Physics Letters B 840 (2023) 137877

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

The role of long distance contribution to the $B \to K^* \ell^+ \ell^-$ in the Standard Model

Massimo Ladisa [a], Pietro Santorelli [b,c,*]

[a] CNR, Istituto per le Applicazioni del Calcolo "M. Picone", Via Amendola 122-D, I-70126 Bari, Italy
[b] Dipartimento di Fisica "E. Pancini", Universita' di Napoli Federico II - Complesso Universitario di Monte S. Angelo Edificio 6, via Cintia, 80126 Napoli, Italy
[c] INFN sezione di Napoli - Complesso Universitario di Monte S. Angelo Edificio 6, via Cintia, 80126 Napoli, Italy

ARTICLE INFO

Article history:
Received 5 August 2022
Received in revised form 25 March 2023
Accepted 25 March 2023
Available online 29 March 2023
Editor: B. Grinstein

Keywords:
Rare decays
Long distance physics
Lepton flavor universality violation

ABSTRACT

We investigate rare semileptonic $B \to K^* \ell^+ \ell^-$ by looking at a specific long distance contribution. Our analysis is limited to the very small values of physical accessible range of invariant mass of the leptonic couple q^2. We show that the light quarks loop has to be accounted for, along with the charming penguin contribution, in order to accurately compute the q^2-spectrum in the Standard Model. Such a long distance contribution may also play a role in the analysis of the lepton flavor universality violation in this process.

In the Standard Model the Flavor-Changing Neutral Current (FCNC) processes are sensitive probes of New Physics (NP) because they arise at loop level and are further suppressed by GIM mechanism. A reliable calculation of the process in the framework of the Standard Model is the first step to highlight effects of NP. The next one consists in comparing the hopefully precise measurements with the theoretical calculations. For a thorough review on the subject we address the reader to the recent paper [1] and the exhaustive bibliography therein. Hereinafter we will discuss one possible Long Distance (LD) contribution to the exclusive process $B \to K^* \ell^+ \ell^-$ which in principle is Cabibbo–Kobayashi–Maskawa (CKM) suppressed. We start discussing the e^2 corrections to the effective Hamiltonian responsible, in the Standard Model, of the $B \to K^* \ell^+ \ell^-$ decay, then we evaluate them.

The effective Hamiltonian for $\Delta B = -\Delta S = 1$ in the Standard Model responsible of the rare transition $b \to s \ell^+ \ell^-$ can be written in terms of a set of local operators [2]:

$$\mathcal{H}_W = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) = \mathcal{H}^{had} + \mathcal{H}^{sl} + \mathcal{H}^{\gamma} \,, \tag{1}$$

where G_F is the Fermi constant and V_{ij} are elements of the CKM mixing matrix. The operators O_i, written in terms of quark, photon and gluon fields, and can be found for example in Ref. [3] and the $\mathcal{H}^{had}$ contains the operators O_i with $i = 1, \ldots, 6$, $\mathcal{H}^{sl}$ contains the operators O_9 and O_{10} whereas $\mathcal{H}^{\gamma}$ contains O_7.[1] In our calculation the main role is played by the operators O_i with $i \in \{1, 2, 7, 9, 10\}$ which we report here for convenience

$$\begin{aligned}
O_1 &= (\bar{s}_{L\alpha} \gamma^{\mu} b_{L\alpha})(\bar{c}_{L\beta} \gamma_{\mu} c_{L\beta}) \,, \\
O_2 &= (\bar{s}_{L\alpha} \gamma^{\mu} b_{L\beta})(\bar{c}_{L\beta} \gamma_{\mu} c_{L\alpha}) \,, \\
O_7 &= \frac{e}{16\pi^2} m_b (\bar{s}_{L\alpha} \sigma^{\mu\nu} b_{R\alpha}) F_{\mu\nu} \,,
\end{aligned}$$

* Corresponding author at: Dipartimento di Fisica "E. Pancini", Universita' di Napoli Federico II - Complesso Universitario di Monte S. Angelo Edificio 6, via Cintia, 80126 Napoli, Italy.

E-mail addresses: m.ladisa@ba.iac.cnr.it (M. Ladisa), pietro.santorelli@na.infn.it (P. Santorelli).

[1] O_8 is the analogous O_7 for the gluon fields.

https://doi.org/10.1016/j.physletb.2023.137877

$$O_9 = \frac{e^2}{16\pi^2}(\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\,\bar{\ell}\gamma_\mu\ell\,,$$
$$O_{10} = \frac{e^2}{16\pi^2}(\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\,\bar{\ell}\gamma_\mu\gamma_5\ell\,. \tag{2}$$

The Greek letters are color indices and, as usual, $b_{R/L} = \left(\frac{1\pm\gamma_5}{2}\right)b$, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. $F_{\mu\nu}$ denotes the electromagnetic field strength tensor and e is the electromagnetic charge.

From here on we shall focus on the $B \to K^*\ell^+\ell^-$ process. In Ref. [4] we will systematically analyze the processes containing the K and the K^*.

