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The limits of limitless debt

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ABSTRACT

While low real interest rates and issuing public debt in fiat money safeguard against rising sovereign debt-to-GDP ratios, evidence shows that high debt often precedes default, and credit spreads may not signal imminent risk.

We offer a simple way to model the trade-offs using local martingales. On the one hand, it acknowledges that large debt overhangs tend to raise default risks. On the other hand, it allows sovereigns to roll over debt regardless of long-term fiscal solvency. The combination allows credit spreads to stay very low for decades, eventually spiral out of control and trigger a default. Hence, neither the reassurance of low spreads nor the alarm from growing overhang should automatically prevail.

To illustrate the trade-offs, we review the ebb and flow of US sovereign debt burdens since World War II. Between record peacetime debt-to-GDP ratios and weakened fiscal discipline, an exemplary double-or-triple-A credit rating for the US no longer seems justified. Moreover, the current outlook is poor, with debt growing much faster than GDP and scant prospects of contraction.

1. Introduction

Over the past fifty years, most prominent states have amassed record or near-record levels of debt (Kose et al., 2020; Yared, 2019). After netting out debt held by central banks, the average sovereign debt-to-GDP ratio in 2012 reached 80 percent globally and 97 percent for G7 countries (IMF, 2022). A mere two decades ago, peacetime debt accumulations that high were widely deemed unachievable; it was presumed that market pressures would compel sovereigns to either restrain their debt ratios or default. Reality has confounded these expectations.

Naturally, this has prompted a reassessment. Relatively few economists currently profess much concern about the default risks of major sovereigns. Most macroeconomic models pay little attention to the stocks of government debt (Barro, 1974; Bhandari et al., 2017); their only concern is to restrain its growth rate relative to GDP (Hellwig, 2021). Low real interest rates decrease funding burdens and suggest that the bond market is optimistic regarding long-term sustainability (Blanchard, 2019). Also, the debt is mostly in fiat currency, whose issuance imposes few direct costs on the government (Panizza et al., 2009; Bolton, 2016).

Presumptions of fully rational expectations encourage complacency. If the fiscal path is unsustainable, the debt-to-GDP ratio will keep rising, thus making public debt a Ponzi scheme. If all investors realize this, none should lend now, and the debt market will collapse. From that perspective, low credit spreads on overnight sovereign debt always testify to 99%+ confidence in long-term sustainability, regardless of accumulated debt, current deficits, or near-term budget outlooks.

Nevertheless, history reveals that lenders frequently underestimate risks. Fiat currency issuers sometimes default, regardless of their technical ability to service their debt. Debt markets seldom give much advance warning of default; they can be complacent

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for long periods and then flip to alarm. Big increases in debt-to-GDP ratios often point to future challenges in servicing regardless of current spreads. Judging from the historical evidence gathered in Reinhart and Rogoff's book *This Time is Different* (2009) and later expanded upon by Abbas et al. (2019) and Badia et al. (2022), current debt levels and trends signal danger.

Two branches of theoretical research support these concerns. One branch highlights that rising debt-to-GDP ratios tend to increase default risks (Debrun et al., 2019; Ghosh et al., 2013; Chatterjee and Eyigungor, 2012). The other branch acknowledges the vulnerability of asset markets to bubbles, where prices overstate fundamental value (Loewenstein and Willard, 2000; Cox and Hobson, 2005; Protter, 2013; Obayashi et al., 2017; Jarrow and Kwok, 2021; Choi and Jarrow, 2022). Regrettably, standard macroeconomic models of debt sustainability have not effectively incorporated either insight, possibly because induction arguments in finite horizon models and a transversality condition in infinite horizon models prevent bubbles (Abreu and Brunnermeier, 2003).

Our work makes three substantial contributions to the existing literature. First, we broaden standard models of sovereign debt to include endogenous credit risk and potential bubbles. Every transaction we model is a fair game with zero expected profit, and bubbles emerge only because otherwise savvy lenders lack long-term foresight. From a technical perspective, we treat debt markets as strictly local martingales rather than the (full) martingales that Samuelson (1965) introduced to economics and have become standard fare. Financially, a martingale assumption amounts to a no-arbitrage condition, but the mathematics clarifies that short-term viability does not guarantee long-term viability. For a strictly local martingale, the viable time horizons can get so short that failure is assured in finite time.

Local martingales have gained traction in finance precisely because they are the neatest way to account for bubbles (Hulley, 2010). However, they are seldom used in macroeconomics, with Phillips (1996), Evans (2011), and Hansen and Sargent (2021) among the exceptions. We hope our tractable implementation will encourage a broader use in macroeconomics.

Second, we show that markets can maintain the rollover of unsustainable debt for extended periods at considerably lower credit spreads than what appears reasonable in hindsight. Indeed, rollover is often so easy that it makes an unsustainable debt look risk-free. However, this cannot be continued indefinitely. As a consequence, policymakers adopting a genuinely long-term perspective should exercise greater prudence than the market typically requires.

Third, we provide evidence that US sovereign debt deserves a downgrade to a single-A or lower credit rating. While the US debt-to-GDP ratio surged to similar levels during World War II, victory removed its main driver. The ratio shrank for decades thereafter, thanks to modest fiscal surpluses and interest rates three or four percentage points below growth rates. Current accumulation reflects higher fiscal deficits and lower discounts on interest rates relative to growth. Current prospects look poor for stabilizing debt-to-GDP ratios, much less shrinking them. While our framework does not predict imminent default, it warns against high confidence that the risks will remain low on a 20-year horizon, as is implicit in the highest credit ratings.

Most models of fiscal sustainability either ignore default risk on fiat sovereign debt or treat it as independent of the debt stock. Section 2 disputes this convention. Almost surely, default risks tend to grow with the debt-to-GDP ratio b. We model this relation as log-linear for analytical simplicity.

In the simplest case, treated in Section 3, the economy's growth rate g, the risk-free rate r, and the primary surplus q all equal zero. Rollover strengthens the government's reputation for servicing debt without depleting real sovereign resources. In effect, each lender gets reassured by others' willingness to lend. Yet, escalating debt gradually boosts the credit spread c, slowly at first but eventually accelerating so fast that default becomes inevitable within a finite time. Section 4 illustrates these dynamics through numerical simulations. Using plausible estimates of core parameters, the market could remain stable for many decades and still experience later a dramatic surge in debt burden and default risk.

Section 5 expands the model to include non-zero values for r, g, q, and post-default salvage value. It is now widely appreciated that r tends to average less than long-term g and that public policy can widen the differential (Reis, 2022). This tends to shrink the relative burden of old debt. However, in line with critiques from Cochrane (2021) and Brumm et al. (2022), our model demands a more comprehensive constraint for long-term sustainability, for which $r < g$ is neither necessary nor sufficient. Our simulations demonstrate that the new burdens from fiscal leniency can easily outpace the reduction in old burdens.

Section 6 illustrates the challenges analysts face by tracing the evolution of US federal debt-to-GDP ratios between two historical peaks. Clearly, the same debt-to-GDP ratios have been associated with very different trends in $r - g - q$. World War II's peak can be considered an emergency blip financed mainly through de facto taxes on savers and whose default risks warranted little concern once the US won. The current peak looks far less sustainable.

The evident switch in US policy exposes the core weakness of any debt-management model that assumes permanently fixed parameters. Section 7 addresses this by incorporating a one-time future Markov switch from imprudence to prudence. This allows bond issuance of any maturity. When the switching rate is low, forward T-bill rates peak at the maximum viable maturity absent switching. However, bond interest rates do not show nearly as sharp a rise and can potentially obscure a looming crisis. As Section 8 shows, uncertainty about switching rates can make the shift from calm to crisis even more abrupt.

Our benchmark simulations suggest that the US needs to tighten fiscal policy substantially within a couple of decades to prevent a debt crisis. In 9, we explore various doubts about our benchmark parameters. Our main conclusion is that the needed tightening is quite feasible in principle but unlikely to command much political backing as long as real interest rates stay low.

Our findings reinforce Reinhart and Rogoff (2009, p. xxv)'s warning that "excessive debt accumulation [...] often poses greater system risks than it seems during a boom." In fact, our models intensify this warning, as the risks emerge from rational assessments rather than the misguided belief that *this time is different*. These outcomes urge increased vigilance in fiscal policy monitoring without relying on the market to express significant distress: merely managing expectations about short-term interest rates proves insufficient. Finally, Section 10 points to avenues for future research.

2. Does the stock of sovereign debt matter?

The core question we investigate in this paper is how the risks associated with accumulated debt affect its subsequent evolution. While there is a vast academic literature on public debt sustainability – see D'Erasmo et al. (2016) for a comprehensive survey – only a small subset probes this particular topic. More commonly, such risks are denied or ignored.

Consider, for example, the copious debates over Ricardian equivalence initiated by Barro (1974). The question is whether rational agents save enough to fully pay for the future tax increases associated with increased public debt. While Barro's affirmation has been challenged on various grounds (Seater, 1993; Stanley, 1998; Ricciuti, 2003), few address the incremental risks of default.

Modern Monetary Theory (MMT) adherents emphasize that governments issuing debt in fiat currency bear no insolvency risk since they can always print enough currency to repay (Mitchell et al., 2019; Mitchell, 2020). Yet governments have motivations to default beyond mere insolvency. For instance, default serves as a surprise tax on debt holders that is remarkably easy to collect, as the government can simply withhold the pledged repayments. This can help check inflation or release funds for other causes. When the perception of lenders as overly privileged prevails, public opinion might favor default over repayment.

Even when sovereign debt is not treated as inherently costless, it is tempting to downplay the significance of stocks and concentrate solely on flows. As Diamond (1965) states, "a fixed absolute amount of debt, in a growing economy, would asymptotically have no effect." Likewise, Blanchard (2019) asserts that "public debt may have no fiscal cost." Such claims rest on four core truths: (1) most sovereign debt is effectively repaid by rollover into new debt, deferring pressure on real resources; (2) the real growth rate g of GDP often exceeds the real interest rate r on sovereign debt, in which case the relative burden of old debt shrinks at rate $g - r$; (3) if the government runs a primary (non-interest) surplus at a fraction q of the debt stock, the aggregate debt-to-GDP ratio b will shrink provided $q > r - g$; (4) governments can temporarily expand b greatly without thwarting long-term fiscal balance.

Governments can easily stretch these truths to defend any fiscal easing. They can defer hard budget choices by rolling debt over. They can portray any jump in b as a one-off event. When b looms high, they can project future g as outstripping r or assure critics that q will soon rise. From this perspective, the sole threat to debt sustainability arises from uncertainty about long-term fiscal balance. This uncertainty manifests as a risk premium c on new debt. A low current c implies the debt path remains sustainable, whereas a high c signals the government should manage expectations more effectively, possibly by urging its central bank to purchase debt. In either case, debt rollover ostensibly eliminates concerns about sustainability.

