%	3,181	388	2,793	4,372	−1,819	241
Illiquid bond	5%	5,554	2,524	3,031	0	369	2,662
Guarantee	30%	8,087	0	8,087	11,638	−4,162	611

This table presents estimates of fire-sale amount and fire-sale cost incurred by insurers with high VA exposures (top tercile), given adverse shocks to stock markets (−19%), illiquid bond prices (−5%), and guarantee gross reserves (or, other determinants of reserves) (+30%), under the baseline assumptions. For each variable (except the gross reserves), the adverse shock roughly represents the 10th percentile of (rolling) 1-year change during the period from 7/1998 to 7/2018. For the gross reserves, the adverse shock comes from the change from 2014 to 2015, a period with very small changes in stock markets (suggesting that the change in reserves is driven by other determinants). The baseline assumptions assume that insurers mark to market 25% of their portfolios, and trade to fully recover the preshock risk-based capital ratios while maintaining the desired stock-bond allocation ($\bar{\alpha}_S$). The price impact (φ) of 18.6 basis points per $10 billion of sale is assumed, following the NSFR estimate for non-agency MBS by Duarte and Eisenbach (2021). The RBC requirements for common stocks, illiquid bonds, and liquid bonds are calculated from the reported NAIC risk category for each position and the RBC requirement for each category. The balance sheet data, including the guaranteed VA liabilities, and the corresponding counterfactual portfolio allocations are as of 2019. The first two columns report the fire-sale amounts and costs for the actual portfolio and Portfolio C1 (no VA). The third column reports the net increase in fire-sale amount and cost attributable to VA, calculated by subtracting the quantities in the second column from those in the first column. The last three columns report the decomposition of the net increases in fire-sale amount and cost in the third column into three components. First, the “Gross VA Exposure” component is calculated by subtracting the fire-sale amounts and costs for Portfolio C1 with no VA from those for the same portfolio but with the actual gross (prehedging) VA exposure. Second, the “Hedging” component is calculated by subtracting the fire-sale amounts and costs for Portfolio C1 with the actual gross (prehedging) VA exposure from those for Portfolio C2, which assumes actual delta hedge coverages but with no reaching for yield (RFY). The last component is the “RFY” component, calculated by subtracting the fire-sale amounts and costs for Portfolio C2 from those for the actual portfolio.

Table 5
Aggregate fire-sale amount and costs in severely adverse scenarios

		Fire-sale amount ($ million)			Decomposition ($ million)		
Type of shock	Magnitude of shock	Actual	No VA	Net increase from VA	Gross VA exposure	Hedging	RFY
Stock	48%	336,068	115,336	220,732	347,038	−139,261	12,954
Illiquid bond	8%	276,494	186,369	90,124	0	13,151	76,973
Guarantee	80%	451,994	0	451,994	521,134	−86,562	17,423
All	As in 2008	571,461	202,983	368,478	364,465	−59,461	63,475
All & downgrades	As in 2008	890,502	527,160	363,342	285,634	−38,079	115,787

		Fire-sale cost ($ million)			Decomposition ($ million)		
Type of shock	Magnitude of shock	Actual	No VA	Net increase from VA	Gross VA exposure	Hedging	RFY
Stock	48%	21,007	2,474	18,533	37,291	−20,346	1,588
Illiquid bond	8%	14,219	6,460	7,759	0	944	6,815
Guarantee	80%	38,000	0	38,000	50,514	−15,387	2,873
All	As in 2008	60,742	7,664	53,078	52,228	−11,894	12,744
All & downgrades	As in 2008	147,497	51,689	95,808	73,189	−13,244	35,863

This table presents estimates of fire-sale amount and fire-sale cost incurred by insurers with high VA exposures (top tercile), given severely adverse negative shocks to stock markets (−48%), illiquid bond prices (−8%), guarantee gross reserves (or, other determinants of reserves) (+80%), and all three variables with magnitudes comparable to what happened in 2008, under the baseline assumptions. For each variable (except the gross reserves), the severely adverse shock represents the worst rolling 1-year change during the period from 7/1998 to 7/2018. For the gross reserves, the adverse shock comes from the change from 2010 to 2011, adjusted for the change in stock markets. The last row of each panel considers also potential rating downgrades in addition to the severe valuation shocks, as observed in 2008; 35% of bonds in each NAIC category are assumed to be downgraded into the next NAIC category (e.g., 35% of bonds in NAIC 2 are downgraded to NAIC 3). The baseline assumptions assume that insurers mark to market 100% of their portfolios, and trade to fully recover the preshock risk-based capital ratios while maintaining the desired stock-bond allocation ($\bar{\alpha}_S$). All other assumptions and calculations are the same as those in Table 4.