There are two classes of contributions to the $B \to K^*\ell^+\ell^-$, the first one comes from the semileptonic part of the effective Hamiltonian, i.e. the operators O_9 and O_{10}. In this case the amplitude factors out

$$A_{\rm SD}(B \to K^*\ell^+\ell^-) = \left\langle K^*\ell^+\ell^-\right|\mathcal{H}^{sl}\left|B\right\rangle =$$
$$\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{e^2}{16\pi^2}\sum_{i=9,10}C_i\left\langle\ell^+\ell^-\right|\bar{\ell}\Gamma^i_\mu\ell\left|0\right\rangle\left\langle K^{(*)}\right|(\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\left|B\right\rangle\,, \tag{3}$$

and it can be written in terms of form factors (e.g. those in Ref. [5], our choice hereafter). This is called the short distance (SD) part of the total amplitude of the process: the hadronic contribution is incorporated in the form factors while the perturbative corrections in the Wilson coefficients (for them we use the same values in [5]).

Moreover, at the same order in e, i.e. e^2, the amplitude contains the contribution of the hadronic effective Hamiltonian multiplied by the QED interaction twice. An amplitude different from zero is obtained when the former interaction produces the leptonic pair in the final state and the latter one factors an hadronic current out:

$$A_{LD}(B \to K^*\ell^+\ell^-) =$$
$$e^2\left\langle K^*\ell^+\ell^-\right|T\int A^\mu(x)\bar{\ell}(x)\gamma_\mu\ell(x)dx\int dy\left[A^\nu(y)j^{e.m.}_\nu(y)\right]\mathcal{H}^{had}(0)\left|B\right\rangle =$$
$$-\frac{ie^2}{q^2}\int d^4xe^{-iqx}\left\langle\ell^+\ell^-\right|\bar{\ell}(x)\gamma_\mu\ell(x)\left|0\right\rangle\int d^4ye^{iqy}\left\langle K^*(p')\right|Tj^\nu_{e.m.}(y)\mathcal{H}^{had}(0)\}\left|B(p)\right\rangle$$
$$\equiv L_\mu\mathcal{H}^\mu(p,p')\,. \tag{4}$$

$\mathcal{H}^\mu(p,p')$ is essentially a non-local term and we call it LD contribution to the decay process although, as discussed recently for example in [6], the hadronic matrix element $\mathcal{H}(p,p')$ contains a factorizable part. We shall clarify this point. In (4) $q^2 = (p-p')^2$ and $j^{e.m.}_\mu = \sum_q Q_q\bar{q}\gamma_\mu q$. By considering the CKM matrix elements and the strength of the Wilson coefficients we can conclude that the leading contribution to $\mathcal{H}$ will come from the operators O_1 and O_2 in the effective Hamiltonian proportional to $V_{cb}V^*_{cs} \approx V_{tb}V^*_{ts}$ and so the T-product is different from zero if and only if $j^{e.m.}_\mu = Q_c\bar{c}\gamma_\mu c$: the contribution of these terms is commonly called the *charm-loop effect*. In other words:

$$\mathcal{H}^\mu = Q_c\int d^4ye^{iqy}\left\langle K^*(p')\right|\mathrm{T}\bar{c}(y)\gamma^\mu c(y)\left(C_1O_1(0)+C_2O_2(0)\right)\left|B(p)\right\rangle\,. \tag{5}$$

The analysis, in QCD -factorization, of the non-local term in the previous equation was done in [7] where the LD contribution results to be essentially proportional to the factorizable part, i.e. our A_{SD}. A systematic study of $\mathcal{H}^\mu$ can be read in [8] where the authors show how to generalize the approach in [7] to the low-q^2 region. The state-of-the-art of these calculations can be found in Ref. [9]. Due to the difficulties to reliably estimate the LD contribution, a different approach relies on the use of data-driven methods to account for the theoretical uncertainties and to quantify possible deviations from the Standard Model [10]. All these papers are devoted to the charm loop contribution. Hereinafter we shall study a contribution which is CKM suppressed although it gives, at very small q^2, a contribution comparable to the short distance one. To this aim, the four quarks operators are

$$O_1^{(u)} = (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{u}_{L\beta}\gamma_\mu u_{L\beta})\,,$$
$$O_2^{(u)} = (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})(\bar{u}_{L\beta}\gamma_\mu u_{L\alpha})\,, \tag{6}$$

and $\mathcal{H}^\mu$ is different from zero for the $j^{e.m.}_\mu = Q_u\bar{u}\gamma_\mu u$:

$$\mathcal{H}^{(u)\mu} = Q_u\int d^4ye^{iqy}\left\langle K^*(p')\right|\mathrm{T}\bar{u}(y)\gamma^\mu u(y)\left(C_1O_1^{(u)}(0)+C_2O_2^{(u)}(0)\right)\left|B(p)\right\rangle\,; \tag{7}$$

in Fig. 1 one can find the Feynman graph of the T-product in Eq. (7) while in Fig. 2 a possible mesonic graph, the one we shall consider hereafter.[2] It should be observed that the T-product in Eq. (7) give rise to more Feynman graphs than the one in Fig. 1. There are three topologies. Nevertheless, the graph in Fig. 1 is the only one that has a meaningful hadronic counterpart. Indeed hadronic counterparts of other quark topologies are ruled out either by the off-shellness of the intermediate particles (e.g. in $B \to B^*\pi$) or by the strong coupling suppression (e.g. $\rho \to \pi\gamma^*$ is smaller than $\omega \to \pi\gamma^*$ [11]) or by the experimental unavailability of some couplings (e.g. $\omega \to \eta^{(\prime)}\ell^+\ell^-$ as explained in Ref. [12]). A numerical comparison with the results of the QCD factorization and Light-Cone Sum Rules [7,13], where

[2] The graph obtained by interchanging ω and π mesons is suppressed by the off-shellness of both the ω meson in the t channel and the K-K^*-ω coupling.