Yet, even in a mostly carefree world, default offers benefits. Funds not paid to debt holders can go to others who might need them more or argue that they do. These benefits typically scale linearly with the defaulted amounts. In contrast, the costs of default – which include contract disruption, limitations on new borrowing, and higher risk premia – tend to scale more slowly than the amounts. Hence, basic economic logic points to default risks rising with the debt stock. This has long been recognized for external debt (D'Erasmo et al., 2016), but it applies to domestic debt too (Reinhart and Rogoff, 2011).

While it is impossible to fully commit not to default, several models suggest that governments might not want to. In Conesa and Kehoe (2017), a government willingly risks a debt crisis during a recession, which forces lenders to choose between lending more and losing what they previously lent. Both lenders and the government anticipate economic recovery, so spending cuts are avoided, and debt increases. However, this can lead to default later on an even greater scale. Bohn (2011) and Casalin et al. (2020) note that real-life taxation might not be flexible enough to deliver the huge increases that sustainability might require, while Bouton et al. (2020) and Cerniglia et al. (2021) emphasize the spending rigidities imposed by public pensions and other entitlements.

If markets price the debt significantly above its fundamental value, issue-now-default-later strategies can also benefit issuers. In practice, this most commonly occurs through deceit: the issue pretends to have more resources or commitment to tap them than it actually does. The most interesting theoretical question concerns whether and when rational agents might knowingly buy debt at bubble prices. Samuelson (1958) and Tirole (1985) shed light on bubble dynamics in infinite-horizon settings where beliefs in sustained overvaluation are self-fulfilling. Harrison and Kreps (1978), Allen et al. (1993), and Scheinkman and Xiong (2003) focused on differentiated beliefs, where savvy investors expect to sell later to someone less well-informed and more optimistic. This is colloquially known as the Greater Fool theory (Whitcomb, 2020), although there may be no clear pecking order of smarts when information is dispersed. In Abreu and Brunnermeier (2003), dispersed information fosters a bubble that lets behavioral and rational agents coexist.

While most models of financial bubbles apply to any security, several focus specifically on government debt. Hellwig and Lorenzoni (2009) demonstrated that public debt bubbles are sustainable through rollover when the only consequence of default is losing the ability to borrow in future periods. Domeij and Ellingsen (2018) used a US-calibrated model to argue that public debt is a welfare-improving Ponzi scheme; in their view, "paying off public debt benefits only a small group of wealthy individuals." Chen and Wu (2018) found that the debt-to-GDP ratios of the G-7 countries plus Denmark and Finland are growing unsustainably fast, which suggests a bubble component. Kocherlakota (2023) demonstrates that public debt bubbles can arise in a wide class of incomplete insurance macroeconomic models.

3. Basic model

Imagine a market economy maintaining perfect stability in real output and prices. The government runs a perfect primary fiscal balance but previously accumulated some debt. The government does not want to default, as that would disrupt the payment system and eliminate credit lines that might be needed later. Yet it does not want to run the fiscal surplus needed to extinguish the debt, as transferring real resources to creditors could reduce perceived domestic welfare. Instead, the government issues overnight T-bills at the market-determined rate and aims to keep rolling over the debt indefinitely without tapping own resources.

The government's creditors are risk-neutral and indifferent to delay: they hold T-bills as long as the expected overnight returns remain non-negative. If creditors were certain of repayment, T-bills could sell at par. However, the transactions are tinged with fear. The government worries that creditors might use T-bill repayments to purchase goods and services, sparking inflation and reducing other consumers' welfare. Creditors worry that the government will default. These fears reinforce each other. The more creditors worry about default, the more tempted they become to switch to goods. The more the government fears a creditor switch, the more likely it considers preemptive default.

Let us assume a fearful equilibrium exists where lenders always correctly perceive the current hazard rate $h(t)$, the probability density of default at time t given no prior default. While all lenders think alike, that is not common knowledge in the sense of Geanakoplos (1992). Instead, they imagine significant variance in market beliefs with their own view just happening to match the current market price.

The greater the T-bill stock $B(t)$, the greater the risk of default since more real resources might be drained upon redemption. The simplest tractable formulation makes $h(t)$ proportional to a positive power α of B. Letting κ denote the hazard rate when $B = 1$,

$$h(t) = \kappa B(t)^{\alpha} \tag{1}$$

The initial time will be numbered 0 to reduce clutter, and all initial values will be subscripted with 0. In particular, $h_0 = \kappa B_0^{\alpha}$. We will see that h_0 and α drive all subsequent calculations. Raising B_0 by one percent is equivalent to raising κ by α percent.

Perceived risk forces the T-bills to offer a credit spread $c(t) > 0$ over the presumed risk-free interest rate of zero. Over the next instant dt, the accrued interest is approximately $c(t)B(t)dt$, while the default risk is approximately $h(t)dt$. When T-bills are worthless after default, the expected rate of return approaches $c(t) - h(t)$. Risk-neutral creditors insist that $c \geq h$, while the government wants c no higher than needed to roll over the debt. Equilibrium requires

$$c(t) = h(t), \tag{2}$$

which is a stripped-down version of standard credit pricing (Duffie and Singleton, 2003, Chapter 5).

How does $B(t)$ behave until a default occurs? Assuming all maturing T-bills are rolled over into new T-bills,

$$\frac{dB(t)}{dt} = c(t)B(t) \quad \text{or} \quad B' = cB, \tag{3}$$

where the second expression uses the prime symbol $'$ to denote the derivative and drops the time arguments. Combine (1)–(3) to obtain $B' = c_0 B^{\alpha+1}$, whose only feasible solution is

$$B(t) = B_0(1 - c_0\alpha t)^{-1/\alpha}. \tag{4}$$

For large values of t, the implied $B(t)$ is imaginary or negative. On inspection, B blows up to infinity in a finite time Ω given by

$$\Omega = \frac{1}{c_0\alpha} = \frac{1}{\kappa\alpha B_0^{\alpha}}. \tag{5}$$

We will call Ω the Debt Apocalypse because no debt gets rolled over past Ω without default. We can confirm this by examining the hazard rate. Since $c = (\log B)'$,

$$h(t) = c(t) = \frac{c_0}{1 - c_0\alpha t} = \frac{1}{\alpha(\Omega - t)}. \tag{6}$$

Result 1. *As $t \to \Omega$, the hazard rate h and credit spread c outstrip any finite bound, ensuring default before Ω.*

Technically, the finality of Ω hinges on the presumed continuity of T-bill issuance and redemption. Otherwise, (3) will not exactly hold. Given any positive floor on duration–inevitable in practice–some high but finite interest rate will generate a zero expected return and some minuscule chance that debt is successfully rolled over past Ω. Since the distinction makes little substantive difference, we will stick with the crisper continuous formulation. We will also relax our assumption of zero-duration T-bills to allow debt of any duration less than Ω provided that interest is paid continuously at the rate specified by (1) and (2) and immediately reinvested in new debt.

How long can debt be rolled over before hazard rates soar? At $t = \frac{1}{2}\Omega$, halfway to the Debt Apocalypse, h only doubles, and another $\frac{1}{4}\Omega$ passes before it doubles again. Many defaults occur well before interest rates get sky-high. The survival probability $S(t)$ satisfies

$$S' = -hS, \tag{7}$$

which together with (2) and (3) imply $(\log S)' = -(\log B)'$. Hence

$$S(t) = \frac{B_0}{B(t)} = (1 - c_0\alpha t)^{1/\alpha} = \left(1 - \frac{t}{\Omega}\right)^{1/\alpha}. \tag{8}$$

As shown in Appendix A.1, the mean time until the default is $\theta = \alpha\Omega/(\alpha + 1)$. It follows that $c(\theta) = (\alpha + 1)c_0 = 1/\theta$ and $S(\theta) = (\alpha + 1)^{-1/\alpha}$. As we will explain shortly, α is unlikely to be as low as 1 or as high as 4, in which case $2c_0 < c(\theta) < 4c_0$ and $0.5 < S(\theta) < 0.67$. Nevertheless, as Appendix A.2 shows, extreme events tend to dominate.

Result 2. *The expected T-bill stock on default is infinite.*

If we were observing a long sequence of games that roll over debt until it defaults, the average B on default would rise over time. Optimists would praise the apparent progress toward perpetual rollovers of debt without default, while pessimists would lament the naïveté of lenders. All would note the bifurcation between long periods of high credibility with low, slowly accumulating debt and short periods where debt balloons and interest rates soar.

Result 3. *The debt market describes a bubble: a price premium over the fair value that inflates until it bursts.*

In mathematical terms, B is a strictly local martingale. Unlike a martingale, which is a fair game with an expected return of zero over any horizon, a strictly local martingale is fair from one moment to the next. Still, it has a long-run expected return that is non-zero. Over the past two decades, researchers have not only revealed close connections between local martingales and financial bubbles (Loewenstein and Willard, 2000; Cox and Hobson, 2005; Protter, 2013) but also found evidence of bubbles in option markets value (Obayashi et al., 2017; Jarrow and Kwok, 2021; Choi and Jarrow, 2022). Yet, as Jarrow and Kwok (2022) acknowledge, mathematical proof "provides limited economic intuition as to why price bubbles exist."

For illumination, let us consider a simpler example of a strictly local martingale. Bet repeatedly on a fair coin, with the Nth toss occurring at time $1-1/N$. Whenever the coin lands heads, the gambler's aggregate stake doubles. Whenever the coin lands tails, the stake is wiped out. As there is zero chance that tails never occur, the gambler is bound by time 1 to lose whatever he bet. Clearly, no reasonable, fully informed gambler would willingly enter this game. Improving the interim prospects, say by tripling the payoff for heads or waiving the penalty for the first tails, will not help either, as the gambler still winds up with nothing. The problem is that the volatility grows without bound as wealth increases so that risk ultimately overwhelms the expected reward. The gambler wants to withdraw some winnings and scale down future bets, but the rules forbid it.

No such prohibitions apply to the debt market modeled here. Lenders roll over their loans only because they are indifferent to stopping. None envision lending past Ω, as they know they will lose, and no contract compels them to. Nothing prevents a lender from committing to stop lending at time $\Omega-\varepsilon$ if no default occurs prior, and any positive ε would convert the strictly local martingale to a (full) martingale. Conversion would not impact the debt market as a whole, provided some other lender picks up the slack or the government runs a primary surplus to pay off the retiring lender. As observers, we know that all lenders think the same and that the government has no intention of tapping its own wealth. But why should we expect every lender to know that? That is not specified in the model or required in real-life markets. A core attraction of short-term debt is that it frees the investor from thinking much about the longer-term consequences for the issuer or the mindsets of other investors.

