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The Mixed Model for the Residential Flow Demand of a Small Number of Users

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Abstract

The WDS hydraulic performances strongly depend on the users water demand, indeed any hydraulic simulation is doomed to fail in absence of a reliable modeling of the flow request.
The paper takes into account the Mixed Model (MM) [1], that has been proposed to predict the water demand for a significant number of users (at least 2 hundred). Herein the MM is instead used to model the water demand of a small number of habitants (10-20). This approach is needed to analyze the phenomenon of null water request, and the relative probability of occurrence has been estimated using real residential water demands. Experimental observations confirm that MM is effective for describing the water demand for few residences.

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Keywords: Water demand; probabilistic approach; probability of null water request.

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1. Introduction

Residential water request is a typical random variable, result of the behavior of each user. Therefore, in the technical literature several methodologies consider the water demand subdivided into different parts: frequency, intensity and duration. These approaches are called rectangular pulse, and the most well-known models are the Poisson Rectangular

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Pulse (PRP), [2 and 3] and the Neymann-Scott Rectangular Pulses (NSRP), [4]. The rectangular pulse approaches are usually used to model the residential water demand of the end users.

These approaches are theoretically very interesting, however they present some difficulties in obtaining reliable estimations of the probabilistic distribution parameters [5]. In addition, the rectangular pulse approaches entail two different kind of problems that can limit the actual use of the same models. The first issue depends on the simplification hypothesis [2]. The second one occurs when it needs to aggregate the residential water demand in order to model the request of several end users [6].

The approaches describing directly the water request flow as a continuous positive random variable are less smart than the end users ones, but they are undoubtedly of wider application [7 and 8].

In fact, demand flow models have to describe only one variable related to the water request for a substantial number of users. Indeed, the flow demand is the result of the overlapping of the residential requests, hence the variable to be modeled becomes continuous and positive, although the single component flows (the demands at the hydraulic appliances) should be effectively modeled by discrete variables [5].

For this reason this approach is spreading in the technical literature as shown by numerous works [e.g. 7, 9 and 10].

Nevertheless, few works give a robust and tested information on the probabilistic models and the relative parameters. Moreover, the suggested probabilistic models in technical literature often use solely the gaussian distributions [e.g. 8 and 11], that are not able to represent the null request, event more likely to happen when the number of users decreases.

Buchberger and Nadimpalli [12] proposed to use a cut distribution to model the null request, that implies a spike in the distribution.

Gargano et al. [1], considering the null request a random event different by the value of the water demand flow have suggest a novel probabilistic model (Mixed Model) that is the mix of two distributions. One which describes the null request event by means of a binary random variable, the second one which models the water demand, obviously, when it is not null.

The mixed model has been tested by means of statistical inferences on real data of several and different residential areas, in which the minimum number of users was more than 2 hundred.

This paper aims to further analyze the effectiveness of the Mixed Model with respect to the end users model for a modest number of users, when the frequency of the water demand decreases significantly and the probability of the null request is high.

2. The field laboratory

The study was developed on the basis of the data provided by the field Laboratory -Laboratorio di Ingegneria delle Acque- of the Università degli Studi di Cassino e del Lazio Meridionale.

The field laboratory consists of nine monitored apartments, where 4 residences are provided with a Woltmann volumetric meter (Fig.1-a), conveniently upgraded to carry out this research. In particular, the system is based on optical detection through an infrared emitter diode illuminating the meter quadrant and eight phototransistors. (Fig.1-b). The water consumption is transformed into electric pulses (4-20 mA), sent to an acquisition board every time a water volume of 0.125 l crosses the flowmeter.

The nine residences are located in two buildings, and the total water flow of the monitored users is measured by means of an electromagnetic meter on the link pipe to the water distribution system.