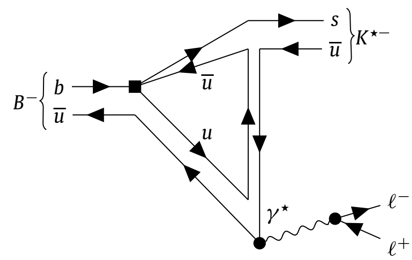


Fig. 1. The Feynman graph obtained by doing the T-product in Eq. (7)

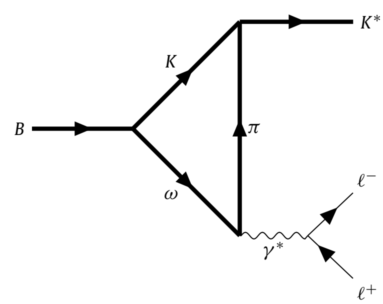


Fig. 2. Hadronic representation of one of the contributions coming from the graph in Fig. 1.

all topologies give contributions, is difficult. Our calculation of the hadronic counterpart of the quark loop scattering topology cannot be considered as an estimation of it, but just an estimation of the hadronic part of it. The same is true for the other topologies where we found a negligible long-distance hadronic contribution. We stress that this is just one of the possible hadronic terms representing $\mathcal{H}^{(u)\mu}$ and so our calculation is just an estimation of one non-factorizable, LD contribution to the one in Eq. (3). Moreover, adding other hadronic contributions of $\mathcal{H}^{(u)\mu}$ in the narrow range of q^2 we have studied is unlikely to result in a complete cancellation. This means that our calculation can be considered as an order of magnitude estimation of such a long distance contribution. On the other hand we cannot use arguments based on the quark hadron duality to give an upper bound to these long-distance effects because of the limited q^2 range, as discussed in the pioneering paper of Poggio et al. [14]. In fact, many examples of violation of the local quark-hadron duality can be found in literature (e.g. in Refs. [15,16]).

The calculation of the triangle graph in Fig. 2 is straightforward. We consider the weak transition $B^- \to K^-\omega$ followed by the electromagnetic one $\omega \to \pi^0\gamma$. The meson-loop is closed by the K^*-K-π^0 strong vertex, $g_{KK^\star\pi}$, computed by the $K^* \to K\pi^0$ decay. The analysis of the $B^- \to K^-\omega$ transition was done, for example, in Ref. [17] where the contribution of the charming penguins was accounted for to improve the factorization approximation prediction. In particular, the branching ratio of $B^- \to K^-\omega$ is enhanced by the charming penguin contribution of about one order of magnitude; our prediction [17], $Br(B^- \to K^-\omega) = 6.19 \times 10^{-6}$, is in excellent agreement with the PDG average $Br(B^- \to K^-\omega) = (6.5 \pm 0.4) \times 10^{-6}$ [18]. In Ref. [17] the so called charming penguins, discussed for the first time in [19], are evaluated by considering the charm rescattering into the charmless two body final state, i.e., for example, $B \to D\,D_s \to K\,\omega$ (cf. also [20–23]).

The last ingredient of the calculation is the $\omega \to \pi\gamma$ transition. The radiative decays of the light vector and axial-vector mesons have been systematically studied in Ref. [24] in the framework of chiral Lagrangian written in terms of the Goldstone boson octet and the nonet of light vector mesons. The electromagnetic form factor relevant to the $\omega \to \pi\ell^+\ell^-$ has been obtained in Ref. [12], where the case with $\ell \equiv e, \mu$ has been considered. It is worth noting that the corresponding widths differ by about one order of magnitude due to the deep decreasing form factor at small dilepton invariant mass. In fact, experimentally, we have $Br(\omega \to \pi^0 e^+e^-) = (7.7 \pm 0.6) \times 10^{-4}$ and $Br(\omega \to \pi^0\mu^+\mu^-) = (1.34 \pm 0.18) \times 10^{-4}$ [18].

Schematically, the LD amplitude can be written as

$$
\begin{aligned}
A_{LD}(\lambda_{K^*}, \sigma_{\ell^+}, \sigma_{\ell^-}) \;=\; & \frac{1}{q^2} \sum_{\lambda_\omega, \lambda_\gamma} \mathcal{A}(B \to K\omega(\lambda_\omega)) \times \\
& \mathcal{A}(K\omega(\lambda_\omega) \to K^*(\lambda_{K^*})\gamma^*(\lambda_\gamma))\, \mathcal{A}(\gamma^*(\lambda_\gamma) \to \ell^+(\sigma_{\ell^+})\ell^-(\sigma_{\ell^-})),
\end{aligned} \tag{8}
$$

where λ's and σ's are the vector particle and fermion polarizations, respectively. Whereas, the weak decay of B into $K\omega$ and the rescattering amplitude can be recast as follows:

$$
\mathcal{A}(B \to K\omega(\lambda_\omega)) = g_{BK\omega}\,(p_K + p_\omega)\cdot \epsilon^*(\lambda_\omega), \tag{9}
$$

$$
\mathcal{A}(K\omega(\lambda_\omega) \to K^*(\lambda_{K^*})\gamma^*(\lambda_\gamma)) = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_{t_{min}}^{t_{max}} \frac{dt}{2|\vec{p}_\omega||\vec{q}|} \times
$$