In our final two exercises, we explore a severely adverse scenario in which the insurers are hit by correlated shocks comparable in magnitude to what happened in 2008.[24] In this scenario, we first explore valuation-only shocks, with no rating downgrades, and assume that the stock market goes down by 36%, the illiquid bond price goes down by 5%, and the guarantee reserve increases by another 67% (in addition to the increase due to the stock market decline). Our exercise reveals a devastating effect, with over \$570 billion of illiquid bonds (over half of the insurers' total holdings) being liquidated, of which \$368 billion is attributable to VA guarantees. The guarantee-induced fire-sale costs would be \$53 billion, or 21% of insurers' equity, with the net-of-hedging guarantee exposures accounting for 76% and RFY accounting for 24% of these costs.

In the last rows of panels A and B of Table 5, we examine the fire-sale amounts and costs under the 2008 valuation shocks plus rating downgrades. We assume that, comparable to the structured finance products in 2008, 35% of bonds, other than Treasury and agency, are downgraded by one NAIC class (e.g., NAIC 2 downgraded to NAIC 3). In this more realistic scenario that resembles what happened in the global financial crisis, the fire-sale amount required to recover the pre-shock RBC ratios would be significantly more devastating than in the case of valuation shocks alone. Interestingly, the fire-sale amount attributable to VA guarantees remains about the same as the downgrades affect insurers with and without VAs almost equally. While insurers with VA guarantees hold more illiquid bonds to start, insurers without VA guarantees experience a greater increase in the amount of illiquid bonds as bonds in NAIC 2 fall into NAIC 3 and become illiquid. Nevertheless, the fire-sale costs indicate that, without VA guarantees, insurers would be able to absorb the shocks as the loss of about \$52 billion would erase less than 20% of insurers' equity. The fire sales induced by the VA guarantees, added on top of the already-stressed insurers, would, however, push them closer to distress.[25] The guarantee-induced fire-sale costs would add another \$96 billion, erasing another 37% of the insurers' equity. About 63% of the costs are directly due to the guarantees, while the remaining 37% are due to RFY.

In sum, our simulations reveals that VAs with guarantees can significantly raise the systemic risk of the insurance industry. Exposures to undiversifiable risk, even after hedging, are the main culprit, but additional illiquid bond investments, or RFY, also play a significant amplifying role. In Internet

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[24] In examining the aggregate fire sales, we ignore individual insurer insolvency (in which case, selling all of the illiquid bonds would not suffice to restore the RBC constraint). This occurs in the case of 2008 shocks both with and without downgrades. We explicitly turn to the issue of individual insurer insolvency in Section 4.4 in which we explore the heterogeneity across insurers.

[25] Due to the quadratic nature of the fire-sale costs, an additional dollar of fire sales generates a greater impact at a higher fire-sale amount.

Appendices E and G, we examine the robustness of our results, and consider some policy experiments to better understand the effects of various assumptions and policy tools.

4.4 Heterogeneity across insurers

In the previous section, we use the aggregate fire-sale costs as a measure of systemic risk. However, such aggregates may mask important variation across insurers. For example, in the case of 2008 valuation shocks without rating downgrades, the aggregate fire-sale costs may seem small, erasing less than a quarter of insurers' equity. Some insurers may nevertheless suffer significant stress, and it is important to examine how prevalent the stress would be and what insurer characteristics determine who would or would not be in such a situation. In addition, a near collapse of even one or two insurers may shake confidence in the system and cause panic, exacerbating the impacts of the shocks beyond our model's prediction.