The fundamental value of an investment strategy is always elusive. This is particularly true with potential local martingales since markets rarely get close to testing their limiting behavior. Each lender might infer from others' lending that default is not imminent or that the borrower has quietly committed to servicing. Furthermore, financial markets often manifest *rational myopia*, where traders focus disproportionately on recent news in hopes of identifying hidden trend changes. When debt has long been reliably serviced, lenders tend to understate default risks looking forward (Osband, 2020, Chapter 16).

Another possibility is that real-life bubbles arise as accidental misreadings and gradually gain self-fulfilling momentum since they make debt servicing cheaper and lower the baseline hazard rate h_0. Critics have little incentive to challenge the consensus until debt swells and mispricing becomes major. The switch from complacency to pessimism then mimics the dynamics of the local martingale but with less continuous recognition of the risks.

In short, a debt bubble always involves bounded rationality, where lenders focus too much on short-term payoffs and miss broader dynamics. Strictly local martingales offer the neatest way to model this. Yet local martingales cannot be fully realistic since they anticipate potential debt explosions with extremely quick turnovers and huge spreads. While we acknowledge these limitations, we prefer a local martingale approximation to assuming that markets instantly see through any would-be debt bubble and trigger immediate default. If the latter were true, low current credit spreads would testify to high confidence that debt bubbles will never again occur. Given the historical evidence and current trends, this implication of full rationality does not seem fully rational.

Our model's biggest deviations from long-term rationality involve the government more than the lenders. The government started with a debt B_0 that it did not want to repay out of its own funds but was averse to defaulting on for fear of causing disruption. The higher interest rates get, the more critics will complain about unfairness, whether the government services or defaults. If lenders enter and exit at different times, some will profit while others lose disproportionately. By the time Ω, lenders have lost in aggregate no more than the B_0, but the government will have caused far more disruption and lost far more credibility than if it had defaulted upfront or revamped its fiscal policy.

4. Simulations of the basic model

Let us reflect on plausible values for α, the elasticity of default risk to debt. From the sovereign's perspective, the main motive for default is to stem redemptions of debt for real goods and services and their resulting infringements on domestic consumption. Applying standard consumer welfare arguments, the marginal disutility rises more than proportionally to the infringements. Hence, incentives for default scale faster than B, which suggests $\alpha > 1$. The upper bound $\alpha < 4$ cited above reflects skepticism that doubling B would increase default risk sixteen-fold.

We can narrow the plausible range further by considering the credit grades that rating agencies use to rank default risks. The eight main letter grades in ascending order of risk can be summarized as triple-A, double-A, single-A, triple-B, double-B, single-B,

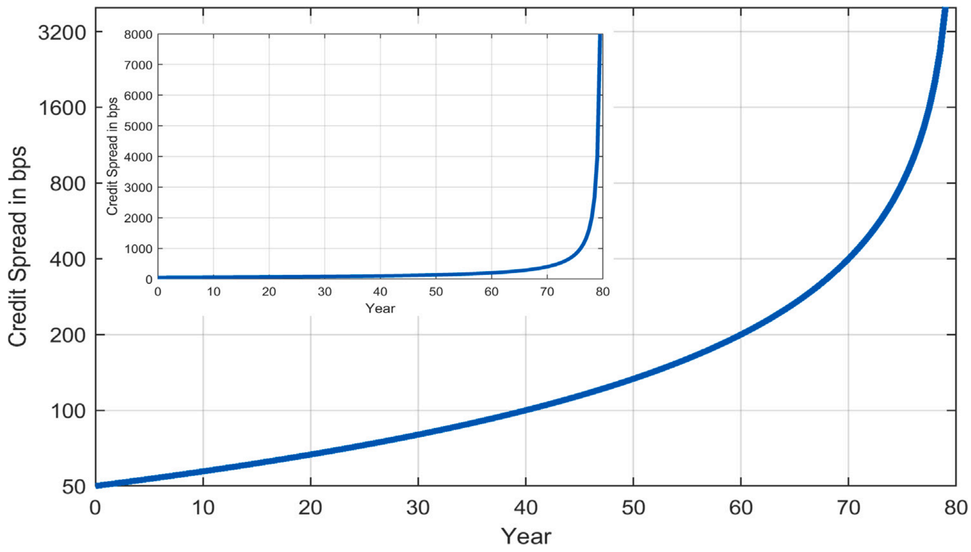

Fig. 1. Credit spreads with $\alpha = 2.5$ and $c_0 = 50$ bps on linear and log scales.

triple-C, and double/single-C. Ratings of triple-B and higher are known as investment grade, while lower ratings are known variously as sub-investment grade, high-yield, or junk.

Although sovereign ratings do not explicitly quantify default risks, they are often likened to their corporate analogues, where data is abundant. Corporate default history suggests that default hazard rates are less than 0.01% or one basis point (bp) for triple-A, around 50 bps at the triple-B/double-B boundary, and over 1000 bps for most triple-C. Each drop in letter grade is associated with roughly quadruple the average default risk; (Osband, 2020) estimates a multiplier of 4.2. By that metric, $\alpha \approx 2$ if doubling debt knocks one letter off the credit grade, while $\alpha \approx 4$ if doubling debt knocks two letters off the credit grade.

Default risks presumably depend far more on the debt-to-GDP ratio b than on the absolute stock B. Reinhart and Rogoff (2009) found that $b > 0.7$ was generally a red flag for default risks. Badia et al. (2022) estimated the same threshold of 0.7 for advanced countries and a lower threshold of about 0.3 for emerging market economies. If we treat $b = 1$ as the average single-B risk and $b = 0.1$ as somewhere between single-A and triple-A risk, then $2 < \alpha < 3$. We will use $\alpha = 2.5$ as a benchmark.

A study by Gabriele et al. (2017) of the credit spreads of eurozone members offers additional support for setting $\alpha > 2$. It found that 10-year spreads rise somewhat faster than linearly with the product of gross financing needs (GFN) and b. When $b \cdot GFN$ was included in a regression, the coefficients were insignificant on b alone and negative on GFN alone. Further investigation found that GFN matters little until b exceeds 60% and that high GFN was highly correlated with high increases in already high b. Hence, GFN appears to rise faster than linearly with b, in which case credit spreads rise faster than quadratically with b. Arguably α itself rises as Ω draws near, as that compels shorter duration issuance. When c_0 is low, the assumed infinitesimal duration is only a mathematical convenience with negligible economic impact.

Granted, domestic GDP is an imperfect metric of sovereign debt capacity. When a country's fiat currency serves as an international reserve, the relevant GDP should include some foreign GDP too. Alternatively, we might deflate debt by potential tax revenue or primary surplus since GDP likely includes many elements the government cannot touch. However, it is hard to estimate potential revenue, which depends on uncertain capacity and contested political will. Also, Ostry et al. (2010) and Ghosh et al. (2013) found evidence of fiscal fatigue, where the primary surplus stabilizes or declines when b gets high.

Our model finesses the complications. Given c_0, the only debt metric it cares about is B/B_0, and since that is endogenous, we can track $c(t)$ without it. Any (κ, B_0) combination that generates the same $c_0 = \kappa B_0^{\alpha}$ will induce the same behavior in c. Fig. 1 charts the evolution of c starting from $c_0 = 50$ bps. Here $\Omega = 80$ years. The inset plots spread on a linear scale, making the spreads for the first 70 years look modest. However, the subsequent surge is so steep, reaching 8000 bps at year 79.5, that it dominates the chart.

A market that went 75+ years without default would look blasé about rollover most of the time only to panic toward the end. For easier identification, the main chart plots spread on a logarithmic scale. It is easy to see that $c = h$ keeps doubling over a duration that progressively halves: 100 bps at year 40, 200 bps at year 60, 400 bps at year 70, 800 bps at year 75, and so on.

For any α and c_0, we can construct a c curve similar to Fig. 1. A higher c_0 does not alter that curve; it just shifts the effective starting point to some later time. The core behavior does not change. The credit spread always doubles as the time to Ω halves.

Fig. 2 plots survival probability S using the same parameters as Fig. 1. There is a 75% chance of rolling over without default for 40 years, a 50% chance of rolling over for 66 years, and a 33% chance of rolling over for 75 years. If observers of credit spreads and default rates in the first 60 years were unaware of the true drivers, they would likely not suspect that the debt is doomed to default. In repeated games, most defaults occur before debt and credit spreads soar.

Since the bond stock B before default is inverse to S, it doubles in 66 years, triples in 75 years, and quadruples in 77.5 years. From a lender's perspective, this is minimally fair compensation for default risk. However, the public will naturally begrudge lenders' rapid

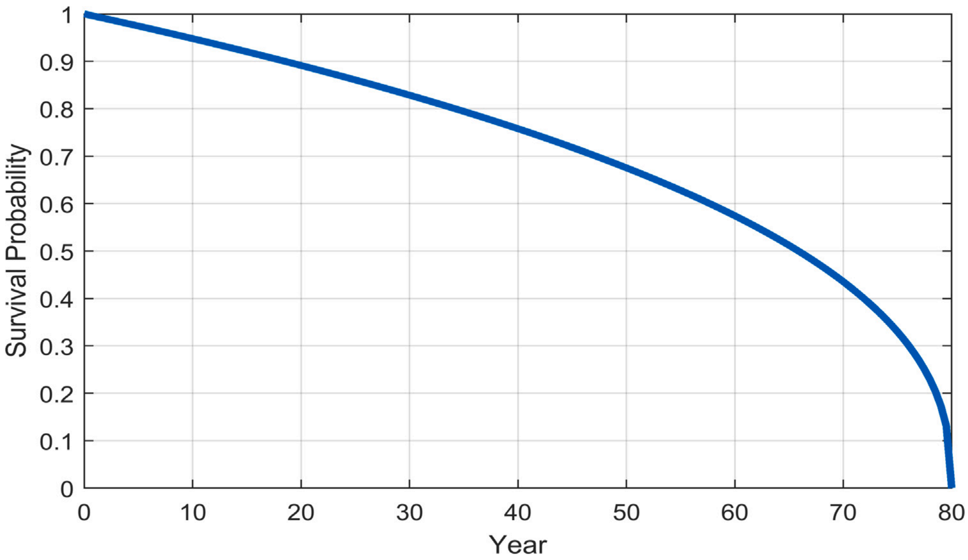

Fig. 2. Survival probabilities implied by Fig. 1.

gains as spreads increase and view them as proof that interest rates are exorbitant. Such sentiment encourages default, reinforcing the risk that credit spreads are expected to cover.