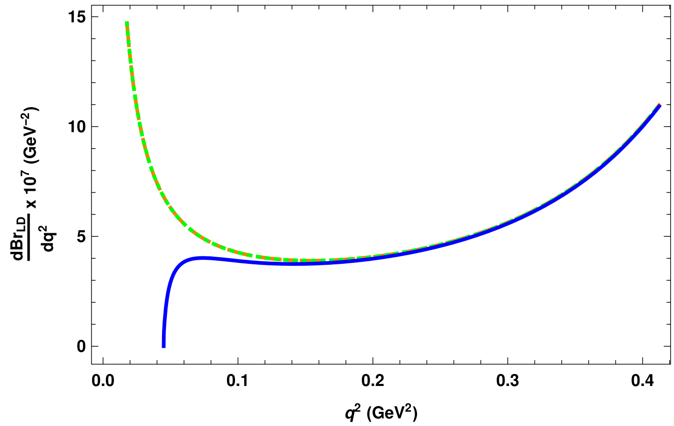


Fig. 3. $B \to K^*\ell^+\ell^-$ long distance differential branching ratios as a function of the dilepton invariant mass squared. Units are GeV^{-2}, while colors refer to $m_\ell = 0$ case in orange (dashed), $m_\ell = m_e$ in green (dotted) and $m_\ell = m_\mu$ in blue (solid).

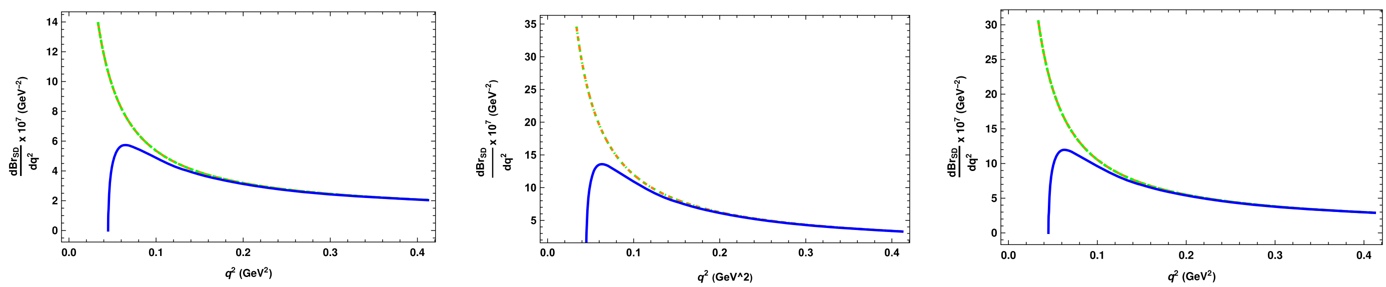


Fig. 4. $B \to K^*\ell^+\ell^-$ short distance differential branching ratios as a function of the dilepton invariant mass squared. Units are GeV^{-2}, while colors refer to $m_\ell = 0$ case in orange (dashed), $m_\ell = m_e$ in green (dotted) and $m_\ell = m_\mu$ in blue (solid). Plots are computed by employing different sets of hadronic form factors (i.e., from left to right, QCD Sum Rules [5], Ligth-Cone SR [25] and Lattice [26]).

$$\left\{ \frac{i\, e\, g_{KK^*\pi}\, f_{\omega\pi^0}}{t - m_\pi^2}\, p_K \cdot \epsilon^*(\lambda_K) \epsilon^{\mu\nu\alpha\beta} p_{\omega\mu} q_\nu \epsilon_\alpha(\lambda_\omega) \epsilon^*_\beta(\lambda_\gamma) \right\}, \tag{10}$$

$$\mathcal{A}(\gamma^*(\lambda_\gamma) \to \ell^+(\sigma_{\ell^+})\ell^-(\sigma_{\ell^-})) = e\, \bar{u}_{\ell^-}(\sigma_{\ell^-}) \left(\gamma \cdot \epsilon_\gamma(\lambda_\gamma)\right) v_{\ell^+}(\sigma_{\ell^+}), \tag{11}$$

being ϕ the euclidean $\vec{p}_\omega$ azimuth (z-axis) and $t = (p_{K^*} - p_K)^2 = (p_\omega - q)^2$.[3] Here $f_{\omega\pi^0}$ is the electromagnetic form factor computed in [12] in the large N_c approximation (where loops are automatically suppressed at leading order), $g_{BK\omega}$ contains the weak coupling and the CKM matrix elements, u and v are Dirac spinors. Our knowledge of the $f_{\omega\pi^0}$ electromagnetic form factor is dictated by the physical range of the $\omega \to \pi^0\gamma^*$ transition, $[4m_\ell^2, (m_\omega - m_\pi)^2]$, and, accordingly, our results are valid in the same range of the dilepton invariant mass. The raising behavior of the branching ratios is due to the ρ meson pole in the $f_{\omega\pi^0}$ electromagnetic form factor. Due to the pseudoscalar nature of the B meson, only the longitudinal polarization of the ω meson contributes to the amplitude; furthermore, after integrating on the azimuth angle (ϕ), the positive (negative) K^* meson polarization selectively couples to the negative (positive) γ^* polarization (while the γ^* longitudinal polarization is ruled out by the ω-γ^*-π Levi–Civita coupling). In Fig. 3 the differential branching ratio $dBr(B \to K^*\ell^+\ell^-)_{LD}/dq^2$ (in unit of 10^{-7}) is plotted vs q^2. The long distance part of the branching ratio is, in the three different cases $m_\ell = (0, m_e, m_\mu)$ (with colors orange, green and blue, respectively), of the same order of magnitude of the short distance part as we shall see later. In this range of q^2 the difference between the electron and the muon case in the $\omega \to \pi^0\ell^+\ell^-$ leads to a small lepton flavor violation. In fact, the branching ratios evaluated in this range of q^2 give $Br(B \to K^*e^+e^-)_{LD} = 2.0 \times 10^{-7}$ and $Br(B \to K^*\mu^+\mu^-)_{LD} = 1.9 \times 10^{-7}$ this effect which is not related to any new interaction violating the lepton flavor universality could mimic the violation.