In Table 6, we track individual insurers in the above fire-sale simulation and report the fraction of illiquid bonds that they sell, the fire-sale costs relative to equity capital, and whether each of them will be under significant stress. Panel A focuses on the 2008 valuation shocks without rating downgrades. We find that the fraction of illiquid bonds sold varies considerably *across* insurers; the 5th and 95th percentiles are 20% and 100%. The average ratio of fire-sale costs to equity capital is 28%, with the 5th and 95th percentiles being 6% and 49%, respectively. Defining stress as the fire-sale costs erasing 50% or more of equity, we identify one insurer that would be under a significant stress from the shocks. In panel A-2, we show that, while the lone stressed insurer under the 2008 valuation shocks is large and has higher RBC ratio than the others, it also has significantly higher guarantee exposures and invests significantly more in illiquid bonds. In addition, its guarantees are also more generous than those of others.

In panel B, we repeat the analysis for the 2008 valuation shocks with rating downgrades. The financial outcomes are much worse, further highlighting the importance of VA guarantees. The average ratio of fire-sale costs to equity is 53%, with the 5th and 95th percentiles being 14% and 94%, respectively. Without guarantees, even the 95th percentile would have been just 35%. Moreover, 16 insurers (or 55% of those in our analysis) would be under significant stress; without guarantees, that number would have been zero. As in panel A-2, panel B-2 shows that the 16 stressed insurers have significantly higher guarantee exposures, write more generous guarantees, and invest more in illiquid bonds. Together, the results show that significant heterogeneity exists among VA-writing insurers. While the industry might be able to collectively weather adverse shocks, some insurers may fail. Variable annuities' guarantees and their associated RFY are important determinants of an individual insurer's risk of failure.

Table 6
Heterogeneity in fire-sale amount and costs across insurers

A-1. (Cross-sectional) Summary statistics for stress-related variables: 2008 shocks

	Actual portfolio and VA exposures			Portfolio C1 and no VA exposures		
	Mean	PCT5	PCT95	Mean	PCT5	PCT95
Fraction of illiquid bonds sold	0.550	0.201	1.000	0.353	0.184	0.994
Fire-sale costs/Capital and surplus	0.275	0.063	0.491	0.041	0.014	0.096
Stress dummy	0.034	0.000	0.000	0.000	0.000	0.000

A-2. Differences between stressed and unstressed insurers: 2008 shocks

Stress dummy	Assets ($ mil.)	Gross reserve ($ mil.)	Gross reserve to cap. (%)	Gross reserve to acc. val. (%)	RBC ratio	Illiquid bond holding
0	81,108	1,408	19.333	3.051	8.801	0.411
1 (1 insurer)	115,546	2,076	35.416	8.317	10.207	0.547
Difference (1 − 0)	34,437**	668	16.083**	5.267***	1.407**	0.136***

B-1. (Cross-sectional) Summary statistics for stress-related variables: 2008 shocks and potential downgrades

	Actual portfolio and VA exposures			Portfolio C1 and no VA exposures		
	Mean	PCT5	PCT95	Mean	PCT5	PCT95
Fraction of illiquid bonds sold	0.569	0.301	1.000	0.406	0.245	1.000
Fire-sale costs/Capital and surplus	0.529	0.142	0.943	0.161	0.056	0.353
Stress dummy	0.552	0.000	1.000	0.000	0.000	0.000

B-2. Differences between stressed and unstressed insurers: 2008 shocks and potential downgrades

Stress dummy	Assets ($ mil.)	Gross reserve ($ mil.)	Gross reserve to cap. (%)	Gross reserve to acc. val. (%)	RBC ratio	Illiquid bond holding
0	72,381	355	4.733	1.821	8.581	0.380
1 (16 insurers)	90,352	2,305	32.200	4.379	9.067	0.445
Difference (1 − 0)	17,970	1,950*	27.467**	2.558**	0.486	0.065*

This table presents cross-sectional summary statistics of fire-sale amount (as fraction of illiquid bond holding) and cost (as fraction of capital and surplus) estimates for insurers with high VA exposures (top tercile) in severely adverse scenarios in which insurers are hit by all three valuation shocks (shocks to stock markets, illiquid bond prices, guarantee gross reserves) with magnitudes comparable to what happened in 2008 (panel A-1), plus potential rating downgrades whereby 35% of bonds in each NAIC category are assumed to be downgraded into the next NAIC category (Panel b-1). All assumptions and calculations are the same as those in Table 5. The first three columns are for the actual portfolios and net guaranteed VA exposures, whereas the last three columns are for counterfactual Portfolio C1 with no guaranteed VA exposures. Stress dummy equals 1 if either the fire-sale costs are greater than 50% of capital and surplus, and 0 otherwise. Panels A-2 and B-2 report means of characteristics, as of the end of 2019, for insurers whose stress dummy equals 0 and those whose stress dummy equals 1. Tests of difference in mean are conducted using heteroscedasticity-robust standard errors. $^{*}p < .1$; $^{**}p < .05$; $^{***}p < .01$.