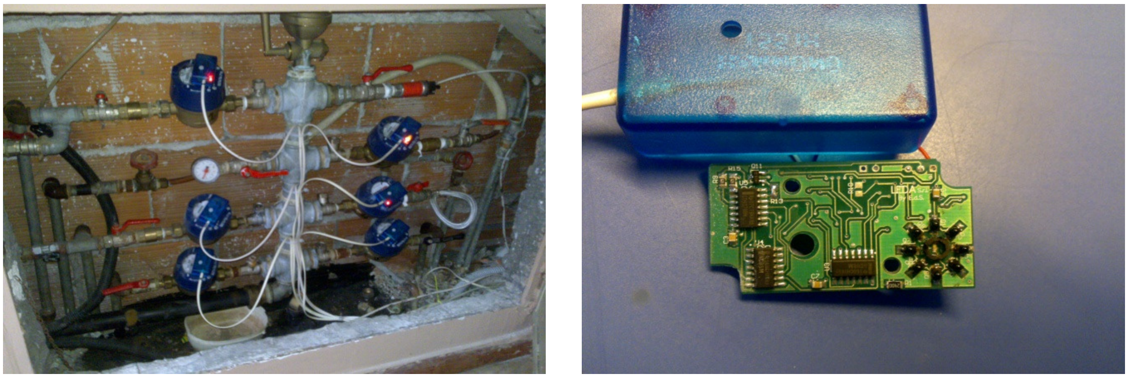
The monitored users represent typical residential indoor water use of families living in apartments of approximately 100 m^2 without garden.

The residential water demand of one building (5 flats for 10 occupiers) and of two buildings (9 flats for 21 occupiers) has been analyzed in the present paper.

The used data -50 days- have been acquired between November the 27th 2012 and February the 5th 2013, Monday through Friday.

A pressure cell installed at the origin of the pipe supplying the building where the monitored residence were located allowed to verify that, during the entire monitoring period, the supplied flows were not influenced by WDS deficit. As a result, the flow measured by the meter can be considered as the actual water demand.

Additional information about the laboratory and the measurements are reported by [13].



(a) (b)

Fig. 1 - a) Equipment employed to monitor Piedimonte San Germano residence; b) Phototransistor board to measure flow rate details.

3. The mixed model

Users life habits in the residential water request is varying in the arch of the day in relation to the needs of any single user. The representation of the water request by an aggregated number of users can be correctly modelled by different probabilistic distributions in function of the entity of the water requested and thus the time of the day. Nevertheless, in the optic of a simplification for WDS modelling implementation of great interest is the possibility of representing the residential water demand by means of a unique probabilistic model, independent from the daily time, the *Mixed Model* [1]. It is based on the combination of two different distributions: a first random discrete variable - which refers to the event of null water demand - and a second continuous one - which is representing the entity of the water demanded.

The random variable of the requested water flow is here represented by means of the dimensionless demand coefficient:

$$C_D(t) = \frac{q(t)}{\mu_q} \tag{1}$$

where for a predefined number of user $q(t)$ is the demanded flow for the daily instant t, and μ_q represents the daily mean water request.

On the basis of the total probability theorem for the random variable C_D, the resulting Cumulative Distribution Function (CDF) of the suggested model is:

$$\mathrm{F}[C_D] = \Pr[C_D = 0] \cdot \Pr[C_D < c_D | C_D = 0] + \Pr[C_D > 0] \cdot \Pr[C_D < c_D | C_D > 0] \tag{2}$$

If F_0 is the probability that the water demand is null ($\mathrm{F}_0 = \Pr[C_D = 0]$), and F^* represents the CDF of the flow demand when it is different from zero ($\mathrm{F}^* = \Pr[C_D < c_D | C_D > 0]$), Eq. (2) can be rewritten as:

$$\mathrm{F}[C_D] = \mathrm{F}_0 + (1 - \mathrm{F}_0)\mathrm{F}^* \tag{3}$$

Eq.(3) has been obtained by observing that the $\Pr[\mathrm{C_D} \leq \mathrm{c_D} | \mathrm{C_D} = 0]$ is the probability of a certain event and the two conditions of flow demand $[\mathrm{C_D} = 0]$ and $[\mathrm{C_D} > 0]$ are mutually exclusive and exhaustive events.