5. General debt traps

The preceding simulations against debt-management complacency are exceedingly tame. If the government could raise enough extra net revenue to retire a small share c_0 of maturing debt, B would stabilize; raise any more and debt would eventually vanish. Nor is change urgent since spreads will stay tiny for decades. To rouse alarm, there must be other factors swelling the debt burdens. To model these factors and potential offsets, this section generalizes the basic model in five ways:

1. Default risk depends not on nominal debt B by itself but on the ratio b of B to GDP Y. This converts (1) into
$$h = \kappa b^{\alpha} \tag{9}$$
2. GDP grows at constant relative rate g:
$$Y' = gY \tag{10}$$
3. Creditors demand a risk-free return r, which bumps up the interest rate i to r plus the credit spread c:
$$i = r + c. \tag{11}$$
4. If default occurs, creditors lose a fraction η of the T-bills' face value. In an instant dt, the expected rate of return is approximately $(1 - h\,dt)(r + c) - \eta h$. As $dt \to 0$ this should equal r, which modifies (2) to
$$c = \eta h. \tag{12}$$
5. The government runs a primary budget surplus that equals a constant fraction q of the bond stock $B(t)$. While not the standard way to depict budget trends, it is the simplest to deal with mathematically, as it lets us replace (3) with
$$B' = iB - qB \tag{13}$$

Since $(\log b)' = (\log B)' - (\log Y)'$, substitution of (10), (11), and (13) indicates that

$$(\log b)' = c + \delta \quad \text{for} \quad \delta \equiv r - g - q. \tag{14}$$

Hence b grows or shrinks at a constant relative rate δ independently of default risk. We call δ the risk-free adjustment. Incorporating $(\log c)' = \alpha(\log b)'$ from (9) and (12) transforms (14) into

$$c' = \alpha c(c + \delta). \tag{15}$$

Result 4. *A risk-free rate r below GDP growth g does not assure zero fiscal cost for outstanding debt b.*

This qualifies a claim of Blanchard (2019). The pressure of b on interest rate i can be ignored only if c is minuscule, α is minuscule, or $q > r - g$. With $\alpha > 1$, as we have argued, the initial value of c strongly shapes its trajectory.

Result 5. *Pending default, b and c accelerate, stabilize, or shrink according to the sign of $c_0 + \delta$.*

Apart from the terminology, the core indicator $c_0 + \delta$ is conventional. One component is $i_0 - g$, which indicates the rate of change in b stemming from previously accumulated debt. The other component is $-q$, which indicates the primary fiscal deficit measured as a share of b. Other parameters matter solely through their influence on $c_0 = \eta\kappa b_0^\alpha$.

Appendix A.3 solves the model. For all feasible paths,

$$c(t) = \frac{c_0 \delta e^{\alpha\delta t}}{c_0 + \delta - c_0 e^{\alpha\delta t}}, \tag{16}$$

$$b(t) = b_0 e^{\delta t} \left(\frac{\delta}{\delta + c_0 - c_0 e^{\alpha\delta t}} \right)^{1/\alpha}, \tag{17}$$

$$S(t) = \left(1 + c_0 \frac{1 - e^{\alpha\delta t}}{\delta} \right)^{1/(\alpha\eta)}. \tag{18}$$

For $c_0 + \delta > 0$, (16)–(18) apply only until a Debt Apocalypse at time

$$\Omega = \frac{\log(1 + \delta/c_0)}{\alpha\delta}, \tag{19}$$

where b and h grow unbounded and no debt fails to default. For $\eta < 1$, default salvages some value for debt holders and a new cycle starts. For $c_0 + \delta < 0$, b and h approach zero while the survival rate S approaches $(1 + c_0/\delta)^{1/(\alpha\eta)}$.

Result 6. *For b to be sustainable long-term, the relative primary surplus q must exceed $i_0 - g$.*

If c_0 is small, there is little difference between i_0 and r. However, the distinction does matter when the government contemplates a large jump in debt as an emergency measure, say, to fight a war or counter a sharp contraction in GDP. For our benchmark $\alpha = 2.5$, doubling b multiplies h by 5.7. Generally, a M-fold boost in debt is sustainable only if $\delta < -cM^\alpha$.

Result 7. *A discrete boost in debt can make the stock unsustainable.*

This result comports with the literature on "fiscal space" that derives a cap on sustainable debt from limits to feasible q (Blanchard, 1984; Bohn, 1998; Ghosh et al., 2013; Casalin et al., 2020). Abrupt tax rate increases and significant power spending cuts encounter substantial limitations. Higher marginal tax rates may hamper economic activity or redistribute wealth in ways that limit public sector revenues (Aiyagari and McGrattan, 1998). Additionally, institutional constraints impose minimum spending requirements on governments for sectors like pensions, health, defense, education, and similar areas. Our emphasis on default risk offers further support.

When $\delta = 0$, the general model collapses into the basic model, and for tiny c_0 no big risks become apparent for 50 years or more. Accordingly, our simulations will focus on δ of at least 1% in absolute value. We will start by assuming c_0 of 50 to 100 bps, which for corporate credits would typically be viewed as high double-B, just below investment-grade status (Osband, 2020). Fig. 3 charts credit spreads for the benchmark $\alpha = 2.5$ and four different δ.

The flat line in the middle of Fig. 3 reflects $c_0 + \delta = 0$ or equivalently $q = i_0 - g$. In the other cases of $\delta < 0$, credit spreads shrink steadily but not quickly: when $\delta = -2\%$, halving a 100 bps spread to 50 bps takes 22 years, while when $\delta = -1\%$, halving a 50 bps spread takes 44 years. Recall that major credit grades vary by factors of four in average default risk. Hence, rating upgrades will likely be slow if the improvement is gauged solely by b.

Similarly, rating downgrades are likely to be slow when $c_0 = 50$ bps and $\delta = 1\%$, as it takes 40 years for fair spread to double. Doubling time shrinks to 16 years when $\delta = 2\%$. When $c_0 = 100$ bps, downgrades come due much faster: spread quadruples in 19 years for $\delta = 1\%$ and 14 years when $\delta = 2\%$. At that point, default is likely within a few years absent dramatic fiscal tightening.

Let us next consider much tighter initial spreads. For corporate borrowers, $c_0 = 1$ bp would typically be identified as triple-A grade and $c_0 = 10$ bps as single-A grade. As noted in Osband (2020, Chapter 14), it is hard to identify such risks without tens of thousands of years of data on similar borrowers. Sovereign default risks are even more challenging to estimate as there are far fewer sovereigns than corporates, and comparisons are suspect. However, we will ignore the complications and assume that the estimates are generally correct.

Over short periods t, (15) implies $\log(c(t)/c_0) \approx (\delta + c_0)\alpha t$. For a one-letter drop in credit grade, the log risk increases by about 1.5, so for $\alpha = 2.5$, $\delta < 150$ bps will not warrant much ratings concern on a 30-year horizon. In contrast, $\delta > 300$ bps would justify a full letter downgrade within 20 years.

Over longer periods, $\log c$ bends upward, and the convexity increases with α and $\delta + c_0$. When $\delta = 2\%$ for 30 years, the credit grade should drop by just over a letter, all else equal. While the default risk would remain investment grade, a single-A credit would sink to triple-B, with default risks on a 30-year horizon of over 10%. Similar concerns would be stirred on a 20-year horizon for $\delta = 3\%$ and on a 15-year horizon for $\delta = 4\%$. Thirty years of $\delta = 4\%$ would warrant over a two-letter drop in credit grade, driving us toward the starting points of Fig. 3. To be clear, none of these scenarios foretell a high default risk at 30 years, or even of likely default over the subsequent 15 years should b restabilize. However, they warn of too much vulnerability from the stock expansion to keep treating the debt as approximately risk-free.

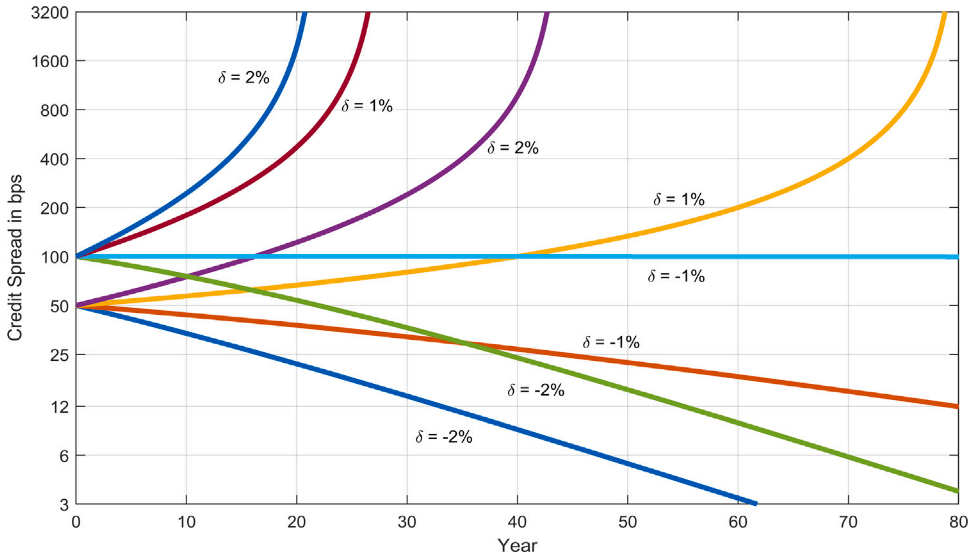

Fig. 3. Credit spreads on log scale. Parameters: $\alpha = 2.5$, $\delta \in \{\pm 1\%, \pm 2\%\}$, and $c_0 \in \{50, 100\}$ bps.

6. A tale of two debt burdens

While debt loads tend to raise default risks, they cannot be the only significant influence on sustainability; otherwise, they would be far more correlated with credit ratings than we observe. One obvious amendment excuses debt expansion during emergencies when coupled with a determination to contract debt quickly after. Here, rollover is not intended to permanently defer repayments, just to spread them out over time and smooth the associated taxes (Barro, 1979).

Distinguishing sound deferrals from unsound procrastination can be difficult. On the one hand, strong GDP growth can potentially shrink the relative debt burden despite a government's deficit. On the other hand, long spells of debt expansion are often depicted as successive emergencies, while promised contractions are repeatedly deferred on flimsy pretexts.

To illustrate the challenge, we describe two debt burdens for the US that reach similarly high peaks without significantly affecting its credit rating yet have very different sources and outlooks. Fig. 4 depicts US government debt held by the public from 1939 to 2021. Here, the public includes the Fed, which arguably could be considered a government agency despite its nominal independence. The dotted line excludes Fed holdings where available (see Appendix A.8 for data sources.)

Debt soared from 1941 to 1946 due to military expenditures in World War II but was financed cheaply thanks to financial repression, patriotic fervor, and the expectation of renewed fiscal discipline. In just the next two years, the gross debt-to-GDP ratio fell by 27 percentage points, partly because the primary fiscal balance flipped into surplus but mostly because inflation soared after wartime price controls were removed. Over the next quarter century, δ averaged -4% annually, with $r - g$ accounting for most of the margin.

