A Risk-Adjusted Analysis of LTC insurance

Di Lorenzo E.[1], Piscopo G.[1], Roviello A.[1], Sibillo M.[2]

[1]Department of Economics and Statistics, Università degli Studi Di Napoli Federico II
[2]Department of Economics and Statistics, Università degli Studi di Salerno

2025 September, 12

AMASES 2025

Introduction

- Homogeneous portfolio of LTC coverages (annual benefit for not self-sufficiency)
- Analysis of the *financial sustainability* considering *systematic risk*

Main references

A. G. Longley-Cook (1998). “Risk-adjusted economic value analysis”. In: *North American Actuarial Journal* 2.1, pp. 87–98

F. W. Lai and M. K. Shad (2017). “Economic Value Added Analysis for Enterprise Risk Management”. In: *Global Business and Management Research: An International Journal* 9.1s

M. Dacorogna (2018). “A change of paradigm for the insurance industry”. In: *Annals of Actuarial Science* 12.2, pp. 211–232

D. F. Babbel and C. Merrill (1998). “Economic valuation models for insurers”. In: *North American actuarial journal* 2.3, pp. 1–15

C. Piney (2003). “Applying utility theory to risk management”. In: *Project Management Journal* 34.3, pp. 26–31

N. Wang (2007). “Optimal investment for an insurer with exponential utility preference”. In: *Insurance: Mathematics and Economics* 40.1, pp. 77–84

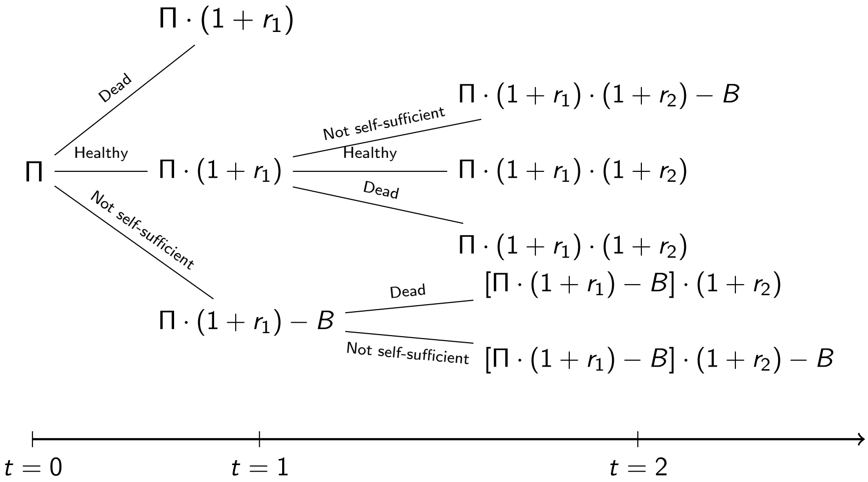
A. Gu, F. G. Viens, and B. Yi (2017). “Optimal reinsurance and investment strategies for insurers with mispricing and model ambiguity”. In: *Insurance: Mathematics and Economics* 72, pp. 235–249

The contract

Model assumptions:

- Homogeneous portfolio of LTC coverages (age, healthy);
- Three-health states (healthy, not self-sufficient, dead);
- No recovery;
- Focus on the benefit period;
- Pre-funded plan with LTC coverage starting at a contractually stipulated age;
- Predetermined and constant benefits.

Individual's financial flows



Portfolio's financial flows

$N(x)$ insureds of age x at $t = 0$

$$\mathcal{F}_t = N(x) \cdot \Pi \cdot \prod_{\tau=1}^{t-1}(1 + r_\tau) - \sum_{d=1}^{N(x)} \left[\sum_{\tau=1}^{t-1} B \cdot X_{\tau,d} \cdot \prod_{t'=\tau}^{t-1}(1 + r_{t'}) + B \cdot X_{t,d} \right]$$

$$X_{\tau,d} = \begin{cases} 1 & \text{if } \Omega_{\tau,d} \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

$$\Omega_{\tau,d} := \{ s_{\tau,d} = 2 | s_{\tau-1,d} = 1 \lor s_{\tau-1,d} = 2 \}$$

* $s_{t,d} = 1$ means healthy and $s_{t,d} = 2$ means not self-sufficient

Systematic risk

$S = \{S_1, S_2, \cdots, S_j, \cdots, S_K\}$ finite set of random scenarios

$p^{(j)}(\Omega_{\tau,d}) = p(\Omega_{\tau,d}|S_j)$ probability given the scenario S_j for $j = 1, \cdots, K$.

What if the insurer make an incorrect choice
to evaluate future outcomes and how to manage ambiguity??