In order to understand to what extent the long distance contribution to $B \to K^*\ell^+\ell^-$ can affect both the branching ratio and the lepton flavor universality violation, it is necessary to compute the short distance amplitude, i.e. the amplitude in Eq. (3). We employ three sets of form factors calculated in Ref. [5], in Ref. [25] and in Ref. [26] and evaluate the $dBr(B \to K^*\ell^+\ell^-)/dq^2$ with lepton finite mass in the final state. In Fig. 4 we plot the short distance differential branching ratio alone for the case of $m_\ell = 0$ in orange, $m_\ell = m_e$ in green and $m_\ell = m_\mu$ in blue, in the left panel we use the QCD sum rules results [5] for the form factors, in the central one and in the right ones LCSR [25] and Lattice [26] calculations are used, respectively. Figs. 4, 5, 6 and 7 have been modified too. As one can see by looking at the Fig. 4 the larger values of the form factors at $q^2 = 0$ in [25,26] respect to the ones in [5] imply a factor of about two in the $d\Gamma(B \to K^*\ell^+\ell^-)/dq^2$.

The Figs. 5, 6, 7 show that the long distance contribution (in orange) is of the same order of magnitude of the short distance one regardless of the lepton flavor. The long distance contribution in all cases increases approaching the $(m_\omega - m_\pi)^2$ upper limit for q^2 because of the pole dominance of the form factor $f_{\omega\pi^0}$. The blue regions represent the band of values of the total differential branching ratios: the lower (upper) curves refer to the constructive (distructive) interference between the short and the long distance contributions. For each q^2

[3] t_{min} (t_{max}) corresponds to t when the euclidean $\vec{p}_\omega$ colatitude θ equals 0 (π).

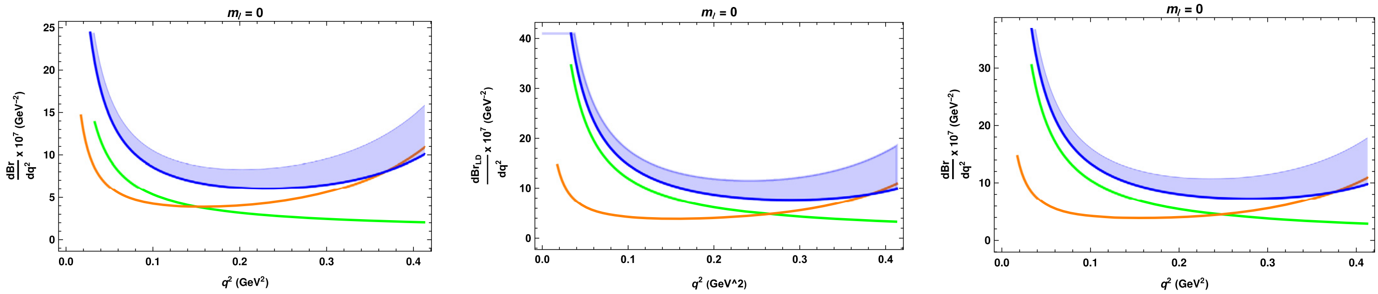


Fig. 5. $B \to K^*\ell^+\ell^-$ total differential branching ratios as a function of the dilepton invariant mass squared when $m_\ell = 0$ (units are GeV^{-2}). Colors refer to the short distance contribution in green, the long distance contribution in orange, while the combination of both contributions spans an area in blue (upper thin bound corresponds to the difference, lower thick one to the sum). Plots are computed by employing different sets of hadronic form factors (i.e., from left to right, QCD Sum Rules [5], Ligth-Cone SR [25] and Lattice [26]).