In contrast, δ has averaged 3.3% per year since 1980 and 5.0% since 2000. Both $r - g$ and q have contributed to the reversal. The interest rate differential surged on financial liberalization and overshooting inflation expectations but has gradually faded. In 2022, it plunged below -8% due to a big inflation shock. The primary balance oscillated between high deficit to debt shares, high surpluses, and even higher deficits.

Fig. 5 displays 10-year moving averages of $r - g$ and q. The signed gap between them is the 10-year moving average δ. Clearly, q has been at least as important as $r - g$ in shaping δ and the path of b. This suggests that the recent emphasis on $r - g$ is misdirected. While r falling below g trims the burden of older debt, its impact is dwarfed by the plunge in q.

Here, q is computed as the difference between $r - g$ and b'/b, although imputations from budget balance plus interest paid give basically the same results. Sometimes, it is suggested that deficits financed by Fed purchases do not count, so Fig. 5 also depicts q without them. Removal reduces average δ to 3.0% since 1980 and 4.4% since 2000, while q oscillates slightly more. Average δ from 2012 through 2021 is trimmed to -2.2% from -3.9% when Fed purchases are excluded.

In short, the World War II-related debt burden was dominated by a sharp surge and sustained retreat. While the retreat was primarily driven by financial repression, it thoroughly justified a top credit rating for the US. Since 1980, the debt-to-GDP ratio has trended upward at an average of 3% annually, with some evidence of acceleration. In our simulations, the 40-year trend warrants at least a two-letter downgrade in credit rating.

Hence, it strikes us as prudent to downgrade sovereign US credit to single-A. Single-A is solidly investment grade; it does not point to significant near-term default risk over a 10-year horizon. However, it does warn of potential deterioration. In contrast, double-A or triple-A ratings are typically considered safe for one generation or longer.

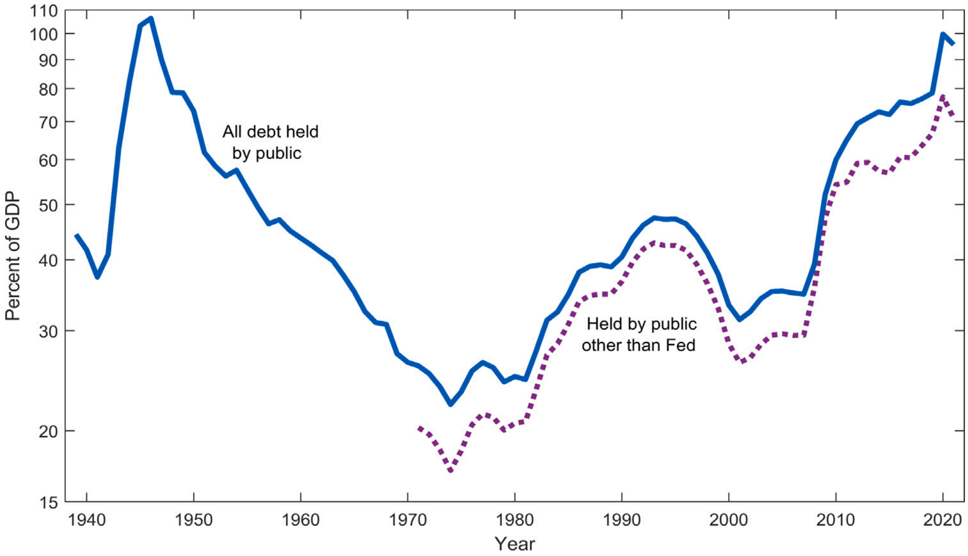

Fig. 4. Publicly held US debt/GDP on a log scale.

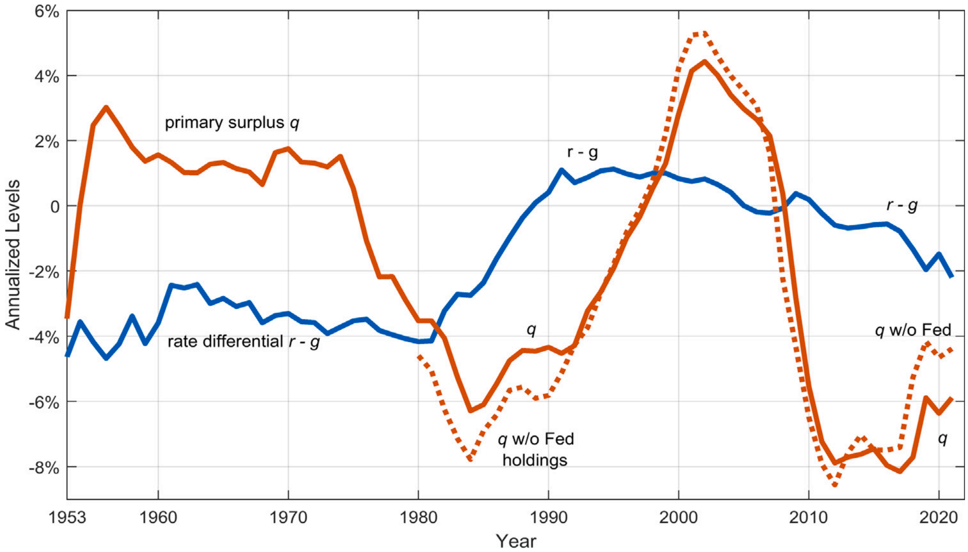

Fig. 5. 10-year moving averages of $r - g$ and q for US.

A recent market scare reinforces our concern. The US has had a formal debt ceiling for most of the past century. Although it has been raised 90 times to accommodate more debt, the negotiations prior have become increasingly hostile. Should they fail, the nominal debt would likely be deflated through accounting tricks or given priority repayment per the Constitution, but neither is certain. The latest round of negotiations was so acrimonious that the market briefly took fright. The cost of one-year "credit swap" insurance against default on US debt soared to 172 bps. For comparison, annual default rates for US corporate bonds have historically averaged less than ten bps for single-A and less than 50 bps for the lowest investment grade of BBB-/Baa3 (Osband, 2020, Chapter 14). Soon after, the Fitch rating agency downgraded US sovereign credit to AA+. While the imputed extra default risk is minuscule, the downgrade is symbolically important and warns of further reductions.

7. Expected future prudence

While our previous model challenges the sustainability of the current US debt path, US debt history challenges the appropriateness of our model. Since the US shifted to prolonged expansion of b from prolonged contraction, why presume that it cannot shift back? Accordingly, this section modifies the model to allow for policy changes. Specifically, we consider potential shifts from an imprudent

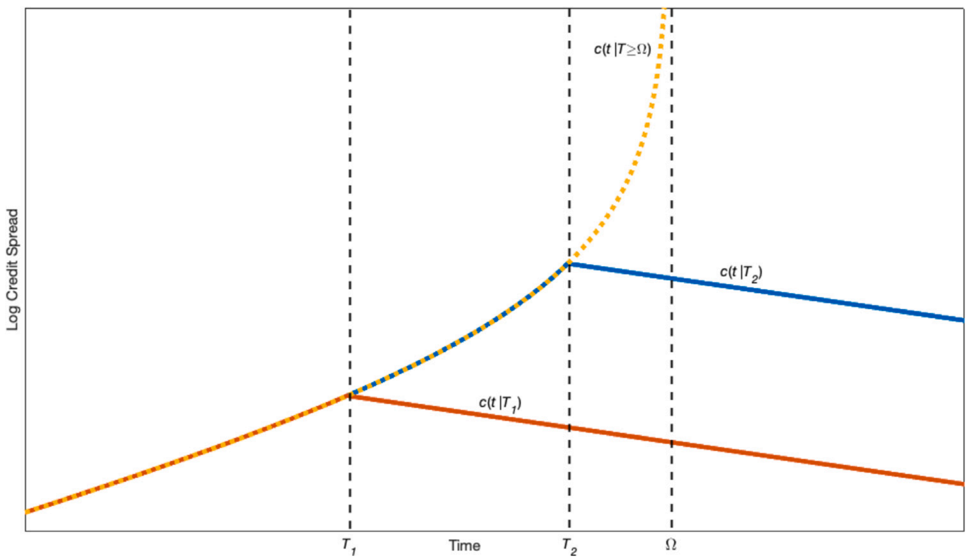

Fig. 6. Log credit spreads given fiscal reform at time T.

policy with $\delta > 0$ to a prudent policy in which $c(t) + \delta(t) = -\epsilon < 0$. Prudence makes b shrink at exponential rate ϵ, which in turn makes c shrink at exponential rate $\alpha\epsilon$. Given transition at time T and Ω defined by (19), Appendix A.4 derives the credit spread

$$c(t|T) = \begin{cases} \dfrac{\delta}{e^{\alpha\delta(\Omega - t)} - 1} & \text{for } t = \min(t, T, \Omega), \\ \dfrac{\delta e^{-\alpha\epsilon(t-T)}}{e^{\alpha\delta(\Omega - T)} - 1} & \text{for } T = \min(t, T, \Omega), \\ \text{undefined} & \text{for } \Omega = \min(t, T, \Omega). \end{cases} \tag{20}$$

The first line is equivalent to (16); it tracks pre-default c pending prudence. The second line incorporates a shift to prudence. If time Ω passes without prudence, default makes c moot.

Fig. 6 charts three paths for $\log c(t|T)$. All start on the path defined by (15), which becomes increasingly convex as c rises. When fiscal policy reforms before Ω, say at T_1 or T_2, $\log c$ declines linearly thereafter with slope $-\alpha\epsilon$. Absent reform, default is certain by Ω.

For simplicity, we will assume that the conditional switching rate to prudence is a constant λ and that prudent policy never relapses. Hence, the fiscal regime is a two-state Markov process with a terminal state. For further simplification, we assume that the defaulted debt pays off the salvage value $1 - \eta$ immediately in cash.

We must weight the various conditional spreads by their likelihoods to compute the expected spreads. Appendix A.5 derives the survival probabilities conditional on fiscal reform at time T:

$$S(t|T) = \begin{cases} \left(\dfrac{c_0 e^{\alpha\delta t}}{c(t|T)}\right)^{\dfrac{1}{\eta\alpha}} & \text{for } t = \min(t, T, \Omega) \\ \left(\dfrac{c_0 e^{\alpha\delta T}}{c(T|T)}\right)^{\dfrac{1}{\eta\alpha}} \exp\left(\dfrac{c(t|T) - c(T|T)}{\eta\alpha\epsilon}\right) & \text{for } T = \min(t, T, \Omega) \\ 0 & \text{for } \Omega = \min(t, T, \Omega). \end{cases} \tag{21}$$

Fig. 7 displays the various $S(t|T)$ for Fig. 6. As we have seen, S given imprudence starts nearly straight but eventually turns steeply concave. The other paths peel away on transition for $T < \Omega$ and turn convex. Despite the common $\eta\epsilon$ rate of decay in $\log c(t|T)$ after prudence, $S(t|T)$ steepens with T because the starting spread $c(T|T)$ gets higher.