Systematic risk

$S = \{S_1, S_2, \cdots, S_j, \cdots, S_K\}$ finite set of random scenarios

$p^{(j)}(\Omega_{\tau,d}) = p(\Omega_{\tau,d}|S_j)$ probability given the scenario S_j for $j = 1, \cdots, K$.

What if the insurer make an incorrect choice
to evaluate future outcomes and how to manage ambiguity??

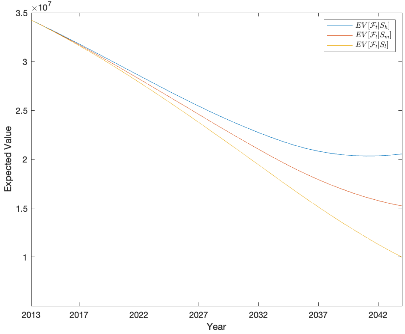
- $E(\mathcal{F}_t|S_j)$ conditional expected periodic profit
- $\sigma_{\alpha,S}(t) = \text{Var}[(E(\mathcal{F}_t|S_j))_{j=1}^{K}, \alpha]$ weighted variance of expected periodic profit with $\alpha = (\alpha_1, \cdots, \alpha_K)$ with $\alpha_j \in [0,1]$ and $\sum_j \alpha_j = 1$ degree of confidence

Assumptions

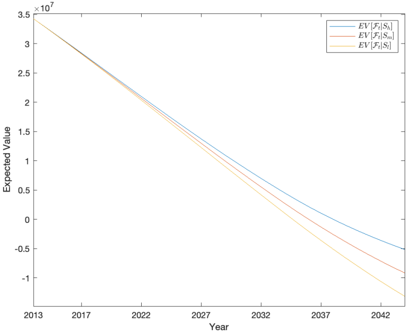
- Three possible scenarios: $S = \{S_h, S_m, S_l\}$;
- Starting age: $x = 70$;
- Time interval: $[2013, 2044]$;
- Annual benefit payout: $B = 24000€$;
- Single premium: $\Pi = 34250€$ according to S_m;
- Technical discount rate: $r' = 2,5\%$;
- $N(x) = 100$;

L. Di Falco and P. De Angelis (2016). *Assicurazioni sulla salute: caratteristiche, modelli attuariali e basi tecniche*. il Mulino
ANIA (2019). *Invalidità e inabilità di tipo previdenziale: evidenze osservate, modelli attuariali e base demografica*. Tech. rep. Studi demografici, http://www.ania.it/it/servizi/studi-e-rapporti\bibrangedashdemografici.html

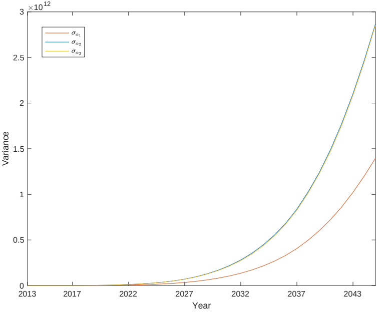
Example



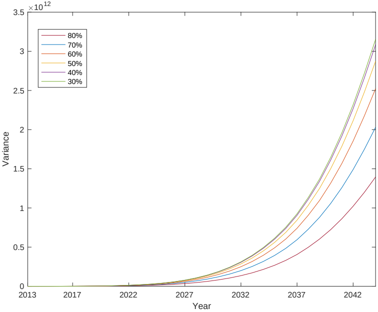
Conditional expected periodic profit with $r = r'$



Conditional expected periodic profit with $r < r'$



Weighted variance with $\alpha_1 = (0.1, 0.8, 0.1)$, $\alpha_2 = (0.8, 0.1, 0.1)$, $\alpha_3 = (0.1, 0.1, 0.8)$



Scenario volatility for different degrees of confidence on S_m

Risk-adjusted expected utility

- Yearly gain/loss function:

$$Y_{t,k} := E[\mathcal{F}_{t-1}|S_k] \cdot (1 + r_{t-1}) - \sum_{d=1}^{N(x)} B \cdot X_{t,d}$$