5. Discussion and Robustness

5.1 Trading behavior during market downturns

How do our model's predictions, both in terms of portfolio de-risking and life insurers' equity valuations, compare to the data we observe during market downturns? Answering this question is important as it can inform us about the real-world application of our exercise. Below, we examine what happened during the global financial crisis in 2008–2009 and the COVID-19 pandemic in 2020. These two events are ideal to validate our fire-sale results, as they engendered significant stress in U.S. financial markets and lie outside our estimation period.

While both events share some common features, there is also at least one major difference. During the initial stages of the COVID-19 pandemic in March–April 2020, the speed with which both the Federal Reserve Board and the federal government moved to support the economy and asset prices was significantly faster and more aggressive than what the markets experienced in 2008. In fact, while the stock market dropped heavily in February–March 2020, it sharply recovered starting in March 23, 2020, when the Fed announced its intervention. By April 2020, half of the drop had been recovered, and by August 2020, the stock market was back to its pre-COVID level. Bond prices show similar, if not even sharper, patterns (Haddad, Moreira, and Muir 2021). On the contrary, the market downturn in 2008 was much more protracted; the stock market decline started to accelerate in August 2008 and the recovery did not materialize until March 2009. Further, the recovery was very slow across most asset markets, with the bond markets taking until June 2009 or later to fully recover and the stock market remaining below its precrisis level even at the end of 2009. It is important to contrast these two events to help understand when fire sales and contagion are likely to occur and which policies could help mitigate system-wide effects.

We study insurers' trading and portfolio reallocations in Table 7 and insurers' equity valuation in Figure 5. In each, we provide statistics for (i) VA-writing insurers with small exposures to guarantees (reserve to capital ratio $< 5\%$) and (ii) those with large exposures to guarantees (reserve to capital ratio $\geq 5\%$). As in our main analysis, we group assets into liquid bonds, illiquid bonds, and common stocks. During the global financial crisis (panel A), both the asset trading behavior and stock price effects are qualitatively and quantitatively similar to what our model predicts in Table 5.[26] First, insurers adjust risk taking by altering the asset allocation within the bond portfolio, while keeping the stock-bond allocation relatively stable. Columns (2)–(5) of Table 7, panel A, show that, from 2007 to 2009, insurers with large (small) guarantee exposures, in aggregate, reduce their allocation to illiquid bonds by 8.5% (3.5%), increase

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[26] Note, however, that our calculation in Table 5 is based on the insurers' balance sheets in 2019.