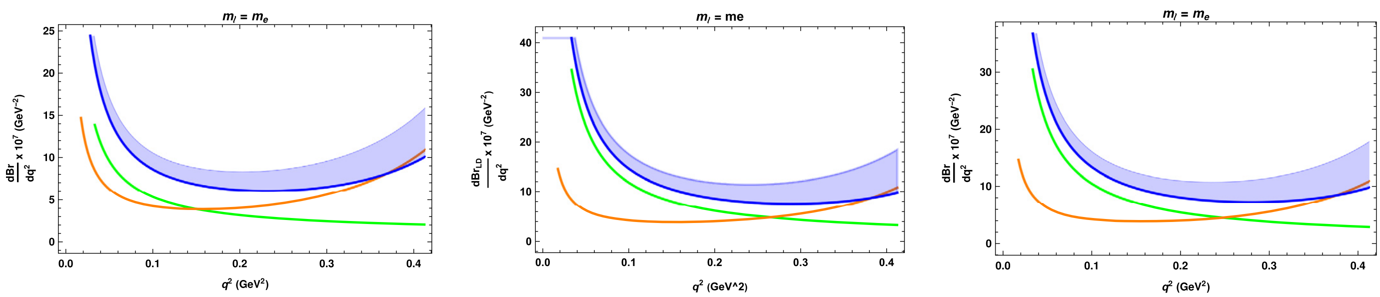
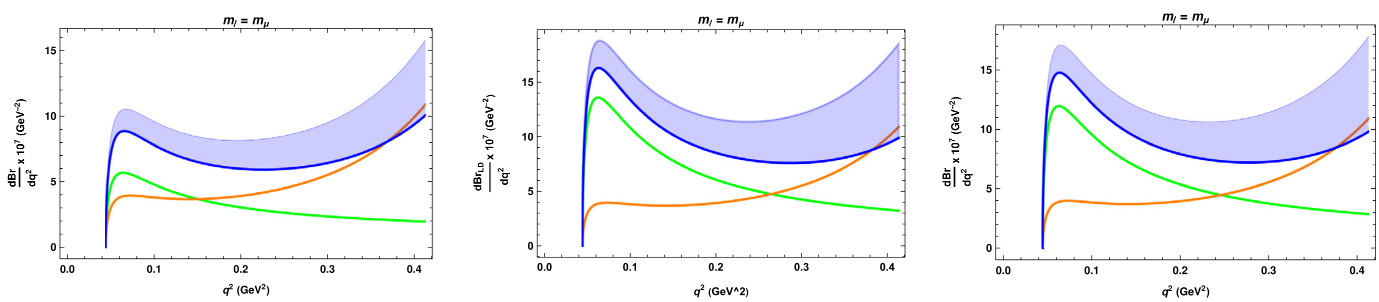


Fig. 6. $B \to K^*\ell^+\ell^-$ total differential branching ratios as a function of the dilepton invariant mass squared when $m_\ell = m_e$ (units are GeV^{-2}). Colors refer to the short distance contribution in green, the long distance contribution in orange, while the combination of both contributions spans an area in blue (upper thin bound corresponds to the difference, lower thick one to the sum). Plots are computed by employing different sets of hadronic form factors (i.e., from left to right, QCD Sum Rules [5], Ligth-Cone SR [25] and Lattice [26]).



(a) (b) (c)

Fig. 7. $B \to K^*\ell^+\ell^-$ total differential branching ratios as a function of the dilepton invariant mass squared when $m_\ell = m_\mu$ (units are GeV^{-2}). Colors refer to the short distance contribution in green, the long distance contribution in orange, while the combination of both contributions spans an area in blue (upper thin bound corresponds to the difference, lower thick one to the sum). Plots are computed by employing different sets of hadronic form factors (i.e., from left to right, QCD Sum Rules [5], Ligth-Cone SR [25] and Lattice [26]).

the width of the band can be considered as an estimation of the theoretical error on the differential branching ratio calculation. In Table 1 the *partial* branching ratios and R_{K^*}, i.e. evaluated in the limited dilepton mass squared range $[4m_\mu^2, (m_\omega - m_\pi)^2]$, are collected to point out the amount of the long distance contribution: it is clear that, in the region of q^2 studied, the long distance contribution increases the tension with the experimental data on R_{K^*}. In fact, the measurement of R_{K^*} in the smallest range was done by LHCb collaboration [27]

$$R_{K^*} = 0.660^{+0.110}_{-0.070} \pm 0.024 \qquad (2m_\mu)^2 < q^2 < 1.1\,\text{GeV}^2 \,. \tag{12}$$

Moreover, the BELLE collaboration presented the following preliminary result [28] obtained by averaging over B^0 and B^+:

$$R_{K^*}[0.045, 1.1] = 0.52^{+0.36}_{-0.26} \pm 0.05. \tag{13}$$

These values have to be compared to the SM predictions [29][4]

$$R_{K^*}^{SM} = 0.906 \pm 0.028 \qquad (2m_\mu)^2 < q^2 < 1.1\,\text{GeV}^2 \,, \tag{14}$$

[4] The authors in Ref. [29] accounted for $\log m_\ell$-enhanced QED corrections for the estimation of R_{K^*} beyond the short-distance contribution within the Standard Model. However, a complete and correct treatment of the QED long distance contribution is missing in the $B \to K^*\ell^+\ell^-$ decay mode while can be found in [30] for the $B \to K\ell^+\ell^-$.

Table 1
Branching ratios and R_{K^*} computed in the dilepton mass squared range $[4m_\mu^2, (m_\omega - m_\pi)^2]$ as a function of lepton mass (m_ℓ) and along with selected amplitude contributions (**SD**,**LD** and combinations thereof). The results have been computed for three sets of hadronic form factors.