The expected survival rate $\langle S \rangle$ is the probability-weighted average of the conditional survival rates. Since the probability of no transition before T is $e^{-\lambda T}$ and the density at T is $\lambda e^{-\lambda u}$,

$$\langle S(t) \rangle = e^{-\lambda t} S(t|t) + \int_0^t \lambda e^{-\lambda u} S(t|u) du \tag{22}$$

For $t > \Omega$, this simplifies to $\int_0^\Omega \lambda e^{-\lambda u} S(t|u) du$, since nothing survives past Ω absent reform.

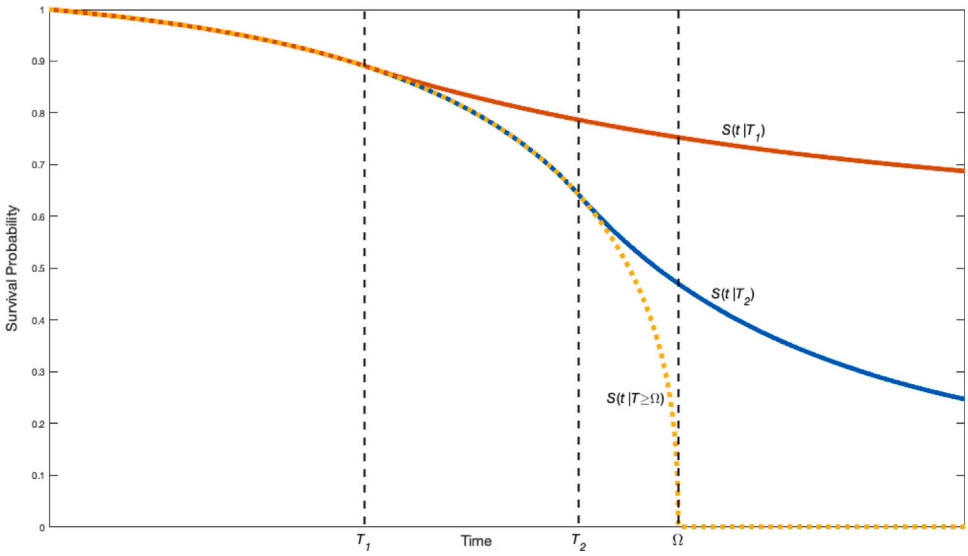

Fig. 7. Survival probabilities given fiscal reform at time T.

The expected hazard rate $\langle h \rangle$ equals the relative decay rate $-(\log \langle S \rangle)'$ of expected survival. The expected credit spread c_{bill} of future T-bills equals η times the corresponding $\langle h \rangle$. Equivalently, it equals a probability-weighted average of conditional credit spreads $c(t|T)$:

$$c_{\text{bill}}(t) = -\eta \frac{\langle S'(t) \rangle}{\langle S(t) \rangle} = \frac{e^{-\lambda t} S(t|t)c(t|t) + \lambda \int_0^t e^{-\lambda u} S(t|u)c(t|u)du}{\langle S(t) \rangle}. \tag{23}$$

The fair price of bonds must compensate for cumulative default risk. Consider a bullet bond issued at time 0 that pays only when it matures at time t. The cumulative credit spread multiplies the cumulative expected hazard rate $\int_0^t \langle h(t) \rangle\, dt = -\log \langle S(t) \rangle$ by η. Hence the mean credit spread c_{bullet} until maturity is

$$c_{\text{bullet}} = -\frac{\eta}{t} \log \langle S(t) \rangle. \tag{24}$$

Ordinary bonds aim to pay enough interest periodically to compensate for ongoing credit risk and delays. While an ideal offset would pay interest continuously at a varying rate $r + c$, most bonds pay a constant interest rate $\bar{i}$ with implied average credit spread $c_{\text{bond}} = \bar{i} - r$. For continuous interest payments, Appendix A.6 shows that

$$c_{\text{bond}} = \eta \left(\frac{1 - e^{-rt} S(t)}{\int_0^t e^{-ru} S(u)du} - r \right) \tag{25}$$

Unfortunately, (22)–(25) have no closed-form solutions. To appreciate the impact, we need to observe simulations. Here are the benchmark values we use, along with brief explanations:

- $dt = 1/250$. Each year is divided into 250 trading days.
- $\alpha = 2.5$. This is the benchmark value discussed previously.
- $\delta = 500$ bps. This roughly matches the growth rate of b in the US over the past 20 years. It implies a doubling of b every 14 years if c stays minuscule.
- $\varepsilon = 200$ bps. This implies a halving of b every 35 years. For comparison, the US shrank b an average of 500 bps annually from 1948 to 1972, but that reflected higher growth rates with more financial repression and less expansive entitlements than seem feasible now.
- $\lambda = 0.1$. This implies an expected waiting time of $1/\lambda$ of 10 years for a switch to fiscal prudence.
- $c_0 = 10$ bps. The implied initial default rate of once in 1000 years reflects the default risks associated with single-A corporate credit grades.
- $\eta = 0.75$. This implies a salvage value of 25% after adjusting for real-life time delays.
- $r = 100$ bps. This impacts c_{bond} through duration mismatch of payments and risks; c_{bill} and c_{bullet} are unaffected.

Fig. 8 plots credit spreads for our benchmark on a log scale. The dotted line on the left indicates the T-bill credit spread $c(t|T \geq \Omega)$ on T-bills absent policy change. It reaches 50 bps after 12 years, 100 bps after 17 years, and 200 bps after 21 years, with default certain before $\Omega = 31.5$ years. The dashed line indicates the expected T-bill spread c_{bill}. Its path is much lower than $c(t|T \geq \Omega)$, reflecting expected switches to prudence. It stays under 100 bps until the last year before Ω, and while it rockets during that year,

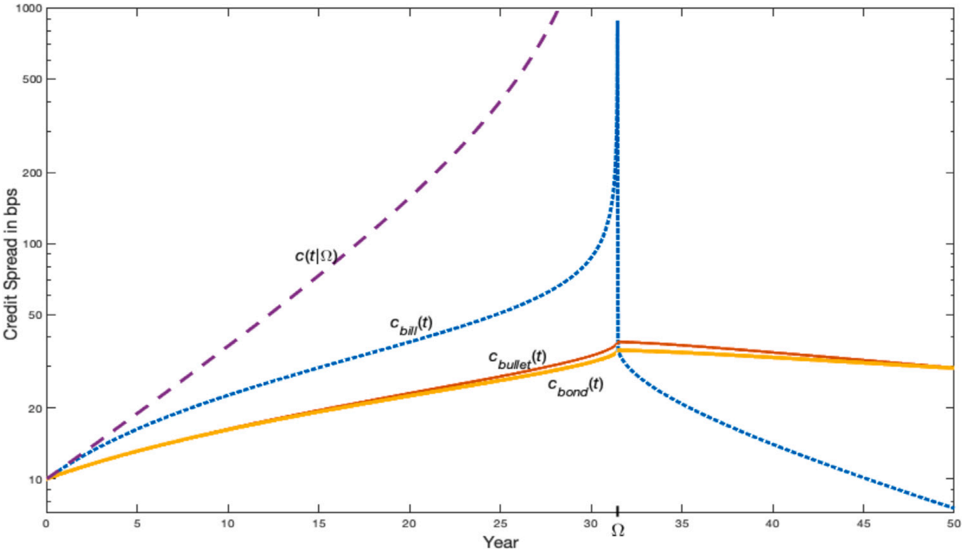

Fig. 8. Credit spreads benchmark.

it immediately plunges when Ω passes and retreats thereafter. One month after Ω, most scenarios with extremely high b have defaulted; c_{bill} drops to 34 bps. Two years Ω, c_{bill} is 24 bps; subsequent shrinkage is nearly exponential at rate $\alpha\epsilon = 0.05$.

Although the average credit spreads for bonds embed the c_{bill} path, they do not warn transparently of crisis. The rise and fall look modest, peaking at 32 years of 38 bps for bullets and 35 bps for ordinary bonds. Since 30-year bonds tend to get bid up on the dearth of longer-term supply, the observed peaks might be even lower. This might make it hard to distinguish the Treasury yield curve from the projections for risk-free rates r, which our model treats as constant. Yet the 15% default risk within 32 years significantly exceeds the 3.2% risk implied by a constant $c = 10$ bps.

If c_0 is significantly lower than ten bps or λ is significantly higher than 0.05, there is little cause for worry. For $c_0 = 1$ bp with no other changes to the benchmark, Ω is 50 years away, with maximal c_{bullet} of 5 bps for a 50-year maturity. For $\lambda = 0.2$ with no other changes to the benchmark, c_{bullet} peaks at 15 bps for a 20-year maturity.

As for other parameters, a 0.1 increase in α shortens Ω by one year and raises peak c_{bond} by 3 bps. A 100 bps increase in initial δ shortens Ω by four years and raises peak c_{bond} by 13 bps. A 0.25 increase in η raises peak c_{bond} by 4 bps. Interestingly, the magnitude of prudent ϵ has relatively little impact on peak risks. A 100 bps increase in ϵ lowers peak c_{bond} by three bps, about the same as a 100 bps increase in r. An early shift to prudence matters far more since it caps risks before they surge.

Suppose policy stays unchanged until $c = 30$ bps. This takes 8.4 years, which leaves Ω 23 years forward. The spread curves shift inward and upward. Peak spreads are $c_{\text{bullet}} = 107$ bps and $c_{\text{bond}} = 96$ bps. Bond spreads past Ω obscure the rapid decline in c_{bill}: c_{bond} at 30 years is 90 bps, just 5 bps below the peak. The best clue to the future crisis is the implied default risk of 32%.

8. Uncertain propensity for prudence

Future spreads are quite sensitive to the expected switching rate to prudence. Suppose the benchmark $\lambda = 0.1$ halves to $\lambda = 0.05$. While $\Omega = 31.5$ stays the same, the probability of remaining imprudent until Ω rises to 23% from 4%. Peak bond spreads are 100 bps for c_{bullet} and 81 bps for c_{bond}, roughly 2.5 times the peaks for the benchmark. Fig. 9 charts the spread curves.

The appropriate switching rate is impossible to measure directly. Estimation is fraught with uncertainty about the risks of relevant comparators. Uncertainty about the mean boosts future spreads due to the heightened risk of fiscal imprudence lasting longer than Ω. For an extreme example, suppose the agent posits a 50% chance that $\lambda = 0.2$ and a 50% chance that $\lambda = 0$. Although the expected switching rate is 0.1, the default risk by time Ω is not the 15% for the benchmark with $\lambda = 0.1$ but over 50%, since default before Ω is certain for $\lambda = 0$.