- CRRA utility function:

$$U(Y_{t,k}) = \frac{Y_{t,k}^{1-\gamma}}{1-\gamma}$$

- Expected utility of gain/loss: $\mathcal{U}_{t,k} = E_{p_k}[U(Y_{t,k})]$

Risk-adjusted expected utility:

$$\mathcal{U}_{h_t,k} = h_t \cdot \mathcal{U}_{t,k}$$

where $h_t \in [0,1]$ is the risk index

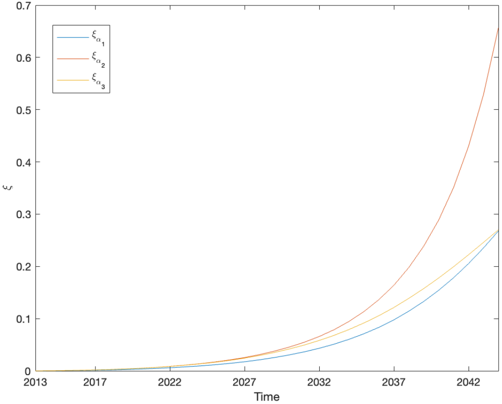
$$h_t = h(\xi_\alpha(t), \nu_t) = e^{-\xi_\alpha(t)(1-\nu_t)}$$

where

$$\xi_\alpha(t) = \frac{\sqrt{\sigma_{w,S}(t)}}{\mu_w(t)} \qquad \text{with} \qquad \mu_w(t) = \sum_{k=1}^{K} E(\mathcal{F}_t|S_k) \cdot w_k$$

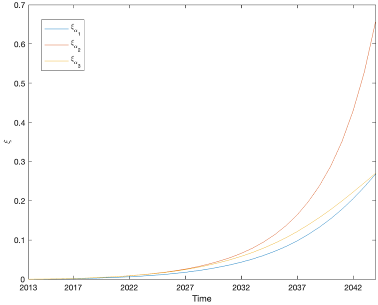
and ν_t is a time-dependent risk score associated by the insurer.

Example



Variation index for $\alpha_1 = (0.1, 0.8, 0.1)$, $\alpha_2 = (0.8, 0.1, 0.1)$, $\alpha_3 = (0.1, 0.1, 0.8)$

Example

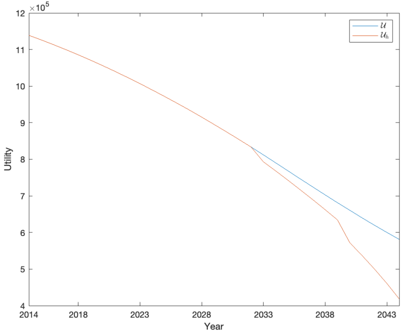


Variation index for $\alpha_1 = (0.1, 0.8, 0.1)$, $\alpha_2 = (0.8, 0.1, 0.1)$, $\alpha_3 = (0.1, 0.1, 0.8)$

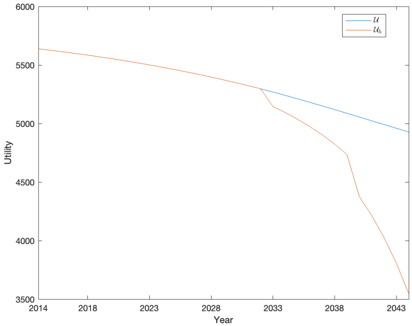
- $\nu_t = 1$ for $t \in [2013 - 2033]$;
- $\nu_t = 0.7$ for $t \in [2034 - 2040]$;
- $\nu_t = 0.5$ for $t \in [2041 - 2044]$.

Example

$$\mathcal{U}_{h_t,k} = h_t \cdot \mathcal{U}_{t,k} = h_t \cdot E_{p_k}[U(Y_{t,k})] \text{ risk-adjusted expected utility}$$



Utility and risk-adjusted utility for $\gamma = 0.5$



Utility and risk-adjusted utility for $\gamma = 0.9$

Further References

J. B. Heaton (2002). "Managerial Optimism and Corporate Finance". In: *Financial Management* 31.2, pp. 33–45

T. Pfeiffer (2004). "Net Present Value-Consistent Investment Criteria Based on Accruals: A Generalisation of the Residual Income-Identity". In: *Managerial and Decision Economics* 25.5, pp. 261–275

M. Acharyya (2008). "In measuring the benefits of enterprise risk management in insurance: An integration of economic value added and balanced score card approaches". In: *ERM Monograph*, pp. 1–25

T. Davidoff (2010). "Home equity commitment and long-term care insurance demand". In: *Journal of Public Economics* 94.1-2, pp. 44–49

M. Levy and A. R. Nir (2012). "The utility of health and wealth". In: *Journal of Health Economics* 31.2, pp. 379–392

A. Aslan and P. N. Posch (2022). "How do investors value sustainability? A utility-based preference optimization". In: *Sustainability* 14.23

P. Bailo, G. Pesel, F. Gibelli, A. Sirignano, and G. Ricci (2024). "Long-term care insurance in Italy: medico-legal and socio-economic profiles". In: *Frontiers in Public Health* 12

Thank you!