Therefore Eq. (3) requires the definition of the two component distributions F_0 -describing the discrete variable (water demand null or different from zero)- and F^* -CDF of the water demand not null.

Gargano et al. [1] have shown the effectiveness of the Bernoulli distribution for F_o, and of the normal distribution for the CDF F^*.

4. The probability of null water request

The probability that the water demand is null depends strongly by the time, as Fig. 2 shows for 21 users. But it depends also by the time discretization and the user number [1 and 12]. Here only the induced effects of the user number N_{us} on F_o has been investigated, while all the following considerations assume the time discretization step equal to one minute.

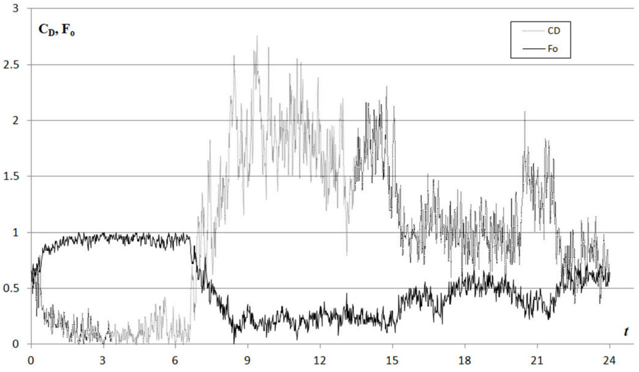
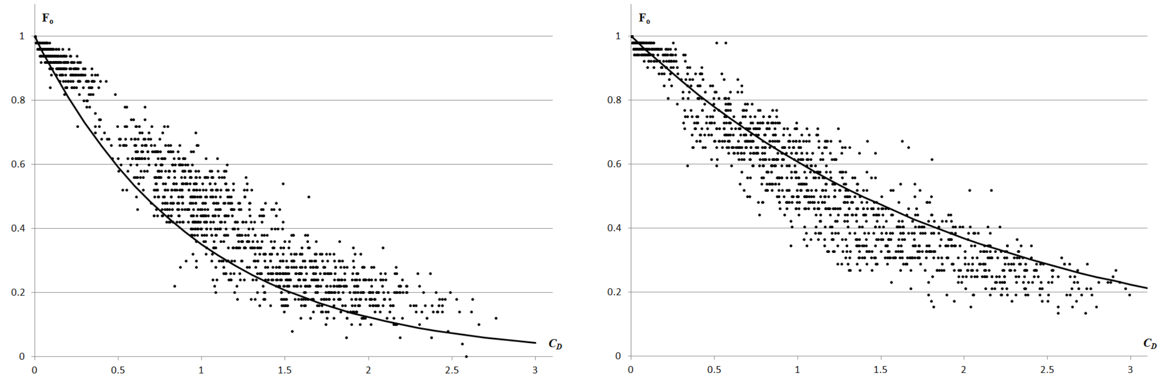


Fig. 2. Daily pattern of the dimensionless request C_D and probability of null request for observed 21 users.

$$F_0 = \exp\left(-5\frac{N_{us}}{1000}\mu_{C_D}\right) \tag{4}$$

where μ_{CD} represents the average value of CD, and it is function of the time as the Fig.2 points out.

The Eq.(4) was obtained by means of an experimental analysis of several data of real users, but it was tested for a number of inhabitants that ranged in a defined interval ($200 < N_{us} < 1200$). Hence, for a small number of users the Eq.(4) is not effective to deduce the F_o.



(a) (b)

Fig. 3. Comparison between Eq.(5) (—) and the real data (●) of 21 (a) and 10 (b) users of Piedimonte San Germano.

For this reason a new relation is proposed in order to estimate the probability of null request for small user number, that presents the same structure of Eq.(4).

$$F_0 = \exp\left(-5\frac{N_{us}}{100}\mu_{C_D}\right) \tag{5}$$

The comparison in Fig. 3 with the experimental data -relative to 21 and 10 users of Piedimonte San Germano- has demonstrated the goodness of Eq.(5) for a small number of users.