To parameterize uncertainty, we assume a gamma distribution. Gamma distributions rule out negative values and multiple peaks but allow any positive mean E and variance V. The shape parameter $k = E^2/V$ generates an exponential distribution when $k = 1$, approximates a Gaussian distribution for $k \gg 1$ and approaches an L-shaped distribution as $k \to 0$.

Fig. 10 displays c_{bond} when the certain $\lambda = 0.1$ ($k = \infty$) of the benchmark is relaxed to allow standard deviations of 0.05 ($k = 4$), 0.071 ($k = 2$), or 0.1 ($k = 1$) around the mean. Appendix A.7 explains the calculations. The more uncertainty around the mean, the

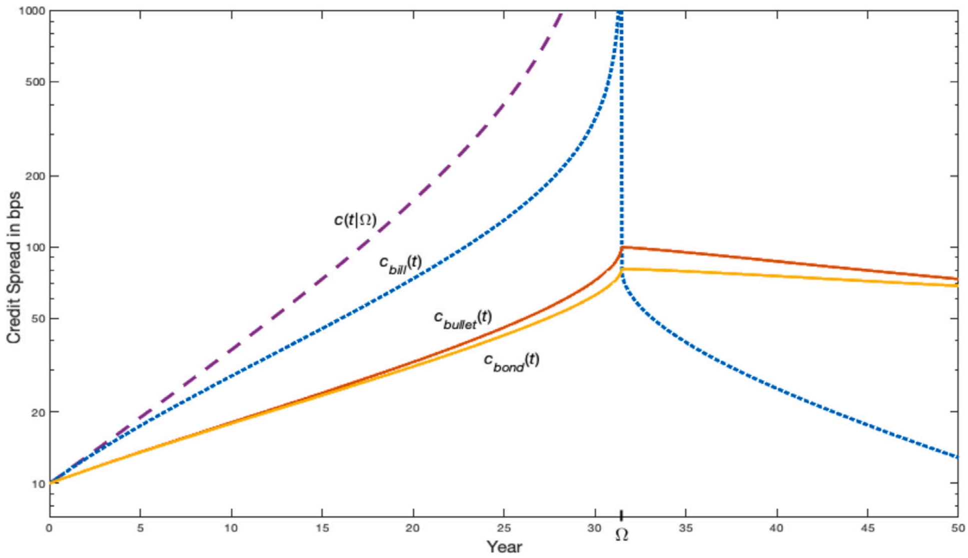

Fig. 9. Credit spreads with $\lambda = 0.05$ substituted into benchmark.

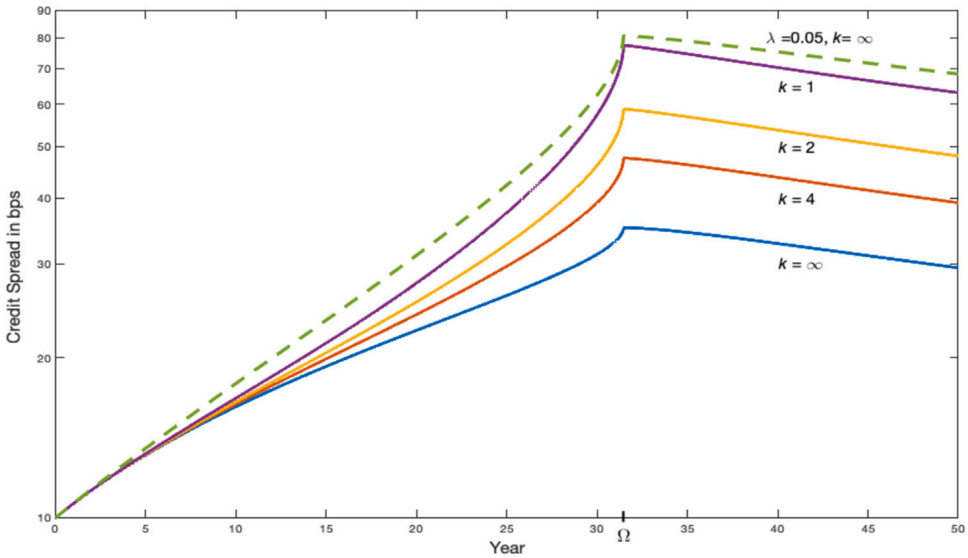

Fig. 10. c_{bond} for various dispersion of beliefs about λ.

more c_{bond} steepens as maturities approach Ω. The probability of default by time Ω is 20% for $k = 4$, 25% for $k = 2$, and 33% for $k = 1$. The corresponding c_{bond} peaks are 48 bps, 59 bps, and 77 bps respectively, compared with 35 bps for $k = \infty$.

Next, suppose that time τ passes without a shift to prudence. Uncertainty remains gamma-distributed with the same shape k, but the mean declines from the original λ_0 to

$$\lambda_\tau = \lambda_0 \frac{k}{k + \lambda_0 \tau}. \tag{26}$$

For $\lambda_0 = 0.1$ and $\tau = 10$ years without a shift to prudence, the additional expected transit time $1/\lambda_{10}$ lengthens from 10 years for $k = \infty$ to 12.5 years for $k = 4$, 16 years for $k = 2$, and 20 years for $k = 1$. This brings the peaks in Fig. 10 a decade closer and more than triples peak spreads. The probability of default by time Ω rises to 42% for $k = 4$, 50% for $k = 2$, and 61% for $k = 1$, compared to 31% for $k = \infty$. Thus, uncertainty about λ considerably increases the delay risks.

9. Feasible fiscal adjustments

Our regime-switching simulations suggest that the US has only a couple of decades left to stave off a sovereign debt crisis. This section offers a broader perspective on the risks and potential mitigation: we present it in terms of doubts about our two posited regimes.

Overstatement of c_0. The US debt has such a high reputation for both redemption in full and liquidity in secondary markets that its credit spread is typically negligible; the recent scare (Rennison, 2023) was a rare exception. If baseline default risk is a few basis points or less—that is, if the US deserves to retain its double/triple-A rating pace our recommendation—then our simulations do not predict much worry on a 30-year horizon.

Our main counter-argument is that debt markets are rational myopic. They naturally find reassurance from past servicing and project its near-term continuation. They naturally discount forecasts of future changes that cannot be immediately tested. By overweighting recent experience, rational myopia can get blindsided by radical change and then overreact. With defaults so rare relative to servicing, swings from big underestimation before crisis to big overestimation after are more the norm than the exception (Osband, 2020, Chapter 15).

Understatement of α. Simulations with overstated c_0 might still reasonably predict future deterioration if α significantly exceeds the benchmark 2.5. Many researchers estimate a large jump in default risk once b passes some threshold b^*, which suggests $\alpha \gg 1$ but does not offer much guidance on the upper bound. Others relate higher b to higher i without attempting to distinguish the components r and c. In particular, r might rise due to downward-sloping demand for perceived risk-free assets. Engen and Hubbard (2004) estimated that a 100 bps increase in Congressional Budget Office (CBO) projections of b five years forward raises the 10-year Treasury rate by 2.8 bps. That implies a 280 bps increase in i should $b = 1$ double, which is far more than our simulations suggest.

Overstatement of δ_0. The CBO (2023) forecasts average two percent annual growth in b for the next thirty years. This is less than half our baseline projection for $\delta + c$ pending a shift to prudence. However, the CBO's mandate forces it to ignore the legislative custom of formally curtailing current spending or tax cuts a few years forward, only to restore them later. For example, (CBO, 2013) projected federal debt to stay under 75 percent of GDP for a dozen more years, but that level was breached in four years. Moreover, CBO projections appear to make no provision for increases in i as b grows.

Improvements in $r - g$. Schmelzing (2020) documented an average decline in real risk-free r of about one bp a year for 700 years with acceleration to two bps a year in recent decades. With trend real r already dropping under 50 bps and trend real g likely to exceed 150 bps, the burden of US debt contracted before 2024 will tend to shrink by 150 bps per year or more. As Reis (2022) emphasizes, the liquidity and hedging conveniences associated with US bonds potentially drive their i below r.

However, the burden from previous bond issues is augmented by the burdens from newly incurred primary deficits and from the rollover of old debt at higher real rates. As the European Commission noted in a 2021 report on fiscal sustainability (Arevalo et al., 2022), the literature does not define $r - g$ consistently. The true rollover cost should include the explicit credit spread c, the boost in r, and potential reduction in g stemming from higher b. Judging from the regression line in Graph II.3.3 (page 170) of the EU report, each percentage point increase in b for EU member countries 2001–21 was associated with an average six bps higher $i - g$.

Reduction in prudent ϵ. Contraction of b at rate ϵ requires $q = \epsilon + i - g$. The estimates above suggest that $\epsilon = 1\%$ is currently feasible without requiring a positive primary surplus. Even if debt grows to twice GDP and i reaches g, $\epsilon = 1\%$ will require only a $qb = 2\%$ primary surplus as a share of GDP. The US averaged nearly this level for 25 years after World War II. The Euro area experienced comparable surpluses from 2014 through 2019 (Arevalo et al., 2022). If maintained for several generations, $\epsilon = 1\%$ could potentially reassure lenders and reduce κ in (1).

The prime role model for this is the UK after the Napoleonic Wars ended in 1815. Over the next century it gradually shrank an enormous debt burden without stoking market fear. However, its average ϵ during this period was 2.1%. Moreover, the UK government was arguably beholden to its lenders through arrangements dating back to the Glorious Revolution of 1688 (North and Weingast, 1989). With the expansion of suffrage and social safety nets, the borrowing classes wield far more influence today, with less commitment to repay in non-depreciated currency. If investor confidence in the US does slip at higher b, sterner measures would likely be needed to restore it.

Increase in prudent ϵ. Even a 100 bps primary surplus could potentially generate $\epsilon > 2\%$ bps if implemented soon. However, the fiscal challenge gets worse the longer prudence is delayed. Our simulations project b to double in 14 years absent prudence. If $i - g$ surges to 100 bps, the primary surplus would need to average over 5% of GDP for 15 years to achieve $\epsilon = 2\%$. According to the (IMF, 2023), no country has maintained that much fiscal restraint for that long.

Furthermore, as noted earlier, a higher ϵ in simulations has a minor impact on peak bond spreads. An early transition to slow contraction of debt will restrain c_{bond} far more than a late transition to fast contraction. This feature is not unique to our model. US debt is rapidly approaching the levels that the fiscal space literature previously warned would bode high default risk.