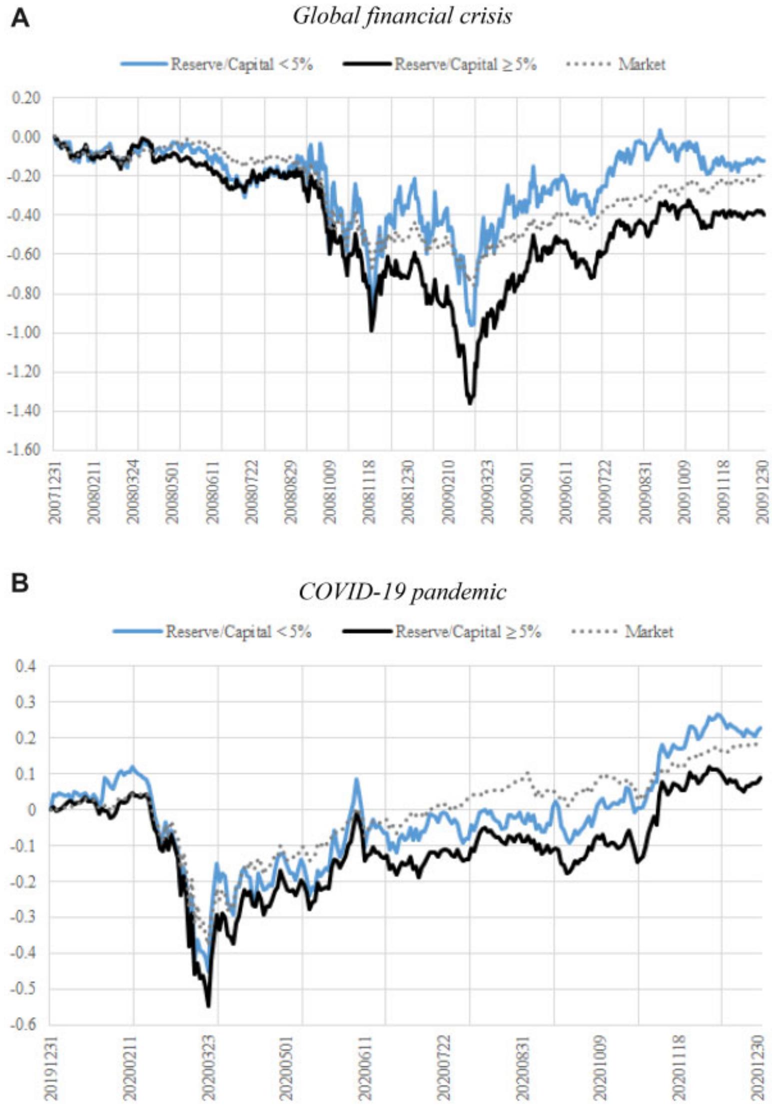

Figure 5
Cumulative stock returns during the global financial crisis and COVID-19 pandemic
This figure plots cumulative logged value-weighted returns during the global financial crisis (2008–2009, panel A) and COVID-19 pandemic (2020, panel B) for (i) publicly traded insurers with small exposures to guaranteed VA ($0 <$ reserve to capital ratio $< 5\%$, light solid line), and (ii) those with large exposures to guaranteed VA (reserve to capital ratio $\geq 5\%$, dark solid line). Cumulative logged returns on CRSP value-weighted index (dotted gray line) are also plotted for comparison. In panel A (B), the reserve to capital ratios used to sort insurers into groups (i) and (ii) are as of the end of 2007 (2019), at which point the cumulative logged returns begin at zero.

their allocation to liquid bonds by 9.8% (5.9%), and reduce their allocation to (cash) stocks by just 0.1% (0.6%).

Table 7
Insurers' asset trading during the global financial crisis and COVID-19 pandemic

A. Net asset allocation changes and trading during the global financial crisis (2008–2009)

		Aggregate allocation changes			Aggregate net trades ($ mil.)		
	Asset change ($ mil.)	Liquid bonds	Illiquid bonds	Common stocks	Liquid bonds	Illiquid bonds	Common stocks
Insurers with reserve to capital < 5%							
2008	20,001	0.073	−0.058	−0.014	−182,434	−62,628	−57,149
2009	58,128	−0.014	0.023	0.008	344,123	−92,402	58,158
Total	78,129	0.059	−0.035	−0.006	161,689	−155,030	1,009
Total as fraction of preshock assets					0.103	−0.098	0.001
Insurers with reserve to capital ≥ 5%							
2008	67,700	0.100	−0.093	−0.008	47,678	−45,231	4,683
2009	46,305	−0.003	0.008	0.007	133,206	−158,352	2,105
Total	114,005	0.098	−0.085	−0.001	180,884	−203,583	6,788
Total as fraction of preshock assets					0.235	−0.264	0.009

B. Net trading during COVID-19 pandemic (2020Q1–2020Q2)

	Aggregate net trades ($ mil.)		
	Liquid bonds	Illiquid bonds	Common stocks
Insurers with reserve to capital < 5%			
2020Q1	33,389	−3,715	6,000
2020Q2	−15,233	−17,759	−2,090
Total	18,157	−21,474	3,910
Total as fraction of preshock assets	0.007	−0.008	0.001
Insurers with reserve to capital ≥ 5%			
2020Q1	36,393	−12,165	1,159
2020Q2	52,941	−8,745	−530
Total	89,334	−20,910	630
Total as fraction of preshock assets	0.069	−0.016	0.000