It is worth noting that the Eqs. (4) and (5) allow to define F_o whatever the daily trend of the water demand. Indeed, the estimation of the null request probability depends only on the dimensionless demand coefficient μ_{CD}.

Eq.(5) reveals that the probability of occurrence exponentially decreases both with the number of the users, and with the requested flow (Fig. 4).

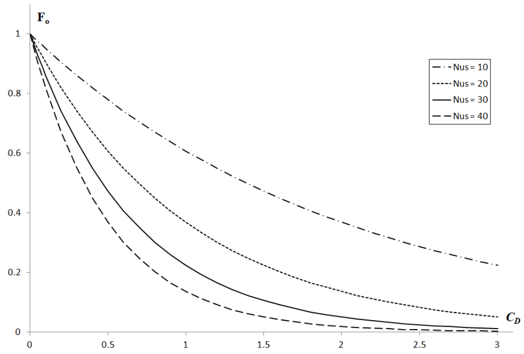


Fig. 4. Trend of F_o versus number of users.

Finally, the effectiveness of Eq.(5) has been tested with the synthetic data generated by means of a calculation program specific for end user water demand. More precisely, the null request for 21 users was generated by means the *Overall Pulse* (OP) model [13].

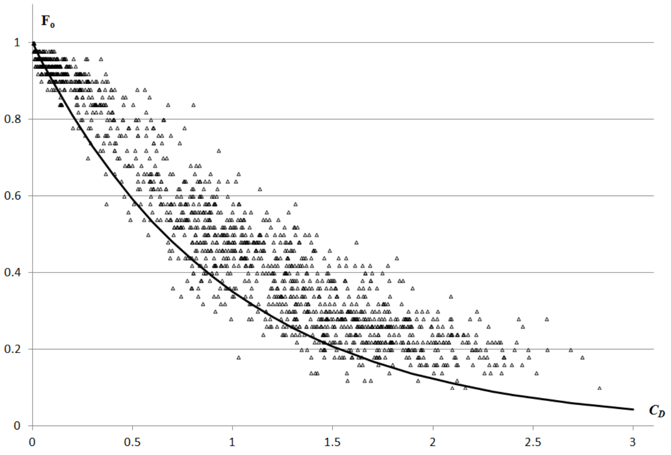


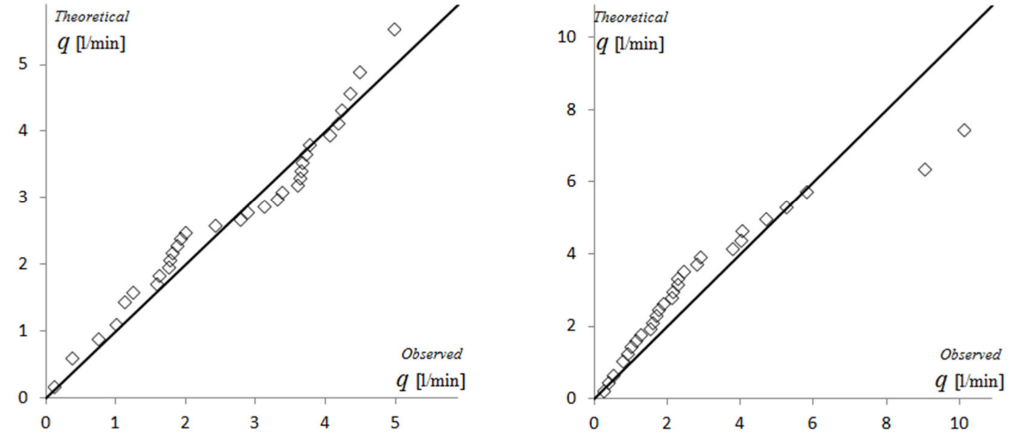
Fig. 5. Eq.(5) (—) and synthetic data (Δ) generated by means of OP model for $N_{us} = 21$.

Fig. 5 shows that Eq.(5) fits well also the generated water demand by means of the OP model.