Inertia in q. The US is wealthy enough to turn primary deficit to surplus without huge privation, but political barriers impede it. No armistice slashes military spending. The public opposes cuts to major entitlements like pensions and health care. The public shows limited support for one-off wealth taxes, fearing they may lead to persistent impositions, disinvestment, and capital flight. Most fiscal debates pit lower taxes again higher benefits; in practice each side wins some of what it wants, which lets both government spending and revenue shortfalls escalate.

The most noticeable obstacle to fiscal prudence is the coupling of growing partisanship with a nearly even electoral split. This encourages each side to begrudge any concessions to the other (Yared, 2019) and focus on short-term advantage. Bond markets, sensitive to fiscal instability, typically alert authorities to unsustainable policies and offset this. Yet the Federal Reserve's post-2008 bond purchases and market interventions to reduce volatility have undermined this early warning system. Many politicians have inferred that debt and deficits are irrelevant.

Between 1980 and 2007, the correlation between $i - g$ and q was a negligible 0.1. Since 2008 the correlation is −0.7, which associates a one bp reduction in $i - g$ with a 1.5 bp increase in the primary deficit. While this has alleviated fiscal pressures, it has also discouraged fiscal restraint. The US government will likely adopt fiscal prudence only when $i - g$ grows.

In short, b is large, rising rapidly, and unlikely to contract absent signs of crisis. Yet our simulations suggest that major fiscal tightening is needed within 15 years to stave off crisis. Granted, our simulations can be faulted in various ways. In particular, the market's long-standing faith in US creditworthiness greatly facilitates debt rollover at low rates and helps keep default risks low. However, we submit that the deterioration to date warrants a letter downgrade, and that the prospects for further deterioration warrant a negative outlook.

10. Closing remarks

In standard macroeconomic models, the credit spread and the default risk of sovereign debt are presumed independent of debt stocks. While this is a reasonable approximation in some contexts, it is dangerous in others. In particular, it has encouraged habits of excusing any debt expansion portrayed as a one-time emergency measure and denying that any debt reduction plan is urgent. Our model of default risk as a power function of debt stock offers a simple remedy.

Our model acknowledges that most sovereign debt can be rolled over into new debt with no significant pressure to repay in goods and services or boost credit spreads. Still, it warns that this cannot be repeated indefinitely without triggering a crisis. The debt-to-GDP ratio must ultimately be restrained through a mix of financial repression, fiscal prudence, or default.

In standard models, rollover is a vote of confidence in long-term sustainability; anyone expecting a Ponzi-like debt bubble should exit unless they are preying on greater fools. Our model imposes no such restriction. It allows short-term credit spreads to stay low for decades even when agents are perfectly rational, realize the debt is a bubble, and anticipate its eventual collapse. Thus, the onus falls on policymakers to pursue a sustainable financing path.

As noted in the Introduction, our results strengthen Reinhart and Rogoff's warnings about the dangers of excessive debt accumulation. Reinhart and Rogoff contended that lenders chronically get lulled into excessive optimism, leaving the possibility that a better-educated lending community could forecast much better. Our model shows unsustainable debt can fester even when lenders are well-informed and fully rational.

As for our assessment of US creditworthiness, we concede that the evidence is not clear-cut and welcome counter-arguments. The downgrade we suggest would implicitly devalue trillions of dollars of debt, with huge knock-on effects as other credit ratings adjust to conform. It would be absurd to accept this without far more research and critical debate.

The only notion we wholeheartedly reject is the claim that low credit spreads guarantee sustainability. A debt burden that continually mounts as a share of GDP is like tinder piling up in a forest. Bond markets sometimes act like deer grazing in the clearings and sometimes like forest rangers scanning for fires. Rarely do bond markets react directly to piles of wood. Still, the tinder matters, as it can turn a controllable fire into an all-consuming conflagration.

CRediT authorship contribution statement

Kent Osband: Conceptualization, Methodology, Software, Formal analysis, Writing – original draft, Writing – review & editing, Visualization. **Valerio Filoso:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing, Visualization. **Salvatore Capasso:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing.

Data availability

Data will be made available on request.

Appendix

A.1. Mean time θ until default

Integrating by parts and applying (8),

$$\theta = -\int_0^{\Omega} t\, dS(t) = -tS(t)\Big|_0^{\Omega} + \int_0^{\Omega} S(t)dt = 0 - \frac{\alpha\Omega}{\alpha+1}\left(1 - \frac{t}{\Omega}\right)^{1+1/\alpha}\Big|_0^{\Omega} = \frac{\alpha\Omega}{\alpha+1}.$$

A.2. Expected bond stock at time of default

Applying (2), (6), and (7),

$$-\int_0^{\Omega} B(t)dS(t) = \int_0^{\Omega} B(t)h(t)S(t)dt = \int_0^{\Omega} B_0 h(t)dt = -\frac{B_0}{\alpha}\log(\Omega - t)\Big|_0^{\Omega} = \infty.$$

A.3. Solutions for primary surplus qB

Rewrite (15) as

$$\frac{dt}{dc} = \frac{1}{\alpha c\,(c+\delta)} = \frac{1}{\alpha\delta}\left(\frac{1}{c} - \frac{1}{c+\delta}\right)$$

and integrate to obtain

$$\alpha\delta t = \log\frac{c}{c+\delta} - \log\frac{c_0}{c_0+\delta}.$$

Exponentiation and rearrangement yields (16). Combining (9) and (12), $b = b_0(c/c_0)^{1/\alpha}$, which implies (17). Calculate (18) by exponentiating

$$\log S = -\int_0^t h(u)du = -\int_0^t \frac{(c_0/\eta)\delta e^{\alpha\delta u}du}{\delta + c_0 - c_0 e^{\alpha\delta u}} = \frac{\log(\delta + c_0 - c_0 e^{\alpha\delta t}) - \log\delta}{\eta\alpha}.$$

A.4. Conditional credit spread

For $t = \min(t, T, \Omega)$, (16) and (19) indicate that $c(t) = \delta/\left(e^{\alpha\delta(\Omega - t)} - 1\right)$. For $\Omega = \min(t, T, \Omega)$, $c(t)$ is undefined. For $T = \min(t, T, \Omega)$, $(\log c)' = -\alpha\varepsilon$, so $\log c(t) = K - \alpha\varepsilon t$ with initial condition $K = \log c(T) + \alpha\varepsilon T$. In that case, $c(t) = c(T)e^{\alpha\varepsilon(T-t)}$. This is summarized in (20).

A.5. Survival rates with switching

When $\Omega = \min(t, T, \Omega)$, $S(t) = 0$. When $t = \min(t, T, \Omega)$, (18) applies. When $T = \min(t, T, \Omega)$,

$$\log\frac{S(t|T)}{S(T|T)} = -\int_0^t \frac{c(u|T)}{\eta}du = -\frac{c(T|T)}{\eta}\int_0^t e^{-\alpha\varepsilon t}dt = c(T|T)\frac{e^{-\alpha\varepsilon t} - 1}{\eta\alpha\varepsilon} = \frac{c(t|T) - c(T|T)}{\eta\alpha\varepsilon}$$

$$\Rightarrow S(t|T) = S(T|T)\exp\left(\frac{c(t|T) - c(T|T)}{\eta\alpha\varepsilon}\right).$$

This is summarized in (21). The unconditional survival rate $S(t)$ sums the various conditional rates weighted by their likelihoods. The chance of no transition before T is $\exp(-\int_0^T \lambda dt) = e^{-\lambda T}$, while the probability density of transition at T is $\lambda e^{-\lambda T}$. Substitution yields (22).

A.6. Credit spread on ordinary bonds

For a bond paying interest continuously at a constant rate $i = r + c_{\text{bond}}$ until maturity at time t, the net present value at time 0 per unit principal is given by

$$\text{NPV} = \int_0^t (r + c_{\text{bond}})e^{-ru}S(u)du + e^{-rt}S(t) + (1-\eta)\int_0^t h(u)e^{-ru}S(u)du.$$

The three terms indicate the respective NPVs of interest, principal, and salvage value. Since $hS = -S'$, the last integral can be rewritten and integrated by parts as

$$-\int_0^t S'(u)e^{-ru}du = -S(u)e^{-ru}\Big|_0^t - \int_0^t S(u)re^{-ru}du = 1 - e^{-rt}S(t) - r\int_0^t e^{-ru}S(u)du.$$

Combining these two equations,

$$\text{NPV} = (\eta r + c_{\text{bond}})\int_0^t e^{-ru}S(u)du + \eta e^{-rt}S(t) + 1 - \eta.$$

Fair risk-neutral pricing requires NPV = 1, which implies (25).

A.7. Gamma uncertainty about λ

A gamma distribution with shape k and inverse scale β has density $f(x) \propto x^{k-1}e^{-\beta x}$ with mean k/β and variance k/β^2. Let F denote the cumulative distribution. In simulations we approximate this as a uniform distribution over $\lambda(n) \equiv F^{-1}(n/100 - 0.005)$ for $n = 1$ to 100. We compute the survival rates for every $\lambda(n)$ and then average them to obtain expected survival rates $S(t)$ for every trading day (250 per year). Substituting S into (23)–(25) generates c_{bill}, c_{bullet} and c_{bond}.

It is readily confirmed that the gamma distribution is a conjugate prior for Poisson events, where m events in time τ increment k by m and β by τ. Hence if we start with mean $\lambda_0 = k/\beta_0$ and no switch to prudence occurs in time τ, the shape remains k while the mean declines to $\lambda_\tau = k/(\beta_0 + \tau)$. Substituting $\beta_0 = k/\lambda_0$ transforms this into (26).

A.8. Data sources on US debt

All data comes from the FRED database maintained by the St. Louis Fed at `fred.stlouisfed.org`. The annual percent change of GDP was taken from A191RP1A027NBEA. The remaining data was reported as a percent of GDP. Yearly data was taken from FYPUGDA188S Gross Federal Debt Held by the Public, FYOIGDA188S Federal Outlays: Interest as Percent of GDP, and FYFSGDA188S Federal Surplus or Deficit as Percent of GDP. Quarterly data was taken from FYGFGDQ188S Federal Debt Held by the Public and HBFRGDQ188S Federal Debt Held by Federal Reserve Banks.

A.9. Symbols used in the text

$_0$	Subscript indicating value at initial time 0
B	Stock of public debt
b	Ratio of B to Y
c	Credit spread
F	Ratio of B to B_0
f	Ratio of b to b_0
g	Growth rate of Y
h	Default hazard rate
I	Annual interest rate
i	Interest rate
q	Primary surplus as share of B or Y
r	Risk-free rate
S	Survival probability
t	Time
Y	GDP
α	Elasticity of h with respect to B
δ	Risk-free adjustment rate $= r - g - q$
ε	Shrinkage rate of b given prudence
η	Fractional loss in debt value after default
κ	Hazard rate when $B = 1$
λ	Switching rate to prudence
θ	Mean time until default
Ω	Time of Debt Apocalypse

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