This table reports aggregate changes in book value of assets and asset allocation, and aggregate net asset trading during the global financial crisis (2008–2009, panel A) and COVID-19 pandemic (2020Q1–2020Q2, panel B) for (i) insurers with small exposures to guaranteed VA (0 <reserve to capital ratio <5%), and (ii) those with large exposures to guaranteed VA (reserve to capital ratio ≥ 5%). Assets are grouped into liquid bonds, illiquid bonds, and common stocks, as described in Table 1, with other assets (e.g., real estate) omitted. The data are from the NAIC, obtained through S&P Global Market Intelligence. The asset allocations (holdings in each group of assets as a fraction of total invested assets) are based on year-end holdings as reported in Schedules A, B, BA, D, DA, and DB. The transactions, used in calculating aggregate net trades, are from quarterly Schedule D filings, which include only bonds and structured products (including mortgage and asset backed securities and collateralized debt/loan obligations, but excluding long-term assets, mortgages, bank loans, and derivatives) and common stocks. The aggregates include all reported transactions, including both market transactions and nonmarket transactions (e.g., calls and prepayments). Only assets in the general account are considered. In panel A, the reserve to capital ratios used to sort insurers into groups (i) and (ii) are as of the end of 2007, and the total preshock assets are $1,576,821 million for insurers with reserve to capital ratios less than 5%, and $770,297 million for insurers with reserve to capital ratios greater than 5%. In panel B, the reserve to capital ratios used to sort insurers are as of the end of 2019, and the total preshock assets are $2,624,037 million and $1,285,782 million for the two groups of insurers, respectively.

The last three columns of Table 7, panel A report aggregate net trades, which confirm the same re-allocation pattern. However, unlike the allocation changes, which can be driven by changes in asset valuation and accounting rules that apply to them, aggregate net trades reflect deliberate actions by insurers to buy and sell different assets. We include all trades reported in Schedule D, including passive trades such as maturity and prepayment. The main reason is that if insurers receive money back from a maturing bond, for example, and decide not to replace it with another bond in the same broad group, then such a reduction in the holding in that category is intentional, just like an active sale.[27] Panel A shows that, over 2008–2009, insurers with large (small) guarantee exposures, in aggregate, sell illiquid bonds worth about 26.4% (9.8%) of their assets in 2007. The difference of 16.6% can be thought of as the incremental impact of VA guarantees, and hence can be benchmarked against the guarantee-induced fire sales in Table 5 for the scenario with 2008 shocks and downgrades. In that scenario, the estimated amount of fire sales due to VA guarantees is about 14.3% of pre-shock assets.

Figure 5, panel A shows that the severity of fire sales was also reflected in the insurers' equity valuation. While both groups of insurers experienced sharp drops in their stock prices, starting in August 2008 and reaching troughs in March 2009, the magnitudes of the drops as well as the speed of recovery significantly differed. At the end of 2009, the stock prices of insurers with large (small) guarantee exposures remained 32.6% (11.5%) below their pre-crisis level.

We now turn to the COVID-19 pandemic. Table 7, panel B shows that while the directions of trades in illiquid bonds, liquid bonds, and stocks were more or less the same as in the global financial crisis, the magnitudes were much more muted. For example, in the first two quarters of 2020, insurers with large (small) guarantee exposures sold illiquid bonds worth about 1.6% (0.8%) of their assets in 2019. The difference of 0.8%, which can be attributed to VA guarantees, is economically insignificant. This is consistent with the fact that the insurers do not need to de-risk in large scale, as stock and bond prices recovered almost fully within just one quarter following the shocks and, as a result, the increase in guarantee reserves was within normal year-to-year variation.[28] Figure 5, panel B, which plots the insurers' stock prices, tells the same story, although the 9.8% difference in equity valuation between insurers with large and small guarantee exposures at the end of 2020 is difficult to reconcile.

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[27] We provide additional reasons and supporting evidence in Internet Appendix F, in which we also show that our conclusion is robust to considering trading in normal times as the benchmark and including only active sales.

[28] According to the NAIC's U.S. Life and A&H Insurance Industry Analysis Report 2020 Results, variable annuities with guarantees, as a line of business, still reported operating gains in 2020, although the gains decreased by about 46% from 2019. Other business lines such as fixed annuities and life insurance performed worse. Due to the nature of periodic regulatory reporting and benefit base calculation (e.g., annual), and the fact that contract anniversaries and events that affect withdrawal and pay-out timing spread over time, asset shocks have to be severe and persistent to significantly impact guarantee liabilities.