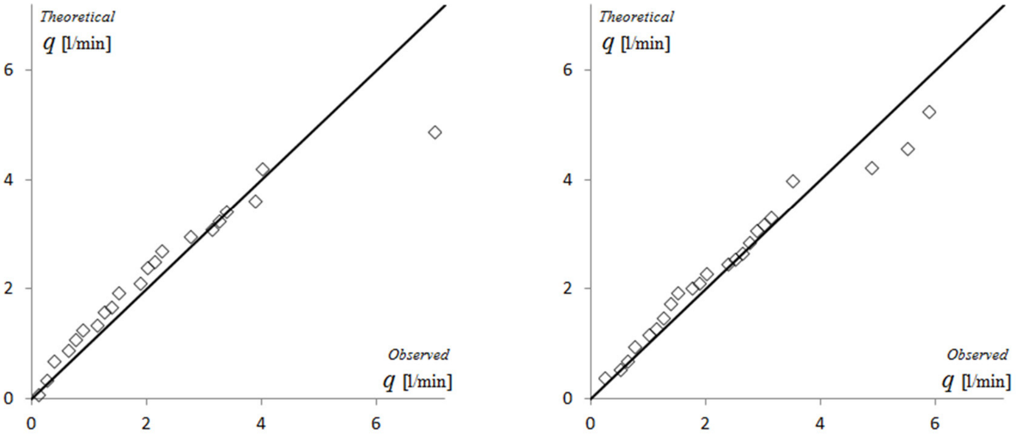
In addition, the comparison of Figures 3(a) and 5 shows that the Eq.(5), originally proposed to describe the probability of null request in the context of the probabilistic MM, is effective also to model F_o, obtained by means a totally different approach. Indeed, the OP model describe the residential water demand considering the single pulse of the request. Therefore, the OP model [13], as a typical end user approach, has to describe frequency, intensity and duration of the pulses.

4. The CDF for the water demand not null

Figures 6 show that the normal distribution, that was demonstrated effective to model F^* for significant user numbers (200< N_{us} <125) [1], fit well the observed cumulative frequencies also for small numbers of inhabitants (Figure 6a and 6b - N_{us} = 21; Figure 6c and 6d - N_{us} = 10).



(a) - *time* = 8.00 (b) - *time* = 13.00



(c) - *time* = 11.00 (d) - *time* = 15.00

Fig. 6 Q-Q plot for Gauss distribution ad flow demand for N_{us} = 21 (a)-(b) and N_{us} = 10 (c)-(d).

More precisely, Figures 6 show the comparison between the theoretical and observed quantiles (Q-Q plot), where the theoretical data have been obtained by means the inverse of Gauss distribution (in Tab.1, the relative parameters).

While, the observed data represent the cumulative frequencies of the data of water demand not null, that have been estimated assuming the plotting position i/(N_{tot}+0,5) - . N_{tot} = total number of data. The captions of Figures 6 point out the different times, during which the observed data of the Q-Q plot were obtained.

5. Conclusion

The ability of the new model -*Mixed Model* (MM)- to predict the residential water demand has been investigated for a small number of users, where the MM describes the water request by means of a random variable of continuous and positive type.

The reduced number of users has implied to study carefully the event of water null request, because the relative probability of occurrence becomes relevant.

Hence a new relation that allows to estimate F_o in function of the demand coefficient has been suggested. The comparison with the real data of residential user shows the effectiveness of the proposed equation.

Finally, the experimental investigations allow to consider the MM a powerful approach also to describe the water request for a small number of users (10-20 users). For these sizes of users, usually in the technical literature the end user models (e.g. PRP, NSRP) are applied.

Table 1. Parameters of the CDFs in Fig.6: average μ and standard deviation σ of flow demand.

An example of a column heading	μ [l/min]	σ [l/min]
N_{us} = 21 (8.00)	2,66	1,32
N_{us} = 21 (13.00)	2,27	2,38
N_{us} = 10 (11.00)	1,62	1,44
N_{us} = 10 (15.00)	2,04	1,43

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