		SD	LD	SD+LD	SD-LD
$m_\ell = 0, m_e$ $(\times 10^{-7})$	[5]	1.333	2.028	2.837	3.885
	[25]	2.731	2.028	3.986	5.531
	[26]	2.405	2.028	3.714	5.151
$m_\ell = m_\mu$ $(\times 10^{-7})$	[5]	1.190	1.919	2.609	3.609
	[25]	2.400	1.919	3.581	5.056
	[26]	2.113	1.919	3.347	4.717
R_{K^*}	[5]	0.893	0.946	0.920	0.929
	[25]	0.879	0.946	0.898	0.914
	[26]	0.879	0.946	0.901	0.916

which is consistent with our value of R_{K^*} by considering SD contribution alone. The inclusion of the LD term increases our prediction of R_{K^*}.
Before concluding, it is essential to emphasize that the contribution we have calculated cannot be rewritten in terms of the SD amplitude because it is essentially a rescattering and so the Lorentz couplings are different from the ones in Eq. (3). This entails that it cannot be estimated by fitting data to the shifts of the Wilson coefficients C_9 and C_{10} with respect to the Standard Model values.

In conclusion, in this letter we have estimated the LD contribution of the light quark loop to the $B \to K^*\ell^+\ell^-$ with $m_\ell \in \{m_e, m_\mu\}$. We have focused on a specific hadronic rescattering channel: $B \to K\omega \to K^*\ell^+\ell^-$. The calculations have been performed in the range of the dilepton mass squared $\left[4m_\mu^2, (m_\omega - m_\pi)^2\right]$, where our hadronic representation of the quark loop is reliable. The LD contribution increases the branching ratios of about a factor 2.5 (1.8) with respect to the QCD sum rules (Ligth-cone and lattice) predictions for the SD results. Our findings also indicate a change in the ratio R_{K^*}.

Added note

After this paper was submitted, the LHCb Collaboration has published a new analysis of the rare B decay discussed in our paper [31,32]. The analysis is based on a higher statistics data sample and, in spite of previous measurements of $R_{K^*} = 0.660$ at low leptonic invariant mass (showing a discrepancy with respect to the Standard Model prediction $R_{K^*} = 0.906$ with a significance of some three sigmas), the new R_{K^*} measured value is set to $R_{K^*} = 0.927$, to be compared to our estimation $R_{K^*} = 0.920$-0.929 (see Table 1 in our manuscript with QCD SR form factors) combining Short-Distance ($R_{K^*} = 0.893$ with QCD SR form factors) and Long-Distance ($R_{K^*} = 0.946$) contributions. The results are in agreement with the SM.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgements

We thank Stefan Leupold for discussions. This study has been partially supported by INFN project (Iniziativa Specifica) QFT-HEP.

References

[1] A. D'Alise, et al., Standard model anomalies: lepton flavour non-universality, g-2 and W-mass, arXiv:2204.03686, 4.2022.
[2] B. Grinstein, M.J. Savage, M.B. Wise, B –> X(s) e+ e- in the six quark model, Nucl. Phys. B 319 (1989) 271–290, https://doi.org/10.1016/0550-3213(89)90078-3.
[3] K.G. Chetyrkin, M. Misiak, M. Munz, Weak radiative B meson decay beyond leading logarithms, Phys. Lett. B 400 (1997) 206–219, https://doi.org/10.1016/S0370-2693(97)00324-9; Phys. Lett. B 425 (1998) 414 (Erratum), arXiv:hep-ph/9612313.
[4] M. Ladisa, P. Santorelli, in preparation.
[5] P. Colangelo, F. De Fazio, P. Santorelli, E. Scrimieri, QCD sum rule analysis of the decays $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$, Phys. Rev. D 53 (1996) 3672–3686, https://doi.org/10.1103/PhysRevD.53.3672; Phys. Rev. D 57 (1998) 3186 (Erratum), arXiv:hep-ph/9510403.
[6] N. Gubernari, D. van Dyk, J. Virto, Non-local matrix elements in $B_{(s)} \to \{K^{(*)}, \phi\}\ell^+\ell^-$, J. High Energy Phys. 02 (2021) 088, https://doi.org/10.1007/JHEP02(2021)088, arXiv:2011.09813.
[7] M. Beneke, T. Feldmann, D. Seidel, Systematic approach to exclusive $B \to Vl^+l^-$, $V\gamma$ decays, Nucl. Phys. B 612 (2001) 25–58, https://doi.org/10.1016/S0550-3213(01)00366-2, arXiv:hep-ph/0106067.
[8] A. Khodjamirian, T. Mannel, A.A. Pivovarov, Y.M. Wang, Charm-loop effect in $B \to K^{(*)}\ell^+\ell^-$ and $B \to K^*\gamma$, J. High Energy Phys. 09 (2010) 089, https://doi.org/10.1007/JHEP09(2010)089, arXiv:1006.4945.
[9] N. Gubernari, M. Reboud, D. van Dyk, J. Virto, Improved theory predictions and global analysis of exclusive $\mathbf{b} \to \mathbf{s}\mu^+\mu^-$ processes, arXiv:2206.03797, 6.2022.
[10] M. Ciuchini, M. Fedele, E. Franco, A. Paul, L. Silvestrini, M. Valli, New physics without bias: charming penguins and lepton universality violation in $b \to s\ell^+\ell^-$ decays, arXiv:2110.10126, 10.2021.
[11] A. Gokalp, O. Yilmaz, The coupling constants g(rho pi gamma) and g(omega pi gamma) as derived from QCD sum rules, Eur. Phys. J. C 24 (2002) 117–120, https://doi.org/10.1007/s100520200934, arXiv:nucl-th/0103033.