An important message emerges when comparing the results in panel A with those in panel B of Table 7 and Figure 5. While this is outside the realm of financial regulators, a potentially costly but effective strategy to halt fire sales in response to a severely adverse shock is to employ aggressive unconventional monetary policies aimed specifically at supporting asset prices. In our framework, these policies work by reducing the price impact parameter φ, which not only limits the impact of fire sales but also limits the fire-sale amount by mitigating externalities. Internet Appendix Figure E.1 shows that the effects of φ on VA-induced fire-sale costs are more than linear; halving the baseline value of φ reduces the VA-induced fire-sale costs by almost 60% (and reducing it by three quarters would push the fire-sale costs below 5% of the insurers' equity, even in the case of 2008 shocks with downgrades). It is important, however, to acknowledge that, while effective against fire sales, policies that support asset prices may have long-term consequences that are not yet known.

On the flip side, Internet Appendix Figure E.1 also shows that if liquidity dries up and price impacts double or triple, guarantee exposures can exacerbate the fire-sale costs to the point of widespread insolvency. Taking the 2008 shocks without downgrades, for example, the combined equity of insurers will be erased by guarantee-induced fire sales if the price impact parameter is 250% of our baseline value. With the potential rating downgrades that often accompany these severe shocks, a doubling of the price impact would be enough to erase the insurers' combined equity.

5.2 Robustness checks

We conduct several robustness analyses to examine the effects of various policy interventions and the sensitivity of our findings to changes in the assumptions we use in our simulation. In Internet Appendix Figures E.1–E.3, we show that the fire-sale costs can be significantly reduced if the price impact coefficient is lower, the regulators relax the minimum RBC ratio (as a form of regulatory forbearance), and/or the insurers use less marking to market accounting. In Internet Appendix Figure E.5, we vary the maturity of the put option we use to simulate the value of the guarantee, as well as the equity premium used to price the put option, and show that our estimates of fire-sale costs are robust. For example, in the case of 2008 shocks with rating downgrades, the fire-sale costs across the assumptions are all in the range of 32–44% of the insurers' equity.

Importantly, Internet Appendix Figure E.4 explores different liquidation rules. In the baseline, we follow Greenwood, Landier, and Thesmar (2015) and Duarte and Eisenbach (2021) and assume that insurers liquidate different types of illiquid bonds in proportion to their holdings. Figure E.4 shows that our baseline estimates of fire-sale costs are in the middle across different alternatives. If the insurers followed the actual sale basket in 2008–2009, the fire-sale costs could increase by 4–46%, depending on nature of the shocks.

On the contrary, the fire-sale costs could be reduced by 25–33% if the insurers sold the illiquid bonds in proportion to the ratio of their RBC charge to price impact coefficient, which we show theoretically in Internet Appendix D.2 will minimize the fire-sale costs. If the insurers were also allowed to sell common stocks, which is inconsistent with reality, the fire-sale costs could be reduced even further.

6. Conclusion

We explore how systemic risk may arise from life insurers' provision of guarantees. We present a theoretical model that captures a novel mechanism through which the guarantees embedded in VA policies interact with insurers' risk taking incentives to create elevated investments in illiquid and risky assets. We calibrate the model using insurer-level data and confirm our model's prediction.

Upon a large, sustained decline in asset markets, the moneyness of VA guarantees increases, the guarantee reserves spike, and VA-writing insurers will need to shore up their capital positions. De-risking by selling illiquid bonds (and buying liquid bonds) will be a likely solution, but one that will cause contagion to other insurers and potentially other parts of the financial system. We show that VAs with guarantees raise the likelihood and severity of asset fire sales not only by exposing life insurers to undiversifiable asset shocks but also by incentivizing them to invest in the same domain of risky and illiquid fixed income assets.

Our findings imply that the transformation of the life insurance industry to focus more on saving and investment products has made life insurers less likely to behave as asset insulators that can absorb severe but temporary market dislocations (Chodorow-Reich, Ghent, and Haddad 2021). This has far-reaching implications for the stability of the financial system.

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