[12] C. Terschlusen, S. Leupold, Electromagnetic transition form factors of light vector mesons, Phys. Lett. B 691 (2010) 191–201, https://doi.org/10.1016/j.physletb.2010.06.033, arXiv:1003.1030.
[13] J. Lyon, R. Zwicky, Isospin asymmetries in $B \to (K^*, \rho)\gamma/l^+l^-$ and $B \to Kl^+l^-$ in and beyond the standard model, Phys. Rev. D 88 (9) (2013) 094004, https://doi.org/10.1103/PhysRevD.88.094004, arXiv:1305.4797.
[14] E.C. Poggio, H.R. Quinn, S. Weinberg, Smearing the quark model, Phys. Rev. D 13 (1976) 1958, https://doi.org/10.1103/PhysRevD.13.1958.
[15] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Penguins with charm and quark-hadron duality, Eur. Phys. J. C 61 (2009) 439–449, https://doi.org/10.1140/epjc/s10052-009-1028-9, arXiv:0902.4446.
[16] P. Colangelo, C.A. Dominguez, G. Nardulli, Violations of local duality in the heavy quark sector, Phys. Lett. B 409 (1997) 417–424, https://doi.org/10.1016/S0370-2693(97)00890-3, arXiv:hep-ph/9705390.
[17] C. Isola, M. Ladisa, G. Nardulli, P. Santorelli, Charming penguins in B —> K* pi, K(rho, omega, phi) decays, Phys. Rev. D 68 (2003) 114001, https://doi.org/10.1103/PhysRevD.68.114001, arXiv:hep-ph/0307367.
[18] R.L. Workman, Review of particle physics, PTEP 2022 (2022) 083C01.
[19] M. Ciuchini, E. Franco, G. Martinelli, L. Silvestrini, Charming penguins in B decays, Nucl. Phys. B 501 (1997) 271–296, https://doi.org/10.1016/S0550-3213(97)00388-X, arXiv:hep-ph/9703353.
[20] C. Isola, M. Ladisa, G. Nardulli, T.N. Pham, P. Santorelli, Charming penguin contributions to $B \to K\pi$, Phys. Rev. D 64 (2001) 014029, https://doi.org/10.1103/PhysRevD.64.014029, arXiv:hep-ph/0101118.
[21] C. Isola, M. Ladisa, G. Nardulli, T.N. Pham, P. Santorelli, Charming penguin contributions to charmless B decays into two pseudoscalar mesons, Phys. Rev. D 65 (2002) 094005, https://doi.org/10.1103/PhysRevD.65.094005, arXiv:hep-ph/0110411.
[22] M. Ladisa, V. Laporta, G. Nardulli, P. Santorelli, Final state interactions for $B \to VV$ charmless decays, Phys. Rev. D 70 (2004) 114025, https://doi.org/10.1103/PhysRevD.70.114025, arXiv:hep-ph/0409286.
[23] A. Deandrea, M. Ladisa, V. Laporta, G. Nardulli, P. Santorelli, Role of final state interactions in the B meson decay into two pions, Int. J. Mod. Phys. A 21 (2006) 4425–4438, https://doi.org/10.1142/S0217751X0603401X, arXiv:hep-ph/0508083.
[24] M.F.M. Lutz, S. Leupold, On the radiative decays of light vector and axial-vector mesons, Nucl. Phys. A 813 (2008) 96–170, https://doi.org/10.1016/j.nuclphysa.2008.09.005, arXiv:0801.3821.
[25] A. Bharucha, D.M. Straub, R. Zwicky, $B \to V\ell^+\ell^-$ in the standard model from light-cone sum rules, J. High Energy Phys. 08 (2016) 098, https://doi.org/10.1007/JHEP08(2016)098, arXiv:1503.05534.
[26] R.R. Horgan, Z. Liu, S. Meinel, M. Wingate, Lattice QCD calculation of form factors describing the rare decays $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$, Phys. Rev. D 89 (9) (2014) 094501, https://doi.org/10.1103/PhysRevD.89.094501, arXiv:1310.3722.
[27] R. Aaij, et al., Test of lepton universality with $B^0 \to K^{*0}\ell^+\ell^-$ decays, J. High Energy Phys. 08 (2017) 055, https://doi.org/10.1007/JHEP08(2017)055, arXiv:1705.05802.
[28] A. Abdesselam, et al., Test of lepton-flavor universality in $B \to K^*\ell^+\ell^-$ decays at Belle, Phys. Rev. Lett. 126 (16) (2021) 161801, https://doi.org/10.1103/PhysRevLett.126.161801, arXiv:1904.02440.
[29] M. Bordone, G. Isidori, A. Pattori, On the standard model predictions for R_K and R_{K^*}, Eur. Phys. J. C 76 (8) (2016) 440, https://doi.org/10.1140/epjc/s10052-016-4274-7, arXiv:1605.07633.
[30] G. Isidori, D. Lancierini, S. Nabeebaccus, R. Zwicky, QED in $\bar{B} \to \bar{K}\bar{\ell}\ell$ LFU ratios: theory versus experiment, a Monte Carlo study, J. High Energy Phys. 10 (2022) 146, https://doi.org/10.1007/JHEP10(2022)146, arXiv:2205.08635.
[31] Test of lepton universality in $b \to s\ell^+\ell^-$ decays, arXiv:2212.09152, 12.2022.
[32] Measurement of lepton universality parameters in $B^+ \to K^+\ell^+\ell^-$ and $B^0 \to K^{*0}\ell^+\ell^-$ decays, arXiv:2212.09153, 12.2022.